Theoretical Analysis of Sound Pressure Distributions inside a Tire Cavity

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Abstract: This paper presents a one-dimensional wave equation that considers the circulating wave and air damping for the sound pressure distributions inside a vehicle tire. The theoretically derived sound pressure distributions inside a tire are compared with the experimental values measured by using a multi-microphone system around the cavity resonance frequency. The theoretical and experimental values show good agreement around the natural frequency of tire cavity resonance.

Keywords: Tire Cavity Resonance, Air Damping, Circulating Wave, Sound Pressure Level

1. Introduction

Vehicle noise is caused by various noise sources. Among noise sources, engine noise, exhaust noise, and tire noise are important. Recently, tire noise has become noticeable because engine and exhaust noises have been reduced over the last 40 years [1].

Vehicle noises are classified into exterior and interior noises. Exterior noise is noise that propagates to the exterior of a car, resulting in noise pollution. Interior noise is noise that transmits to the cabin of a car, which affects passenger comfort in the cabin.

Tire noises are generated by two mechanisms: the vibration of exterior air and the vibration of vehicle structures. Air pumping into and out of the tread cavities causes noise by air vibration, and this noise contributes mainly to exterior noise [2]. Exterior noise is well understood and restricted by the sound pressure level (SPL) limits described in UN-ECE R117-01 and UN-ECE R117-02 [3]. In contrast, interior noise is mainly produced by structure vibrations, and it is not well understood owing to the difficulties faced in identifying its transfer paths.

In particular, tire cavity resonance is one of the main causes of interior noise in the low-frequency domain. Tire cavity resonance is basically the air column resonance inside a tire. Sakata et al. [4] measured SPL in a vehicle and showed that a noticeable peak exists at the cavity resonance frequency. They also showed the SPL distributions inside a tire through finite element method (FEM) analysis, and showed that the direction of the SPL distributions inside a tire affects interior noise [4]. Tanaka et al. used eight microphones to measure the SPL distributions inside a deformed tire [5]. They also showed the cavity resonance frequencies inside a deformed tire [5].

Some theoretical approaches have applied a one-dimensional wave equation to the sound inside a tire. Thompson [6] approximated the tire cavity as a straight acoustic tube and calculated the theoretical value of the cavity resonance frequency of a tire. Yamauchi et al. [7] approximated the radius and cross-sectional area as a Fourier series and used this to calculate a theoretical value of the cavity resonance frequency of a tire. These studies focused on the cavity resonance frequency only. However, it is important to estimate the direction and amplitude of the sound pressure inside a tire at other frequencies as well because the direction and amplitude of spindle vibration affect passenger comfort.

From these previous studies, the sound pressure distributions are derived by FEM analysis [4], and the cavity resonance frequencies inside a tire are predicted by a 1D wave equation [6, 7]. However, it is necessary to solve complicated equations in the FEM analysis, and the conventional analysis by the 1D wave equation cannot predict the sound pressure distributions except at the cavity resonance frequency.

In this study, as the initial stage in developing of a theory of sound pressure distributions inside a deformed tire, we theoretically determine the sound pressure distribution inside an undeformed tire by solving a 1D wave equation that considers the circulating wave and air damping to express the sound pressure distributions inside a tire without using FEM analysis. We also compare the theoretically obtained sound pressure distribution with the experimental result, and we clarify that the theory well expresses the sound pressure inside a tire around the tire cavity resonance frequency.

2. Theory

Figure 1 shows the tire that is studied in this paper. Lc (circumferential length of tire) is 1.51 [m], Lw (width of tire) is 0.195 [m], and La (height of tire cross-section) is 0.119 [m]. A (cross-sectional area) and S (perimeter of tire cross-section) are shown in Table 1.

![Fig. 1 Tire studied in this paper.](image)

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References


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The acoustic cavity resonance inside a tire is modeled as a finite length tube, as shown in Fig. 2, because $L_c$ is sufficiently greater than $L_e$ and $L_o$. This model has a constant cross-sectional area with continuity at each end.

![Fig. 2 Tire model for a 1D wave equation.](image)

The sound inside a tire is treated as a plane wave when the wavelength is much greater than the cross-sectional length. In this case, the continuity equation and motion equation are written as Eqs. (1) and (2), respectively.

