Effect of Fluid Acceleration due to Area Reduction of a Wind Tunnel on Temporally and Spatially Decaying Grid Turbulence

Hiroki SUZUKI1, Shinnosuke MATSUO2, Shinsuke MOCHIZUKI1 and Yutaka HASEGAWA2

1 Graduate School of Sciences and Technology for Innovation, Yamaguchi University, Ube 755-8611, Japan
2 Department of Electrical and Mechanical Engineering, Nagoya Institute of Technology, Nagoya 466-8555, Japan

(Received 9 December 2015; received in revised form 15 April 2016; accepted 3 June 2016)

Abstract: We examine the effect of fluid acceleration on the decay characteristics of temporally and spatially decaying grid turbulence, where a spatially decaying turbulence is similar to a grid-generated turbulence. The derived forms from the governing equation of the turbulent kinetic energy show that there is a quantitative difference in the acceleration effects between the decaying turbulences. Using the observed forms, the negligible effect of the fluid acceleration in the present small wind tunnel is validated.

Keywords: Theoretical Observation, Wind Tunnel Blockage, Grid Turbulence, Wind Tunnel, Fluid Acceleration

1. Introduction
1.1 Background
Flows seen in the field of mechanical engineering are often turbulent. The principles of experimental mechanics are applied to analyze these turbulent flows. Turbulent flows are classified roughly according to the presence or absence of a mean velocity gradient. In contrast to flows with a mean velocity gradient, the turbulent kinetic energy of flows without a mean velocity gradient decreases due to the action of viscous dissipation if the uniformity of the kinetic energy is spatially sufficient. This uniformity is often seen in the flows far from the wall. The decay of turbulent flows reflects one of the most fundamental characteristics of turbulent flows.

The turbulent kinetic energy in decaying homogeneous turbulence decays due to viscous dissipation [1]. The turbulent kinetic energy is defined as a summation of the diagonal components of the Reynolds stress tensor, where the diagonal components represent the intensity of the velocity fluctuation. In contrast to the decaying characteristics of the turbulent kinetic energy, the components of the intensity do not necessarily decrease for homogeneous turbulence because of the presence of the pressure-strain term.

Because the summation of the pressure-strain term is zero due to the incompressibility, the decaying characteristics of the turbulent kinetic energy are certainly found, even if anisotropy is present. This kind of turbulent flow has been previously studied experimentally using grid turbulence [2–4]. Grid turbulence is generated using a turbulence-generating grid set in a uniform flow. Although the turbulent kinetic energy of grid turbulence is smaller than that of the turbulence generated by an active grid, grid turbulence is still a current research area mainly because of its classical nature. The form of the turbulent kinetic energy in grid turbulence is known and is considered to be expressed by a power law. Because the idealized grid turbulence has no production term, its rate of change is equivalent to the viscous dissipation.

1.2 Brief review
The power law formulating the turbulent kinetic energy in grid turbulence includes the decay exponent $n$ and the decay coefficient $A$ as $k = A t^n$ or $k = A x^n$, where $k$, $t$, and $x$ are non-dimensional turbulent kinetic energy, time and streamwise direction, respectively [1-5]. The latter is directly related to the drag coefficient of the turbulence-generating grid [5]. Therefore, the magnitude of $A$ depends on the shape of the turbulence-generating grid. In contrast to $A$, the magnitude of $n$ is of the order unity. The value of $n$ is unity in the linear decay law [5]. Values of $n$ that are larger than unity were often found in previous experiments [2–4]. The value of the decay exponent characterizes the fundamental nature of the decaying homogeneous turbulence. In high-Reynolds-number turbulence, the values $n = 6/5$ and $n = 10/7$ are theoretically derived. The difference between the values of $n$ is caused by the nature of the large-scale velocity fluctuations. The value of $n$ may be 6/5 for grid turbulence. For low-Reynolds-number turbulence, the value of $n$ increases as the Reynolds number decreases. Therefore, the value of $n$ could be larger than 1.5.

