A Missing Piece of Mutual Learning Model of March (1991)

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Abstract: The mutual learning model described in “Exploration and Exploitation in Organizational Learning” (March, 1991) concludes that “slow learning on the part of individuals maintains diversity longer, thereby providing the exploration,” based on the results of computer simulations. However, the simulations of March (1991) excluded both ends of the socialization rate domain. When compensating for those missing portions, there is an optimal socialization rate that actually maximizes the average knowledge level. This is because low learning on the part of individuals actually causes frequent lock-ins and impedes learning. This optimal socialization rate may be a common rate for socialization, and we cannot deny this possibility by using only computer simulations. Moreover, this high knowledge level is achieved in a non-equilibrium state.

Keywords: exploration and exploitation, organizational learning, mutual learning model, computer simulation, socialization rate

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Introduction

When multiple learners (including the organization) learning simultaneously affect each other, March referred to the learning process as the ecology of learning (Takase, 1991, p. 60). The ecology model of learning of March featured a selection process of organizational routines under the influence of other learners. A learner acquires other learners’ experiences codified as technology, codes, procedures, and routines (Levitt & March, 1988). Furthermore, the diffusion (Rogers, 1962) of learners’ experiences and routines in the organization makes this model even more complex.

Mathematical analysis is typically abandoned and replaced by computer simulation analysis due to the complexity of the model. (e.g., Levinthal & March, 1981; Lounamaa & March, 1987). Throughout the 1980s, March was the primary proponent of this type of research (Huber, 1991). Levinthal and March (1993) called the phenomenon of preferring exploitation to exploration a “myopia of learning” (Sato, 2012). March (1991) entitled “Exploration and Exploitation in Organizational Learning” primarily developed and analyzed two computer simulation models: (A) a mutual learning model and (B) a competitive ecology model.

However, Takahashi (1998) mathematically analyzes model (B) and does not require a computer simulation. In model (B), a reference organization having the normal performance distribution with mean $m$ and variance $v^2$ competes with $N$ organizations having a standard normal performance distribution with mean 0 and variance 1. Let $u$ be a value with an upper probability of $1/(N + 1)$ in the standard

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1 Computer simulations are not only used in organizational learning but also have various uses in the garbage can model (Cohen, March, & Olsen, 1972; Inamizu, 2012; Takahashi, 1997), the communication competition model (Kuwashima, Takahashi, & Tamada, 2005; Takahashi, Kuwashima, & Tamada, 2006), or theoretical shaping of learning curves (Takahashi, 2013).
normal distribution table \( u \approx 0.44 \) for \( N = 2 \), \( u \approx 1.34 \) for \( N = 10 \), \( u \approx 2.33 \) for \( N = 100 \), Takahashi (1998) obtains the following equation:

\[
m = u(1 - \nu) \quad (1)
\]

Let \( P^* \) be the probability of the reference organization having the best performance within the group. March (1991) constructs the “competitive equality lines” by conducting 5,000 simulations for each value \( \nu^2 \) from 0 to 2 in steps of 0.05 and plotting points \((m, \nu^2)\) where \( P^* = 1/(n + 1) \).

The three lines of \( N = 2 \), \( N = 10 \), and \( N = 100 \) illustrated in Figure 6 in March (1991, p. 82) cross the vertical axis of mean \( m \) at approximately 0.2, 0.8, and 1.7, respectively. However, using Equation (1) the mean must be, respectively, 0.44, 1.34, and 2.33

![Competitive equality lines](image)

**Figure 1.** Competitive equality lines

*Source: Takahashi (1998, Figure 6)*
when variance is 0. More specifically, Figure 1 illustrates the parabolas that touch the vertical axis at these points. Takahashi (1998) is suspicious regarding the validity of March’s (1991) simulation program at least at the end point 0 of domain of variance.

As March’s (1991) programs have not been published, this paper creates a computer simulation program for model (A) (Appendix A) in free software language R for statistical computing, and checks the validity of the program at end points of the domain wherein we can analyze mathematically without simulation. We then compensate for the missing piece of March’s (1991) mutual learning model (A) to identify the true conclusion.

**Mutual Learning Model**

March’s (1991) model is vaguely formulated because of less use of mathematical notation. This paper fully uses Takahashi’s (1998) notation and reformulates March’s (1991) model mathematically.

**Reality**

There is an external reality, which is called the “state of nature” in statistical decision theory. A reality is an $m$-dimensional vector, with each component having a value of 1 or $-1$.

$$
\mathbf{r} = (r_1, r_2, \ldots, r_m)
$$

*Assumption 1*: Each component $r_i$ of a reality is given an initial value of 1 or $-1$ with a probability of 1/2.