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial x} = 0$$  \hspace{1cm} (1)

$$\rho \frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} = 0$$  \hspace{1cm} (2)

Here, $t$ denotes the time [s]; $\rho$, the air density [kg/m$^3$]; $v$, the sound particle velocity [m/s]; $\gamma$, the specific heat ratio of the air; and $x$, the distance along the tube [m]. From Eqs. (1) and (2), the relation between the air density and the sound pressure is written as follows:

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}$$  \hspace{1cm} (3)

The wave propagation in air is regarded as an adiabatic change:

$$\frac{p}{\rho^\gamma} = \text{const.}$$  \hspace{1cm} (4)

where $\gamma$ is the specific heat ratio of the air. From Eqs. (3) and (4), the 1D wave equation is rewritten as follows:

$$\frac{\partial^2 p}{\partial t^2} - \frac{\gamma \rho}{\rho} \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial x^2} - c^2 \frac{\partial^2 p}{\partial x^2} = 0$$  \hspace{1cm} (5)

where $c$ is the sound velocity [m/s]. In this study, we examine the case in which the vehicle velocity is sufficiently less than the sound velocity. If the vehicle velocity is 60 km/h (=16.7 m/s), it is around 5% of the sound velocity at 20 °C. We consider that the tire rotation has no influence if the vehicle velocity is less than this velocity.

The pressure of the clockwise and counterclockwise moving waves is expressed by Eqs. (6) and (7), respectively.

$$p_{cw}(x,t) = p_0 \exp[j(\omega t - kx)]$$  \hspace{1cm} (6)

$$p_{ccw}(x,t) = p_0 \exp[j(\omega t + kx)]$$  \hspace{1cm} (7)

Here, $j$ is the imaginary unit ($j = \sqrt{-1}$); $p_0$, the pressure amplitude; $k$, the wavenumber; and $\omega = 2\pi f$, the angular frequency of sound. The resultant pressure at arbitrary points in the model can be expressed as the superposition of the clockwise and counterclockwise moving waves.

$$p(x,t) = p_{cw}(x,t) + p_{ccw}(x - L_c t)$$  \hspace{1cm} (8)

$$= p_0 \exp(j\omega t)[\exp(-jkx) + \exp(jk(x - L_c))]$$

(0 ≤ $x ≤ L_c$)

Here, $L_c$ is the circumferential length of a tire [m]. The natural frequency of a tire cavity is derived as follows:

$$f_i = \frac{c}{2L_c} i (i = 1, 2, ...).$$  \hspace{1cm} (9)

However, a tire is a ringed acoustic tube, and so it is necessary to consider the influence of the circulating wave. When we consider the circulating wave, we can express the sound field in the tire model as shown in Fig. 3.

![Fig. 3 Tire model including circulating wave.](image)

When the sound pressure is superposed at arbitrary positions, it is possible to express the pressure of the clockwise and counterclockwise moving waves including consideration of the circulating wave as follows.

$$p_{cw}(x,t) = p_0 \exp(j\omega t) \sum_{i=0}^{n} [\exp(-jk(x + iL_o))]$$  \hspace{1cm} (10)

(0 ≤ $x ≤ L_c$)

$$p_{ccw}(x,t) = p_0 \exp(j\omega t) \sum_{i=0}^{n} [\exp(jk(x - iL_c))]$$  \hspace{1cm} (11)

(-$L_c$ ≤ $x$ ≤ 0)

We superpose the clockwise and counterclockwise moving waves at arbitrary positions inside a tire, from which the sound pressure distribution inside a tire is written as follows.

$$p(x,t) = p_{cw}(x,t) + p_{ccw}(x - L_o t)$$  \hspace{1cm} (12)

$$= p_0 \exp(j\omega t) \times \sum_{i=0}^{n} [\exp(-jk(x + iL_o)) + \exp(jk(x - i + 1)L_c))]$$

(0 ≤ $x$ ≤ $L_c$)
We introduce a distance attenuation to model air damping. In the description of the wave motion in a viscous medium, it is convenient to introduce a complex quantity for the equivalent density. The complex density is written as follows \[ \tilde{\rho} = \rho - j\rho' = \rho - j\frac{AR}{\omega} \] \[ R = \frac{S}{A^2\sqrt{\gamma}} \],

where \( \tilde{p} \) is the complex density; \( A \), the cross-sectional area \( \text{[m}^2\text{]} \); \( S \), the circumferential length of the cross-section \( \text{[m]} \); and \( \mu \), the coefficient of viscosity of air \( \text{[Pa}\cdot\text{s]} \). The wave propagation in air is regarded as an adiabatic change:

\[ \frac{p}{\rho'} = \text{const.} \] \[ (15) \]