The observed value of $n$ should be accurate in studies of grid turbulence; however, accurate measurement of $n$ is rather challenging for two reasons. The first is due to the inaccuracy and uncertainty of the experimental technique. The second is due to the nature of the generated flow. The area of the wind tunnel used in the experiments is limited. There is a boundary layer on the side walls. The displacement thickness of the boundary layer increases because the boundary layer evolves in the streamwise direction. This increasing displacement thickness, in turn, causes the wind tunnel area to decrease. Therefore, it increases the streamwise velocity because of the incompressibility. The growing streamwise velocity causes the production of turbulent kinetic energy, which should be negligibly small. The resulting production of turbulent kinetic energy affects the decay characteristics of the grid turbulence. The magnitude of the production should be negligibly small to improve the accuracy of the observed decay exponent [4]. A similar issue is found with
experiments on a turbulent boundary layer with a zero-pressure gradient [6–7]. In the experiments on the boundary layer, the acceleration parameter is used to quantify the effect of the fluid acceleration.

1.3 Purpose of this study
It is often appropriate to use homogeneous turbulence to address the effect of fluid acceleration. The simplicity of the governing equation of homogeneous turbulence will be helpful in this kind of discussion. There is a rather large difference between homogeneous and grid turbulences. Grid turbulence decays spatially, in contrast to homogeneous turbulence, which temporally decays. The convection velocity based on Taylor's assumption relates the decay of grid turbulence to that of homogeneous turbulence. A temporally decaying turbulence of the homogeneity convected by a freestream velocity approximately a grid-generated turbulence [6] and is considered to be a spatially decaying turbulence. The difference in the effect of fluid acceleration between spatially decaying and temporally decaying turbulences is still not well understood. The production of turbulent kinetic energy due to fluid acceleration should be negligibly small in the experiments. The flow in the wind tunnel used for the experiments should be validated.

The present study examines the flow in the present wind tunnel from both aspects.

The aim of the present study is to address the difference in the effect of fluid acceleration between spatially decaying and temporally decaying turbulences. The present study addresses this issue using the governing equation of the turbulent kinetic energy. By focusing on the condition where the fluid acceleration is small, the governing equation of the turbulent kinetic energy is analytically solved. Using the derived solutions of the equation, the effect of fluid acceleration is formulated. The present form of the effect shows that the effect of fluid acceleration is slightly different between the temporally decaying and spatially decaying turbulences except for the negligible fluid acceleration. The slightly larger effect of the fluid acceleration may be found in the spatially decaying turbulence. Moreover, the present study validates the flow in the present wind tunnel for the experiments. A negligible acceleration effect is observed in the present wind tunnel by using the derived form of acceleration.

2. Production Term and Governing Equation
2.1 Production term
Previous experiments that measure the turbulent boundary layer use the acceleration parameter to quantify the effect of fluid acceleration. The definition of the acceleration parameter [6] is given as follows:

\[ K = \frac{\nu \frac{dU'}{U'^2}}{dx'} = -\frac{\nu \frac{dP'}{\rho U'^3}}{dx'} \] (1)

Here \( \nu \), \( U' \), and \( x' \) are the kinematic viscosity [m²/s], the mean streamwise velocity [m/s], and the streamwise direction [m], respectively. Here the origin of the streamwise direction \( x' \) is set to a point at which a turbulence-generating grid is set. As shown above, the definition using the streamwise pressure gradient is also derived. The quantities \( \rho \) and \( P' \) in this definition are the density and the mean static pressure, respectively. Although this definition was used in the previous experiments, we consider it unsuitable for grid turbulence because the velocity \( U' \) normalizing the acceleration parameter is not the characteristic velocity of the bulk flow. Therefore, we use the following modified definition of the fluid acceleration in this study:

\[ K_o = \frac{\nu \frac{dU'}{U_o^2}}{dx'} \] (2)

