Effect of Tensile Stress on Transformation Plastic Deflection under Combined Bending-Tensile Loading System

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Abstract: In previous studies, the experimental results for the transformation plasticity coefficient in terms of three-point bending system and tensile-torsional-compressive system were different with the same heat treating case for the same tested material. Therefore, in this study, bending-tensile loading system have been proposed to analyze the transformation plasticity behavior and determine the transformation plasticity coefficient. The specimens of S45C steel were heated to austenitization temperature and kept at this temperature for several minutes, then naturally cooled to room temperature. During cooling process before transformation start, bending and tensile stresses have been applied on specimen and have continued to the end of experiment. The maximum bending deflections due to austenite-pearlite transformation have been examined with different loading conditions and then transformation plasticity coefficients were determined. The obtained transformation plasticity coefficient of pearlitic transformation were compared with the results of other test methods such as three-point bending system and tensile-torsional-compressive system.

Keywords: Bending-Tensile Loading, Austenite, Pearlite, Transformation Plasticity, Bending Deflection, Transformation Plasticity Coefficient

1. Introduction
In heat treatment process solid to solid phase transformation occurs due to some process such as quenching and type of phase transformation exactly depends on cooling rate and alloying content. In particular it is well known that during cooling process with applying extremely small stress to material, a new mechanical phenomenon, named transformation plasticity (TP) is observed. The thermo-mechanical behavior of materials undergoing a solid-solid phase transformation has been largely investigated experimentally and numerically [1–13]. The transformation plasticity behavior under uniaxial loading system have experimentally studied by many researchers with different materials [1–7]. Simon et al. [8, 9] and Inoue et al. [10, 11] have numerically analyzed the effect of applied tensile stress during pearlitic phase transformation for the eutectoid carbon steel.

Gautier et al. [1] investigated the effect of tensile stress on the start and finished times of the pearlitic transformation for the eutectoid carbon steel. The experimental results show that the pearlitic transformation is accelerated with increases in the applied stress significantly. Tokizane et al. [2] studied the effect of tensile stress on the martensitic transformation start temperature (Ms point) in the Fe-Ni-0.21C alloy. The results show that the Ms point increases linearly with increases in applied stress. Taleb et al. [3] studied the effect of pre-hardening tests on transformation plasticity behavior with 16MND5 steel under tensile and compressive loading system. The specimens pre-deformed with 5% plastic strain before bainitic transformation plasticity show the TP strain occurred to opposite to the load direction at the time of transformation. The experimental results of pearlitic TP coefficient were different with tensile-compressive-torsional loading system [6], a new constitutive equation which depends on hydrostatic stress was proposed to consider the relationship between tensile and compressive experimental results.

Investigations to analyze the transformation plasticity have been carried out in terms of four-point bending system [12] and pearlitic TP coefficient was determined from the measured deflection. Nagaki et al. [13, 14] proposed a three-point bending system with simple supports and obtained TP coefficient of martensitic transformation [13, 14].

In previous studies, the TP coefficients in terms of three-point bending system with simple and fixed supports [15] and tensile-torsional-compressive system [6] were different with the same heat treating case for the same tested material of S45C steel. In uniaxial system, the TP coefficient depends on the applied stress. However in bending case, it is assumed that the TP coefficient is constant and is independent of applied stress. Moreover, bending stress causes tensile and compressive deformations at lower and upper parts of a beam, respectively and TP deformation will occur in tensile and compressive directions as well. Therefore, to reduce the effect of compressive stress on upper part and to increase
the effect of tensile stress on lower part of specimen, bending-tensile loading system as schematically shown in Fig.1 has been proposed. With this proposition, we could evaluate the effect of tensile stress on TP deflections and the pearlitic TP coefficients under different loading conditions have been determined.

2. Theoretical Approach

2.1 Transformation plasticity (TP)

The constitutive equation for total strain rate $\dot{\varepsilon}$ is given by the sum of elastic, plastic, thermal expansion, transformation expansion and TP strain rates [13].