**The beliefs of $n$ members and an organizational code**

Each of $n$ members has a belief for each component of the reality at each time period. This belief is not a probability, whereas the belief of statistical decision theory represents a probability. For each
component in an \( m \)-dimensional vector of the reality, each belief has a value of 1, 0, or \(-1\). For member \( j \), an \( m \)-dimensional vector of belief can be expressed as

\[
b_j = (b_{j1}, b_{j2}, \ldots, b_{jm})
\]

Similarly, the belief of the organizational code is expressed as

\[
c = (c_1, c_2, \ldots, c_m)
\]

with each component having a value of 1, 0, or \(-1\).

**Assumption 2:** Each component of an \( m \)-dimensional vector \( b_j \) that expresses the belief of each organizational member is given an initial value of 1, 0, or \(-1\), each having an equal probability of \( 1/3 \).

**Assumption 3:** The initial value of each component of an \( m \)-dimensional vector \( c \) that expresses the belief of the organizational code is 0.

**Learning from the superior group**

For each component of a belief, we examine whether it matches the component of the known reality. Thereafter, we define knowledge level \( L \) as the number of matching components divided by \( m \). The group of individuals whose knowledge level is higher than that of the organizational code is called a “superior group.”

**Assumption 4:** 1) If the superior group is empty or there is no majority opinion regarding the \( i \)th component of the superior group, the \( i \)th component \( c_i \) of an organizational code’s belief does not change. 2) If the majority opinion regarding the \( i \)th component of the superior group is the same as the \( i \)th component \( c_i \) of an organizational code’s belief, \( c_i \) does not change. 3) If the majority opinion regarding the \( i \)th component of the superior group differs
from the $i$th component $c_i$ of an organizational code’s belief, $c_i$ changes with probability $q = 1 - (1 - p_2)^{k_i}$ in accordance with the majority opinion, where $k_i$ ($k_i > 0$) is the number of members whose $i$th component of belief differs from $i$th component of organizational code’s belief minus the number of members who do not (within the superior group). Changes in components are independent of each other.

Probability $p_2$ is expressed as the learning rate of the organizational code.

**Learning from the organizational code**

As a result of socialization, it is possible for the value of each component of a member’s belief to change to either 1, 0, or $-1$ due to influence from the organizational code (for example, changing from 1 to $-1$, or 0 to 1).

**Assumption 5:** 1) If the $i$th component of the organizational code $c_i = 0$, the $i$th component $b_{ji}$ of organizational member $j$’s belief continues to have the same value. 2) If $c_i = b_{ji}$, $b_{ji}$ continues to have the same value. 3) If $c_i \neq b_{ji}$, $b_{ji}$ takes on the same value of $c_i$ with probability $p_1$.

Probability $p_1$ is called the socialization rate, and is common to all members.

In our simulation program in R, the number $m$ of dimensions of reality is set at 30, the number $n$ of members is set at 50, and simulations were conducted in 80 time periods as same setting as March’s (1991) original simulation. However, we set seeds of the random number generator to 501, 502, ..., 600 for a total of 100 different random number settings, and then we average the results of 100 simulations.
Equilibrium and Lock-in

March (1991) defines the state of sharing\(^2\) same belief among all organizational members and the organizational code as an equilibrium, and thus states “this equilibrium is stable” (March, 1991, p. 75). As a backdrop to this definition, March assumed that “the beliefs of individuals and the code converge over time” (March, 1991, p. 75). Based on the definition of an equilibrium, in such a state the beliefs of organizational members and an organizational code are identical, and the equilibrium is certainly stable. On the other hand, although the state may be stable, it is not an equilibrium; rather, this is called “lock-in.”

Lock-in at the learning rate \(p_2 = 0\) of the organizational code

A typical case of lock-in occurs when an organizational code’s learning rate \(p_2 = 0\), and then when \(q = 0\). In this case, the organizational code is unchanged according to Assumption 4. From Assumption 3, the initial value of each component of an organizational code is 0, and thus it is unchanged throughout. In actuality, from Assumption 5 (1), if each component of an organizational code is 0, then the beliefs of organizational members will not be affected by an organizational code at all. In other words, the beliefs of organizational members are unchanged from the initial state. Indeed, this is not an equilibrium, though it is a lock-in from the outset.

In this case, reality component \(r_i\) and belief component \(b_{ji}\) of member \(j\) are the same when the initial values \((r_i, b_{ji}) = (1, 1)\) or \((-1, -1)\). Thus, from Assumptions 1 and 2, the probability of reality and

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\(^2\) However, some vagueness remains; according to March (1991, p. 75), these do not necessarily have to be entirely the same. As can be seen later in this paper, in an 80 period simulation, an equilibrium is often not attained, and March’s statement may be in reference to that fact.
belief being the same is as follows: $(1/2)(1/3) + (1/2)(1/3) = 1/3$. In other words, if learning rate $p_2 = 0$, then the average knowledge level of members should be $1/3$ regardless of socialization rate $p_1$. On the other hand, for the organizational code, each component remains unchanged from the initial value 0 and thus the knowledge level remains unchanged from 0.