In equation (2), the velocity \( U_o = U'(0) \) is the characteristic velocity of the flow of a grid-generated turbulence. In this definition, the fluid acceleration is normalized by \( U_o \). A velocity normalizing velocity fluctuations of a grid-generated turbulence is generally set to the characteristic velocity \( U_o \) rather than \( U' \). Therefore, we consider the modified definition to be more suitable for the grid turbulence experiment. These definitions are related by the following simple relation:

\[ K_o = \frac{U'^2}{U_o^2} K \] (3)

These definitions are identical when the fluid acceleration is negligible. The following relation specifies an effect of the velocity increment \( dU'/U_o \) on the difference in the value between the definitions of the fluid acceleration:

\[ K_o = K \left(1 + 2 \frac{dU'}{U_o} + \cdots\right) \text{, where } U' = U_o + dU' \] (4)

The difference between the definitions is calculated by the double magnitude of the velocity increment and decreases as the magnitude of the velocity increase decreases.

An axisymmetric-homogeneous turbulence is applied to the present study. In the present flow, the production term of the turbulent kinetic energy is derived as follows:

\[ P = -\left(\langle u'^2 \rangle - \langle v'^2 \rangle \right) S, \text{ where } S = \frac{dU}{dx} \] (5)

Here, \( u \), \( v \), and \( w \) are the velocity fluctuations normalized by the characteristic velocity for the streamwise, transverse, and spanwise directions, respectively. Also \( \langle \cdot \rangle \) denotes ensemble average. In addition, a non-dimensional acceleration rate: \( S = dU/dx = (dU'/dx') / (U/M) \), where \( M \) is the mesh size of a turbulence-generating grid and \( U \) and \( x \) are non-dimensional mean streamwise velocity and the streamwise direction. Here the derivation of Eq.(5) is specified in the appendix. Because of the axisymmetric nature of the turbulence, a relation, \( \langle w'^2 \rangle = \langle v'^2 \rangle \), can be used. Using the definition of the turbulent kinetic energy in axisymmetric-homogeneous turbulence, which is \( k = (1/2)(\langle u'^2 \rangle + \langle v'^2 \rangle + \langle w'^2 \rangle) = (1/2)(\langle u'^2 \rangle + 2\langle v'^2 \rangle) \) due to \( \langle w'^2 \rangle = \langle v'^2 \rangle \), the production term takes the following form:

\[ P = -\frac{2 - 2a}{2a} Sk(t), \text{ where } \langle v'^2 \rangle = a \langle u'^2 \rangle \] (6)
2. Production Term and Governing Equation

of fluid acceleration. The definition of the acceleration uses the acceleration parameter to quantify the effect of fluid acceleration. Previous experiments that measure the turbulent boundary layer can be used to study the flow in the present wind tunnel for the experiments on a turbulent boundary layer with a zero-mean streamwise velocity. A decaying turbulence. Moreover, the present study validates decaying and spatially decaying turbulences except for the acceleration being slightly different between the temporally and spatially decaying cases.

The present study addresses this issue using the governing equation of the turbulent kinetic energy. The aim of the present study is to address the effect of fluid acceleration. The simplicity of the governing equation allows us to examine the effect of fluid acceleration. The governing equation of the turbulent kinetic energy in the condition where the fluid acceleration is small, the effect of fluid acceleration is negligible. The magnitude of the fluid acceleration is small in the present flow. The small acceleration in the flow gives $U' = U_o + (dU'/dx) x'$ and therefore yields the following linear form for the convection velocity:

$$U = 1 + \frac{dU}{dx} x = 1 + K_o \text{Re}_M \times x.$$  \hspace{1cm} (11)

Here the unity in Eq.(11) is derived by using the characteristic velocity. Using the formulated convection velocity, the governing equation in the spatially decaying turbulence results in the following form:

$$(1 + K_o \text{Re}_M \times x) \frac{dk}{dx} = P - \epsilon.$$  \hspace{1cm} (12)

For the spatially decaying turbulence, the resulting equation of the turbulent kinetic energy is used to examine the effect of fluid acceleration.