The following equation is a 1D wave equation that considers air damping.

\[ \frac{\partial^2 p}{\partial t^2} - \frac{\gamma}{\rho} \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial x^2} = 0 \] \[ (16) \]

Here, \( \tilde{c} \) is the complex sound velocity, defined as

\[ \tilde{c} = \sqrt{\frac{\gamma p}{\rho}}. \] \[ (17) \]

The clockwise and counterclockwise moving waves are expressed by Eqs. (18) and (19), respectively.

\[ p_{cw}(x, t) = p_0 \exp \left\{ j \left( \omega t - \frac{\omega}{\tilde{c}} x \right) \right\} \] \[ (18) \]

\[ = p_0 \exp(j\omega t) \exp(-j\tilde{k}x) \] \[ (0 \leq x \leq L_c) \]

\[ p_{ccw}(x, t) = p_0 \exp \left\{ j \left( \omega t + \frac{\omega}{\tilde{c}} x \right) \right\} \] \[ (19) \]

\[ = p_0 \exp(j\omega t) \exp(j\tilde{k}x) \] \[ (-L_c \leq x \leq 0) \]

Here, \( \tilde{k} = \omega/\tilde{c} = \omega/c \times \sqrt{\rho/p} \) is the complex wavenumber. We consider the circulating wave in Eqs. (18) and (19), from which the pressure inside the tire model is expressed as follows.

\[ p(x, t) = p_{cw}(x, t) + p_{ccw}(x - L_c, t) \] \[ (20) \]

\[ = p_0 \exp(j\omega t) \times \]

\[ \sum_{i=0}^{n} \exp[-j\tilde{k}(x + iL_c)] + \exp[j\tilde{k}(x - (i + 1)L_c)] \] \[ (0 \leq x \leq L_c) \]

3. Experimental Apparatus for Measuring SPL Distribution

We measured the SPL distributions inside a tire by using a multi-microphone system [5]. The experimental apparatus for measuring sound pressure inside a tire is shown in Fig. 4. The tire size is 195/65R15 91S and the diameter is 634 mm; the tire is loaded at 200 kPa. A signal generator (FGA5050GC, Kikusui electronics) is used to automatically generate a sine wave sound in the range from 200 to 260 Hz, and the speaker vibrates the tire as an acoustic vibrator.

![Fig. 4 Multi-microphone system for tire cavity resonance.](image)

The microphone unit (WM-61A, Panasonic) is attached to an air-proof sensor head by a \( \phi \) 3.5 mm monaural jack, as shown in Fig. 5. The diameter and length of the microphone unit are 9.5 mm and 25.6 mm, respectively. The sound affected by the microphone is that with frequency more than 13 kHz; it is thought that the size of the microphone does not influence the sounds in the frequency range of interest.

![Fig. 5 Air-proof sensor head.](image)

4. Results

In this section, we compare the theoretical and experimental sound pressure distributions. The parameters used in the comparison are the pressure \( \text{[Pa]} \) and SPL \( \text{[dB]} \). SPL is expressed by Eqs. (21) and (22).

\[ P_s(x) = \frac{1}{T} \frac{1}{f_0} p(x, t)^2 dt \] \[ (21) \]
\[
\text{SPL} = 20 \log_{10} \frac{P_e(x)}{P_{\text{ref}}}
\]

Here, \(P_e(x)\) is the effective value of the sound pressure [Pa], and \(P_{\text{ref}}\) is the reference sound pressure \((2.0 \times 10^{-5} \text{ Pa})\).

4.1 SPL spectrum distributions

We compare the theoretical and experimental values of SPL inside a tire. The theoretical and experimental conditions are shown in Table 1. \(S\) and \(A\) are measured by using a contour gauge [6] (see Fig. 1). SPL at 0\(^\circ\), as shown in Fig. 4 in the frequency range of 200-260 Hz, is plotted using a contour gauge [6] (see Fig. 1). SPL at 0\(^\circ\), as shown in Table 1.

![Fig. 6](image)

Fig. 6 SPL spectrum distribution of the theoretical value without air damping and the experimental value.