In the same setting as March (1991), reality dimensions $m = 30$, organizational members $n = 50$, and 80 periods, we conduct 100 simulations for each socialization rate $p_1$ in steps of 0.1, and the average knowledge level of 100 simulations are indicated in Table 1. As with the theoretical value, regardless of the value of the socialization rate $p_1$, the average knowledge level is 0.334, or approximately $1/3$. Indeed, lock-in occurs from the outset, and this average knowledge level is a reflection of the probability for initial values of $(r_i, b_{ji}) = (1, 1)$ or $(-1, -1)$ being $1/3$.

**Lock-in at the socialization rate $p_1 = 0$**

Another typical case of lock-in occurs when the socialization rate $p_1$ is 0. In this case, the belief of organizational members is not affected by the organizational code from Assumption 5. Accordingly, from Assumption 2, each component of each organizational member’s beliefs is unchanged, with an initial value of 1, 0, or $-1$, each with an equal probability of $1/3$. The average knowledge level in this case is $1/3$.

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**Table 1. Average knowledge levels at the learning rate $p_2 = 0$**

<table>
<thead>
<tr>
<th>$p_2$</th>
<th>Socialization rate $p_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.334 0.334 0.334 0.334 0.334 0.334 0.334 0.334 0.334 0.334</td>
</tr>
</tbody>
</table>

*Note: $m = 30$, $n = 50$, 80 periods, 100 simulations*
On the other hand, from Assumption 3, the initial value for each component of an organizational code’s belief is 0 and the initial knowledge level is 0; however, from Assumption 4, when the $i$th component of the majority opinion of the superior group is either 1 or $-1$, the $i$th component $c_i$ of an organizational code’s belief changes with probability $q = 1 - (1 - p_2)^{k_i}$ in accordance with the majority opinion. This change continues until the $i$th component of the organizational code’s belief matches with the (unchanging) $i$th component of the majority opinion of the superior group’s belief. However, even upon stabilization, the belief of the organizational code will certainly not be the same as the minority opinion of the superior group. In any case, this is not an equilibrium but a lock-in.

In the same setting as March (1991), reality dimensions $m = 30$, organizational members $n = 50$, and 80 periods, we conduct 100 simulations for each learning rate $p_2$ in steps of 0.1, and the average knowledge level of 100 simulations are indicated in Table 2. As with the theoretical value, regardless of the value of the learning rate $p_2$, the average knowledge level is 0.334, or approximately $1/3$.

### Table 2. Average knowledge levels at the learning rate $p_1 = 0$

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>Learning rate $p_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.334</td>
</tr>
<tr>
<td>0.1</td>
<td>0.334</td>
</tr>
<tr>
<td>0.2</td>
<td>0.334</td>
</tr>
<tr>
<td>0.3</td>
<td>0.334</td>
</tr>
<tr>
<td>0.4</td>
<td>0.334</td>
</tr>
<tr>
<td>0.5</td>
<td>0.334</td>
</tr>
<tr>
<td>0.6</td>
<td>0.334</td>
</tr>
<tr>
<td>0.7</td>
<td>0.334</td>
</tr>
<tr>
<td>0.8</td>
<td>0.334</td>
</tr>
<tr>
<td>0.9</td>
<td>0.334</td>
</tr>
<tr>
<td>1.0</td>
<td>0.334</td>
</tr>
</tbody>
</table>

*Note: $m = 30$, $n = 50$, 80 periods, 100 simulations*  

Follow-up Simulations and True Conclusion

Based on the above considerations, our program created in R reproduced our theoretical predictions, we therefore conduct simulations in the same setting as March (1991): reality dimensions
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\[ m = 30, \quad n = 50, \quad 80 \text{ periods, and for each socialization rate } p_1 \text{ from 0.1 to 0.9 in steps of 0.1.} \]

However, unlike March (1991), the simulation was conducted 100 times for each setting. These are follow-up simulations to find whether we could reproduce the results illustrated in Figure 1 of March (1991); the average knowledge level of 100 simulations are illustrated in Figure 2.\(^3\)

As with Figure 2, Figure 1 of March (1991) appears to be a graph of a monotonically decreasing function, causing March to conclude that “slower socialization (lower \( p_1 \)) leads to greater knowledge at

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\(^3\) Figure 1 of March (1991) has an overall average knowledge level that is higher than Figure 2 by approximately 0.1, with greater variance and no convergence of average knowledge level as the socialization rate \( p_1 \) approaches 1. Takahashi (1998) proposes modifications to more closely resemble Figure 1 of March (1991); see Appendix B for more detail.
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equilibrium than does faster socialization, particularly when the code learns rapidly (high $p_2$)” (March, 1991, p. 75).