The resulting governing equations of the turbulent kinetic energy are not closed. Therefore, we introduce an assumption into the governing equation to solve it. There are two conditions for solving the governing equation. In the first condition, the form of the scale in the decaying turbulence is assumed to be:

$$k(t)/t = \frac{1}{n} \times \frac{k(x)}{x} = \frac{1}{n} \times \frac{x}{1 + K_o \text{Re}_M x}.$$  \hspace{1cm} (13)

Here,

$$t = \frac{x}{1 + K_o \text{Re}_M x}.$$  \hspace{1cm} (14)

This form of the scale is available in the decaying turbulence with a negligible effect of the fluid acceleration. This assumption is considered to be accurate because the effect of fluid acceleration is small in the present flow. Following the results of numerical simulation based on k-ε model carried out by the first author, we found that the magnitude of $K_o \text{Re}_M$ should be smaller than 1/(11 t) to hold the present assumption with the deviation which is smaller than 1%. where $P_o = 0.5$ and $n = 1.2$.

When the effect of fluid acceleration is negligible, the form of the turbulent kinetic energy is known and as follows:

$$k(t) = k_o t^{-n} \text{ or } k(x) = k_o x^{-n}.$$  \hspace{1cm} (15)
This form is acceptable as shown in previous works (e.g., [1-5]). In the above, \( k_0 = k(1) \). When considering the effect of fluid acceleration, the turbulent kinetic energy becomes

\[
k(t) = k_0 f(t) t^{-n} \quad \text{or} \quad k(x) = k_0 g(x) x^{-n}. \tag{16}
\]

Here, the two non-dimensional functions \( f(t) \) and \( g(x) \) characterize the effect of the fluid acceleration on the turbulent kinetic energy. In the above, \( f(1) = g(1) = 1 \). This condition is derived from \( k_0 = k(1) \) and corresponds to that there is no fluid acceleration at the state.

3. Effect of Fluid Acceleration

3.1 Derived effects of the fluid acceleration

The resulting governing equations of the turbulent kinetic energy are each linear ordinary differential equations. Therefore, the solutions of the governing equations can be written.

In temporally decaying turbulence, the effect of fluid acceleration is solved as follows:

\[
f(t) = \exp(-\frac{\kappa_K \Re_M}{\kappa_0} (t-1)) \\
= 1 - \frac{\kappa_K \Re_M}{\kappa_0} (t-1) + \cdots \tag{17}
\]

As shown above, the fluid acceleration with an assumption of the scale is an exponential form, which includes a coefficient characterizing the effect of fluid acceleration, \( \frac{\kappa_K \Re_M}{\kappa_0} \). The form of \( f(t) \) is approximated by the linear form, which is given by expanding the form around a zero point. Here, \( \kappa_0 = k(1) \).

In spatially decaying turbulence, the effect of fluid acceleration is also solved. The solutions for the two conditions are as follows:

\[
g(t) = \left( \frac{1 + \kappa_0 \Re_M}{1 + \kappa_0 \Re_M \times x} \right)^{P_0} \\
= 1 - \frac{\kappa_0 \Re_M}{\kappa_0} (t-1) + \cdots \tag{18}
\]

Here the second form is derived by applying the Taylor series expansion for the small magnitude of \( \kappa_0 \) to the first form. As shown in the solutions, there is a notable difference in the effect between temporally decaying and spatially decaying turbulences. In contrast to the solution in temporally decaying turbulence, the acceleration effect in spatially decaying turbulence is formulated based on the power law. The exponent of the acceleration effect is the magnitude of the production term \( P_0 \). The form of \( g(x) \) in spatially decaying turbulence is also approximated by the linear form, which is derived for the small magnitude of the coefficient. Although the original form of the acceleration effect is different between the decaying turbulences, the approximated linear forms are the same. This agreement in the linear forms shows that the effect of fluid acceleration on the turbulent kinetic energy is the same between the decaying turbulences.