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p + \dot{\varepsilon}^T + \dot{\varepsilon}^{ph} + \dot{\varepsilon}^{TP}$$ (1)

In this study, since the applied stress is smaller than yield stress, so it will not cause plastic deformation and the plastic strain rate $\dot{\varepsilon}^p$ is 0. It is assumed that the specimen is so small that the temperature distributed uniformly across the cross-section area during cooling. Therefore, the transformation expansion and thermal expansion don’t have effect on total deflection. The constitutive equation for TP strain rate is defined by Greenwood-Johnson [16] as:

$$\dot{\varepsilon}^{TP} = 2K(1 - \xi)\sigma\xi$$ (2)

where $K$ is the TP coefficient, $\xi$ is the volume fraction of generated phase and $\sigma$ is the applied stress. Substituting each component of the strain rates in Eq. (1) and solving for the stress rate, the equilibrium equation of the moment is given as:

$$\int_A \dot{\varepsilon}^T dA - 2K(1 - \xi)\int_A \sigma\eta dA = 0$$ (3)

where $\eta$ and $A$ are the distance from a neutral plane and the cross-sectional area of the beam, respectively.

2.2 Bending equation

The proposed model of bending-tensile loading system has been shown in Fig.1. The distances between fixed support and lower pin is $l$ equal to 120.0 mm, lower pin and upper pin is $a_1$ equal to 20.0 mm, upper pin to end of specimen is $a_2$ equal to 17.0 mm and the space between upper pin and lower pin is 1.1 mm. The reaction forces at fixed support is $R_A$, at lower pin is $R_B$, at upper pin is $R_C$, the moment at fixed support is $M_A$, bending load $W$ has been applied at point of $l/2$ and tensile load $P$ has been applied at free end of specimen. The bending moment along the beam can be calculated from the moment equilibrium as below equation.

$$M = -M_A + R_A x - W \left(1 - \frac{x}{l}\right) + R_B(x - l) - R_C(x - l - a_1)$$ (4)

where

$$\begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

According to the moment and force equilibrium equations, the moment, $M_A$ and reaction forces, $R_A$, $R_B$ and $R_C$ were determined as:

$$M_A = \frac{Wl(9l + 18a_1)}{24(3l + 4a_1)} + \frac{6E'\delta_c}{a_1(3l + 4a_1)}$$ (5)

$$R_A = \frac{W(6l + 11a_1)}{4(3l + 4a_1)} + \frac{18E'\delta_c}{a_1(3l + 4a_1)}$$ (6)

$$R_B = \frac{9Wl^2}{24a_1(3l + 4a_1)} + \frac{6l + 5a_1}{4(3l + 4a_1)}W - \frac{6E'\delta_c}{a_1(3l + 4a_1)} \left(\frac{2}{a_1} + \frac{3}{l}\right)$$ (7)

$$R_C = \frac{9Wl^2}{24a_1(3l + 4a_1)} - \frac{12E'\delta_c}{a_1(3l + 4a_1)}$$ (8)

where $E'$ is the Young’s modulus.

In above equation the effect of tensile load $P$ at horizontal direction has not considered and the bending moment has been calculated according to only vertical loads (bending load $W$ and reaction forces at fixed support and upper and lower pins). Here we assume that the deflection $y$, which is a function of the position $x$ and the time $t$ from loading start, is written as:

$$y(x, t) = \delta(t)g(x)$$ (9)

$$0 \leq g(x) \leq 1$$ (10)

$$\delta^e \leq \delta(t) \leq \delta_{max}$$ (11)

where $\delta^e$ and $\delta_{max}$ are elastic deflection and maximum deflection, respectively, and $g(x)$ can be considered as a function of dimensionless deflection. Then

$$\frac{d^2y}{dx^2} = \delta(t)\frac{d^2g(x)}{dx^2}$$ (12)

Using the Euler-Bernoulli bending theories for strain rate and beam curvature, integrating Eq. (3), where the integration of second terms became equal to the bending moment $M$, the moment equilibrium equation can be as:

$$l\frac{d\delta}{dt} \frac{d^2g(x)}{dx^2} + 2K(1 - \xi)\xi M = 0$$ (13)

where $l$ is the geometrical moment of inertia.

In bending deformation, the total strain can be expressed only by TP strain. Therefore, the deflection caused by the bending strain is due to transformation plasticity.