![Fig. 7](image)

Fig. 7 SPL spectrum distribution of the theoretical value with air damping and the experimental value.

4.2 Time change of sound pressure distribution

We compare the time change of the sound pressure distributions to observe the direction of the sound pressure inside a tire. Figures 8-10 show the experimental and theoretical values derived using Eq. (20) for the time change of the sound pressure distributions at the resonance frequency \(f_n (=229 \text{ Hz})\), at the frequency following resonance \((220 \text{ Hz})\), and at the frequency above resonance \((240 \text{ Hz})\). \(\varphi (=\omega t)\) is the phase of the time change of the sound pressure distributions inside a tire. In both the theoretical and experimental results, the sound pressure inside the tire varies greatly in the vertical direction at the tire cavity resonance frequency \(f_n (=229 \text{ Hz})\). The sound pressure distribution vibrates a vehicle spindle in the vertical direction.

Moreover, there is a difference between the theoretical and experimental values at the tire cavity resonance frequency. In theory, we regard the tire structures as a rigid body. In the experiment, the tire structures, such as side walls and wheels (see Fig. 1), are elastic bodies that vibrate due to the time change of sound pressure distributions inside the tire. We have concluded that the difference can be attributed to the energy consumed by the vibration of the tire structures. Therefore, it is possible to predict the cavity resonance frequency and the direction of the sound pressure distributions inside a tire, although the amplitude of the sound pressure at the cavity resonance frequency is underestimated.

4.3 SPL distributions

The SPL distributions inside a tire are plotted in Fig. 11. In both the theoretical and experimental results, the SPL shapes have antinodes at 0\(^\circ\) and 180\(^\circ\), and nodes at 90\(^\circ\) and 270\(^\circ\) in the range of 200-260 Hz. Therefore, it is seen that the SPL mode distributions around the cavity resonance frequency vibrate the vehicle spindle in the vertical direction.

The difference between the theoretical and experimental values is evaluated by the RMS error in the range of 200-260 Hz. The RMS error at 0\(^\circ\) is 1.14 dB, and the theoretical value shows reasonable agreement with the experimental value. Therefore, we are able to express the sound phenomenon without using FEM analysis.
We compare the theoretical and experimental values of the SPL derived from Eq. (20) (pressure amplitude: $A = 0.010 \text{ Pa}$; $\phi = 0 \text{ rad}$) to the experimental value. Therefore, we are able to express the theoretical value shows reasonable agreement with the experimental values is evaluated by the RMS error in the range of 200-260 Hz. The RMS error at 0° is 1.14 dB, and Eq. (21) expresses the SPL distributions inside a tire. We have concluded that the difference can be expressed by solving the proposed 1D wave equation.

Moreover, there is a difference between the theoretical and experimental results, the SPL pressure distribution vibrates a vehicle spindle in the vertical direction. In theory, we regard the tire structures as a rigid wave equation with air damping $\rho \cdot c \cdot A \cdot S$ and $\mu \cdot ��$. In this study, a 1D wave equation that considers the circulating wave and air damping is applied to the sound pressure distribution inside a tire. The theoretical value of the SPL derived by the proposed equation is roughly the same as the experimental value in terms of the shapes of the SPL distributions around tire cavity resonance. Therefore, it is concluded that the sound pressure distributions around the tire cavity resonance can be expressed by solving the proposed 1D wave equation.

5. Conclusions
In this study, a 1D wave equation that considers the circulating wave and air damping is applied to the sound pressure distribution inside a tire. The theoretical value of the SPL derived by the proposed equation is roughly the same as the experimental value in terms of the shapes of the SPL distributions around tire cavity resonance. Therefore, it is concluded that the sound pressure distributions around the tire cavity resonance can be expressed by solving the proposed 1D wave equation.

Fig. 8 Time change of theoretical and experimental sound pressure distribution (220 Hz).

Fig. 9 Time change of theoretical and experimental sound pressure distribution (229 Hz).

Fig. 10 Time change of theoretical and experimental sound pressure distribution (240 Hz).
Experimental value

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References

[3] Regulation No.117 Revision 2: Uniform provisions concerning the approval of tyres with regard to rolling sound emissions and to adhesion on wet surfaces and/or to rolling resistance, UN-ECE, (2011).

Fig. 11 Theoretical and experimental SPL distributions inside a tire around tire cavity resonance.