We examine the difference in the effect using the following relative deviation of \( g(x) \) from \( f(t) \). These coordinate systems relate using Eq. (14). The form of this relation is modified as follows:

\[
x = \frac{t}{1 - \kappa_0 \Re_M \times t}. \tag{19}
\]

By using this relation, the effect of fluid acceleration on the spatially decaying turbulence \( g(x) \) is transformed as follows:

\[
g(t) = ((1 + \kappa_0 \Re_M) (1 - \kappa_0 \Re_M \times t))^{P_0}. \tag{20}
\]

Figure 2 shows the temporal and spatial distributions of the effect of fluid acceleration, where \( \kappa_0 \Re_M = 3 \times 10^4 \) and \( P_0 = 0.5 \). This value of \( P_0 \) is derived from \( a = 0.5 \), where the minimum value of \( a \) would be about 0.5 in a grid-generated turbulence, because \( (\bar{v}^2)_{1/2} / (\bar{u}^2)_{1/2} \leq 1.4 \) as shown in previous experiments (e.g., [2]).

Here, \( f(t) \) and \( g(t) \) decrease as the flow evolves. In the temporally decaying and spatially decaying turbulences, the derivations of \( g(t) \) from unity are slightly larger than that of \( f(t) \). Therefore, the effect of fluid acceleration on the spatially decaying turbulence should be examined to quantify the fluid acceleration because of the larger effect of the fluid acceleration. The form of \( f(t) \) is approximated by the linear form. The linear forms agree with those of the distribution of \( f(t) \). This agreement shows that the linear forms are accurate.

The absolute deviation of \( g(t) \) in the spatially decaying turbulence is slightly larger than that in the temporally decaying turbulence, as shown in Fig. 2. Here, we will discuss the difference between the two cases. Specifically, we attempt to derive a prediction of the difference observed in the effect of fluid acceleration between the decaying turbulences.
Using the transformed form of $g(t)$, the relative deviation of $g(t)$ from $f(t)$ is determined. The following approximation is derived for a small coefficient.

$$
\frac{g(t) - f(t)}{f(t)} = -P_0(K_\alpha Re_M)^2 \left(1 + t^2\right) + \cdots
$$

(21)

As shown in the above prediction, the relative deviation is proportional to the square of the coefficient of fluid acceleration. The relative deviation is found to be a quadratic function of time. The negative magnitude of the relative deviation shows that the spatially decaying turbulence is more sensitive to fluid acceleration than the temporally decaying turbulence. Figure 3 validates the derived form of the relative deviation. The distributions of the derived relative deviation agree well with those of the numerical results. This agreement shows that the derived form is sufficiently accurate.

### 3.2 Decay exponent and decay coefficient

The decay exponent $n$ and decay coefficient $A$ characterize the nature of the decaying turbulence. The fluid acceleration will affect the value of these quantities. We derive the expression that describes the dependence of $n$ and $A$ on the fluid acceleration. We use the spatially decaying turbulence to examine the effects on the decay exponent and decay coefficient because the spatially decaying turbulence is found to be more sensitive to fluid acceleration.

In decaying turbulence without production, the decay exponents of the turbulent kinetic energy and the dissipation are calculated using the following forms:

$$
n = -\frac{x}{k(x)} \frac{dk(x)/dx}{k(x)}
$$

(22)

Here Eq.(22) is derived from Eq.(15). Using these forms, the decay exponents of the quantities are formulated as follows:

$$
n = -\frac{x}{g(x)} \frac{dg(x)/dx}{g(x)}
$$

(23)

As shown above, the effect of fluid acceleration on the decay exponent is the same between the turbulent kinetic energy and the dissipation. Using the form of $f(t)$, the deviation of the decay exponents due to the effect of fluid acceleration is derived as follows:

$$
\frac{x}{g(x)} \frac{dg(x)/dx}{g(x)} = P_0K_\alpha Re_M \times x
$$

(24)

Here Eq.(24) is derived from the first form of Eq.(18).

In decaying turbulence without production, the decay coefficient is calculated by the following form:

$$
A = x^n k(x)/g(x)
$$

(25)

Here Eq.(25) is derived from the second form of Eq.(15) with $A = k_c$. Although we use the notation of $k_c$ to emphasize that $k_c$ is an initial value of $k$, the notation of $A$ is more conventional for the decay coefficient [2-3].