It is considered that single type of transformation occurs and assuming that Young’s modulus of each phase is equal. Finally by integrating the above equation from loading start (cooling start) time $t = 0$ to transformation end $t = t_0$, with consideration of $\delta = \delta^e$ and $\xi = 0$ at $t = 0$, and $\delta = \delta_{max}$ and $\xi = 1$ at $t = t_0$, the differential equation for dimensionless deflection function is obtained.

$$\frac{d^2g(x)}{dx^2} = -\frac{KM}{(\delta_{max} - \delta^e)l} = -\frac{KM}{\delta^e l}$$ (14)
where, $\delta_{TP} = \delta_{\text{max}} - \delta^e$ is TP deflection due to bending stress. Since, the Young's modulus and other parameters are independent of temperature. By introducing
$$E' = \frac{\delta_{\text{max}} - \delta^e}{\delta_{TP}} = \frac{\delta_{TP}}{K}$$
then Eq. (14) can be rewritten as:
$$\frac{d^2g(x)}{dx^2} = -\frac{M}{E'I}$$
Solving the differential Eq. (16), the deflection function $g(x)$ can be obtained as:
$$g(x) = \frac{1}{EI} \left\{ \frac{M_A}{2} x^2 - \frac{R_A}{6} x^3 + \frac{W}{6} (x - \frac{l}{2})^3 - \frac{R_B}{6} (x - l)^3 + \frac{R_C}{6} (x - l - a)^3 \right\}$$
where
$$\langle x \rangle = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Determining the reactions and moment of the beam, then solving Eq. (17) at loading point, the TP coefficient $K$ can be as:
$$K = \frac{192I(3l + 4a_1)}{W^3(3l + 7a_1)} \left( \delta_{TP\text{measure}} - \frac{3\delta_d^2}{8a_1(3l + 4a_1)} \right)$$

Therefore, given the beam shape and applied bending load, TP coefficient $K$ can be calculated by measuring the TP deflection $\delta_{TP\text{measure}}$ of bending loading point.

### 3. Experimental Method and Material

The experiments are performed under bending-tensile loading system. The specimen is fixed at one end and simply supported at the other, and subjected to bending and tensile loads. The bending stress is applied between the fixed support and lower pin and tensile stress is applied at the free end of specimen. The specimens are fabricated from S45C steel rods of volume 1.0×8.0×165.0 mm$^3$. The chemical compositions of the tested material (S45C) is shown in Table 1. Figure 2 shows the schematic view of bending-tensile loading system. Specimen is fixed on holders inside the furnace and bending load $W$ has been applied from top of the furnace and tensile load $P$ has been applied to one end of specimen.

The specimens were heated from room temperature to austenitization temperature of $830 \, ^\circ C$ in an electric infrared radiation furnace and then were held for 5 min at this temperature. After holding process, the specimen was naturally cooled to room temperature. When its temperature had reduced to $800 \, ^\circ C$ (before the transformation start point), each specimen was subjected simultaneously to its maximum bending stress and tensile stress. Different patterns of bending stresses (22.6, 28.4 and 34.2 MPa) have been applied. With each mentioned bending stresses different values of tensile stresses (0.0, 2.5, 5.0 and 10.0 MPa) have been applied. The deflection of bending loading point ($l/2$ from the supports) was measured with a laser sensor and the temperature was measured by type K thermocouples spot-welded to the lower surface of the specimens. In the temperature measurements, one thermocouple was fixed at the bending loading point of every specimen; two more thermocouples were placed ±40 mm from the bending loading point on one of specimens at each loading conditions. For precisely determining the specimen temperature, the Alumel and Chromel terminals were welded ±0.1 mm apart. To ensure accurate bending deflections, the experiments were repeated four times at each loading conditions.

### 4. Experimental Results and Discussion

The experiments were performed on S45C steel under different loading conditions as mentioned in previous section. The maximum deflections due to transformation plasticity at the bending loading point were measured. Since the specimens were naturally cooled, their phases transformed from austenite to pearlite in all experiments. Figure 3 shows the cooling path of S45C at the bending loading point and ±40.0 mm from bending loading point to simple and fixed supports of one sample on a continuous cooling transformation (CCT) diagram [17].