Although a closer examination indicates that the socialization rate is a probability in definition, and should take on a value of between 0 and 1, Figure 1 of March (1991) is limited to a range of 0.1 to 0.9. Thus, either end of the domain, particularly the graph with a range of 0 to 0.1, which has particularly large changes, is missing. When we compensate for that missing graph, we arrive at an entirely different conclusion. In fact, we know the theoretical results when socialization rate $p_1$ is 0, we experience lock-in, and the theoretical knowledge level is 1/3, as has already been demonstrated. In the results of our simulations indicated in Table 2, the average knowledge level, 0.334, is right about at the theoretical value. In other words, there should theoretically be no monotonically decreasing function.

Thus, we use the same settings of Figure 2: reality dimensions $m = 30$, organizational members $n = 50$, and 80 periods. However, this time we conduct 100 simulations for each socialization rate $p_1$ from 0 to 0.1 in steps of 0.01. When we tie this to Figure 2, we obtain Figure 3 (we have also compensated for $p_1 = 1$). As is clearly illustrated in Figure 3, there is a peak where socialization rate $p_1$ is between 0 and 0.1. When the learning rate $p_2$ is 0.1, the highest average knowledge level is 0.756 at $p_1 = 0.06$. When the learning rate $p_2$ is 0.5, the highest average knowledge level is 0.868 at $p_1 = 0.07$. When learning rate $p_2$ is 0.9, the highest average knowledge level is 0.901 at $p_1 = 0.07$.

In other words, the statement that “slower socialization (lower $p_1$) leads to greater knowledge at equilibrium than does faster socialization” (March, 1991, p. 75) is clearly a mistake, as can be seen from the shape of the graph. The conclusion that “slow learning on the part of individuals maintains diversity longer, thereby providing the exploration that allows the knowledge found in the
organizational code to improve” (March, 1991, p. 76) cannot be arrived at using this model. Actually, there is an optimal socialization rate that maximizes the average knowledge level. This is because low learning on the part of individuals causes lock-ins frequently, and impedes learning. The optimal socialization rate of 0.06 to 0.07 may be a common rate for socialization, and we cannot deny this possibility by using only computer simulation method.

Figure 3. Average knowledge levels of March’s (1991) model with a wide range of 0 to 1

Note: $m = 30$, $n = 50$, 80 periods, 100 simulations
Moreover, maximization of the knowledge level does not require an equilibrium. Figure 4 illustrates the equilibrium rate, which is the number of equilibriums at the end of the 80th period divided by 100. As we have already stated, when socialization rate $p_1$ is 0, all simulations are lock-in’s and the equilibrium rate should be 0. Even when the socialization rate $p_1$ is 0.1, the equilibrium rate is low, at between 0.20 and 0.31. In other words, when the average knowledge level is highest in Figure 3, the equilibrium rate is only around 20%.

The vertical axis of Figure 1 in March (1991) was intentionally labeled “average equilibrium knowledge.” However, a high knowledge level was actually obtained without equilibria. In actuality, the fact that an equilibrium is not a desirable state is one important finding from our simulations, and an important conclusion of Takahashi et al. (2006) who conducted simulations using a communication competition model.
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References


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Appendix A: R Program

(supplementary materials, http://dx.doi.org/10.7880/abas.14.5)
Appendix B: Revised Model by Takahashi (1998)

Takahashi (1998) attempted revisions to the model used by March (1991) to reproduce March’s Figure 1. Takahashi (1998) corrected the majority opinion of the superior group in Assumption 4 to the majority opinion of the superior group “except for 0” by removing 0 from the belief components to avoid lock-in as much as possible from the outset. In addition, he proposed that Assumption 4 (3), “$k_i (k_i > 0)$ is the number of members whose $i$th component of belief differs from $i$th component of organizational code’s belief minus the number of members who do not (within the superior group)” be modified to “$k_i$ is the number of the majority minus the number of the minority (within the superior group)”. When conducting simulations using this modification as in Figure 2, the result is Figure 5, and the shape is quite similar to Figure 1 of March (1991).

![Graph showing average knowledge levels of Takahashi's revised model](image)

**Figure A1.** Average knowledge levels of Takahashi’s (1998) revised model

*Note: $m = 30$, $n = 50$, 80 periods, 100 simulations*