**Fig. 3 Temporal evolution of $-(g(t) - f(t))/f(t)$, where $K_\alpha Re_M = 1 \times 10^{-3}$ and $1 \times 10^{-4}$, respectively.** The solid lines show the numerical results of $-(g(t) - f(t))/f(t)$. The value of this quantity increases with $t$. The larger value of this quantity is found in the larger value of $K_\alpha Re_M$. The derived approximations are also shown in the figure. The dotted lines show the distribution of the approximation. The accuracy of this approximation is improved as the value of $K_\alpha Re_M$ decreases.

Using this form, the relative deviation of the decay coefficient $e_A$ is formulated as follows:

$$
e_A = \frac{A - x^n k(x)}{x^n k(x)} = \frac{1}{g(x)} - 1
$$

(26)

As shown in the above form, the relative deviation of the decay coefficient can be formulated by the acceleration effect $g(x)$. Using the form of the relative deviation, for the spatially decaying turbulence, the dependence of the decay coefficient on fluid acceleration is derived as follows:

$$
\frac{1}{g(x)} - 1 = \frac{1}{((1 + K_\alpha Re_M)/(1 + K_\alpha Re_M x))^{P_0} - 1
$$

$$
= \left(\frac{1 + K_\alpha Re_M x}{1 + K_\alpha Re_M}ight)^{P_0} - 1
$$

$$
= P_0K_\alpha Re_M (x - 1) + \cdots
$$

(27)

As shown above, although the forms of the decay exponent and the decay coefficient are different, the gradients of the linear forms approximating the effect of fluid acceleration are the same.

### 4. Small Wind Tunnel and Experiment

#### 4.1 Experiment

A small wind tunnel is used in the present study. The cross-sectional area at the entrance ($D \times D$) is 40,000 mm$^2$. The streamwise length of the test section $L$ is 2650 mm.
The origin of the coordinate system is located at the center of the entrance. The streamwise, transverse, and spanwise directions are taken to be the x, y, and z directions, respectively. The freestream velocity is about 20 m/s. We set a diffuser at the downstream end to reduce the loss of fluid energy. This diffuser has a two-dimensional expansion with an adjusted expansion ratio.

There are boundary layers developing on the side walls. Because the displacement thickness of the boundary layers increases, the streamwise velocity at the centerline increases and causes fluid acceleration. We introduce two-dimensional spanwise expansion to the test section in order to reduce the fluid acceleration. The rate of expansion of the width per 1 m is about 6.7 percent and is constant. The value of the rate was set by calculating the development of the displacement thickness.

The measurement system is composed of a personal computer, a data logger (KEYENCE, NR-600), a constant temperature anemometer (SOKKEN, HC-30) based on an I-type hot wire probe, a differential manometer (SIBATA SCIENTIFIC TECHNOLOGY LTD, ISP-750), and a JIS-type Pitot tube (TSUKUBARIKASEIKI, F202). The hot wire consists of a tungsten wire with a diameter of 5 μm. The aspect ratio of the wire is about 200. Here, the frequency response of the hot wire measurement is confirmed to be sufficiently high by using a square wave test. The coefficient of the Pitot tube is guaranteed by calibrating the coefficient. We put either the hot wire probe or Pitot tube into the test section during the experiments.

We validate that sufficient uniformity is found in the quantities. The spatial distribution of the mean velocity is highly uniform, where the maximum deviation is smaller than 1%. This uniformity is held in the entire region of the test section. The rms of the velocity fluctuation is also highly uniform and is measured to be as large as 0.2–0.3%. We compare the present results of the uniformity with that of the previous experiments [7,8], which measure grid-generated turbulence. The present value is comparable with the previous values and is included in the range of previous rms values. Therefore, the present value is considered to be small enough for the measurement of grid turbulence.

### 4.2 Negligible effect of fluid acceleration

We measure the mean streamwise velocity along the centerline. We show the observed value of the acceleration parameter is sufficiently small for the measurement of a grid-generated turbulence in this section. We calculate the characteristic velocity $U_\infty$ and the velocity increment $dU'/dx'$ by fitting $U' = U_\infty + (dU'/dx')x'$ to the mean streamwise velocity, where a least-square fitting method is used. The rms of the mean velocity deviation from the streamwise average is about 0.5%. This value is comparable to the uncertainty of the velocity measurement. We calculate $K$ by taking the mean value of the definition (Eq. (1)) because the definition includes $U'$. The value of $K_\infty$ is calculated from the observed value of $U_\infty$ and $dU'/dx'$.

This result yields a small fluid acceleration magnitude. The value of the acceleration parameter is evaluated as follows: $K = 1 - 2 \times 10^{-5}$. This value is smaller than that of previous experiments, in which $K$ is on the order of $1 \times 10^{-7} - 1 \times 10^{-9}$ [8-12] as shown in Fig. 4 (a).

We validate the present wind tunnel using the derived form of the acceleration effect, where $K_\infty = 1 - 2 \times 10^{-5}$.
We use the form of the spatially decaying turbulence because of the large effect of the fluid acceleration. To quantify the fluid acceleration, the value of the parameter needs to be set. We set $P_o = 0.5$ and $x = 200$, respectively. From the observed value of the fluid acceleration, the value of $K_pRe_u$ is set to $3 \times 10^5$. Figure 4(b) shows the calculated effect of the fluid acceleration on the decay exponent and decay coefficient in the present wind tunnel. The absolute magnitude of the fluid acceleration is smaller than 1%, which is smaller than the uncertainty of the constant temperature anemometry measurement. From these values, the effect of fluid acceleration in the present wind tunnel is considered to be negligible. Figure 4(b) compares the effects of fluid acceleration on the quantities between different values of the acceleration parameters, where the larger magnitude of the acceleration parameter is set to $K_p = 1 \times 10^5$, which is comparable to that of the previous experiments.

5. Conclusions

In the experiments on grid turbulence, the effect of fluid acceleration should be negligibly small. The grid turbulence is a spatially decaying turbulence. There will be a difference in the effect of fluid acceleration between the temporally decaying and spatially decaying turbulence. The present study examines the effect of fluid acceleration on the decay characteristics of temporally decaying and spatially decaying turbulence.

Using the governing equation of the turbulent kinetic energy, the effect of fluid acceleration on these decaying turbulences is derived. To close the governing equation, the scale of the decaying turbulence is held. Using the derived solutions of the effect of fluid acceleration, the decay exponent and decay coefficient, $n$ and $A$, affected by the fluid acceleration are derived. These derived forms show that the spatially decaying turbulence will be more sensitive to the effect of fluid acceleration. Using the derived forms of the acceleration effect, we validate the background flow in the present wind tunnel. The acceleration parameter in the small wind tunnel is on the order of $1 \times 10^9$. The presently observed forms show that this acceleration value has a negligibly small effect on the fluid acceleration.

Using this wind tunnel, we will attempt to measure the decay characteristics of grid turbulent flows. The negligible production term is confirmed in the present wind tunnel, as shown in this study. Therefore, the value of $A$ will be measured to two decimal places.

Nomenclature

- $a$: anisotropy parameter [-]
- $A$: decay coefficient [-]
- $f$: influence function of the turbulent kinetic energy [-]
- $g$: influence function of the dissipation [-]
- $k$: non-dimensional turbulent kinetic energy [-]
- $K$: acceleration parameter of a boundary layer [-]
- $K_p$: acceleration parameter of a grid-generated turbulence [-]
- $M$: mesh size of a turbulence-generating grid [m]
- $n$: decay exponent [-]
- $P_o$: coefficient characterizing the production term [-]
- $Re_u$: mesh Reynolds number [-]
- $U_{\infty}$: free stream velocity [m/s]
- $U$: non-dimensional mean streamwise velocity [-]
- $U'$: dimensional mean streamwise velocity [m/s]
- $t$: non-dimensional time [-]
- $x$: non-dimensional streamwise direction [-]
- $x'$: dimensional streamwise direction [m]
- $\varepsilon$: non-dimensional dissipation [-]
- $\nu$: kinematic viscosity [m²/s]

Acknowledgement

Part of this study is supported by the Japanese Ministry of Education, Culture, Sports, Science, and Technology through Grants-in-Aid (Nos. 25420115, 15K05792, 15K13871, and 15K17970).

References

Appendix

A.1 Derivation of Equation (5)

We focus on a grid-generated turbulence with a fluid acceleration. The production term of the transport equation for the turbulent kinetic energy, which is normalized by the characteristic velocity, and length is given as follows:

\[ P = - \left( \langle u^2 \rangle \frac{dU}{dx} + \langle v^2 \rangle \frac{dV}{dy} + \langle w^2 \rangle \frac{dW}{dz} \right). \]  

(A1)

Here \( x, y, \) and \( z \) are streamwise, transverse and spanwise directions, respectively. \( u, v, \) and \( w \) are the velocity fluctuations normalized by the characteristic velocity. \( U, V, \) and \( W \) are mean velocity normalized by the characteristic velocity. In an axisymmetric-homogeneous turbulence, the following relation is held:

\[ \langle w^2 \rangle = \langle v^2 \rangle. \]  

(A2)

Applying Eq.(A2) to Eq.(A1), Eq.(A1) results in the following equation:

\[ P = - \left( \langle u^2 \rangle \frac{dU}{dx} + \langle v^2 \rangle \left( \frac{dV}{dy} + \frac{dW}{dz} \right) \right). \]  

(A3)

The continuity equation of the mean flow gives the following relation:

\[ \frac{dU}{dx} + \frac{dV}{dy} + \frac{dW}{dz} = 0. \]  

(A4)

Using Eq.(A4), the following equation is yielded from Eq.(A3):

\[ P = - \left( \langle u^2 \rangle - \langle v^2 \rangle \right) S, \]  

where \( S = \frac{dU}{dx}. \)  

(A5)

Equation (A5) is equal to Eq.(5).

A.2 Derivation of Equation (18)

The governing equation of \( g(x) \) is derived from Eqs.(8), (12) for \( x, \) (13) and (15), and is yielded as follows:

\[ \frac{dg(x)}{dx} = - \frac{P_o K_o R_e M}{1 + K_o R_e M} \rho(x). \]  

(A6)

General solution of Eq.(A6) is yielded as follows:

\[ g(x) = C_g \exp(-P_o \ln(1 + K_o R_e M x)). \]  

(A7)

Here \( C_g \) is an integral constant. Applying the setting \( g(1) = 1 \) to Eq.(A7), the constant \( C_g \) is formed as follows:

\[ C_g = \exp(-P_o \ln(1 + K_o R_e M)). \]  

(A8)

Applying Eq.(A8) to Eq.(A7), Eq.(A7) results in the following equation, which is equal to Eq.(18):

\[ g(t) = \left( \frac{1 + K_o R_e M}{1 + K_o R_e M x} \right)^{P_o}. \]  

(A9)

A.3 Derivation of Equation (21)

We have the form of \( f(t) \) and \( g(t) \) as Eqs.(17) and (20). Using the two equations, \( (g(t) - f(t))/f(t) \) is yielded as follows:

\[ \frac{g(t) - f(t)}{f(t)} = \left( 1 + K_o R_e M \right) \frac{(1 - K_o R_e M \tau t)^{P_o}}{1+K_o R_e M \tau t} \times \exp\left( P_o K_o R_e M \left( \tau - \tau \right) - 1 \right). \]  

(A10)

Applying the Taylor series expansion to Eq.(A10) around the zero magnitude of \( K_o \), the form of Eq.(A10) results in the following approximation:

\[ \frac{g(t) - f(t)}{f(t)} = \frac{P_o (K_o R_e M)^2}{2} (1 + \tau^2) + \cdots. \]  

(A11)

Here, Eq.(A11) is equal to Eq.(21).