Figures 4–7 show the relationships between the TP deflection and cooling temperature for the tests which subjected to only bending loading condition and bending-tensile loading conditions, respectively (Bending stress = 22.6 MPa, Tensile stress = 0.0, 2.5, 5.0 and 10.0 MPa,

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Table 1 Chemical compositions of S45C (wt. %)

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cu</th>
<th>Ni</th>
<th>Cr</th>
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<tr>
<td>0.46</td>
<td>0.16</td>
<td>0.83</td>
<td>0.14</td>
<td>0.17</td>
<td>0.04</td>
<td>0.03</td>
<td>0.15</td>
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</table>
respectively). The horizontal axes show the temperature during cooling and vertical axes show the TP deflections at the bending loading point. During cooling when the specimen is subjected to a bending stress, elastic deflection occurs in the austenite phase and then phase transformation begins (at approximately 680 °C, pearlite start temperature) with a rapid increases in deflection and finish to the final deflection. Therefore, the plastic deflection due to the phase transformation is obtained by subtracting the elastic deflection from the final deflection. As previously mentioned, experiments were performed four times under each loading condition (Tests 01–04). Figures 4–7 show that the TP deflections of the quadruplicated tests almost match and decreased with increases in the value of tensile stress.

For other patterns of bending-tensile loading conditions (Bending stress = 28.4 and 34.2 MPa, Tensile stress = 0.0, 2.5, 5.0 and 10.0 MPa), the quadruplicated experiments of each loading conditions have close values of TP deflection, the average values have been plotted in Figs. 8 and 9. In these figures the horizontal axes represent the average values of cooling temperature and vertical axes represent the average of TP deflection of each loading case. In Figs. 4–9, comparing the TP deflections, with increases of tensile stress, the deflections reduced. Moreover, when the bending stress increased, the TP deflections increased as well.

The TP coefficients under the each loading condition were calculated by Eq. (18) and presented in Table 2. According to the experimental results, the TP coefficient decreased with decreasing TP deflection. The relationship between the TP coefficient K according to bending deformation and tensile stress σTens are plotted in Figs. 10–12 for all three patterns of bending loads.

Comparing the TP coefficients obtained in this study with tensile tests [6, 7], four-point bending tests [12] and three-point bending tests with simple and fixed supports [15], these results are found to be large with only bending stress and much small at bending-tensile cases.

The TP coefficients in terms of tensile and compression systems [6] were different. Since the applied bending stress cause tensile and compression stresses at lower and upper parts of the specimen, if the combined bending-tensile stresses are applied to the specimen, the tensile stress at lower part of the beam will be increased and the compressive stress at upper part will be decreased. When the tensile stress applied on the free end of specimens, it will cause negative bending moment between fixed support and lower pin or simple support, this moment caused reduction on TP deflection.

Furthermore, when the external stresses are applied on the specimens during cooling, the deformation occurs according to applied stress direction. It means that the applied bending and tensile loads will deform the specimens plastically in bending and tensile directions during phase transformation. In this study, the TP coefficient have been calculated according to only bending deformation and since the applied tensile stress at the free end of specimen have its own plastic deformation during phase transformation, therefore it is necessary to consider the effect of tensile stress at free end of specimens on TP coefficient. It need more works and investigations to consider the effect of both bending and tensile stresses and in future we will study these effects.

![Fig. 3 Cooling path plotted on CCT diagram](image)

**Fig. 3 Cooling path plotted on CCT diagram [17]**

M-martensite, Ms-martensite start temperature, P-pearlite, F-ferrite, B-bainite, Ac1-austenite formation start temperature and Ac3-austenite formation finish temperature

![Fig. 4 Relationship between measured transformation plastic deflection and cooling temperature](image)

**Fig. 4 Relationship between measured transformation plastic deflection and cooling temperature**

(Bending stress = 22.6 MPa, Tensile stress = 0.0 MPa)

![Fig. 5 Relationship between measured transformation plastic deflection and cooling temperature](image)

**Fig. 5 Relationship between measured transformation plastic deflection and cooling temperature**

(Bending stress = 22.6 MPa, Tensile stress = 2.5 MPa)
**Fig. 6** Relationship between measured transformation plastic deflection and cooling temperature (Bending stress = 22.6 MPa, Tensile stress = 5.0 MPa)

**Fig. 7** Relationship between measured transformation plastic deflection and cooling temperature (Bending stress = 22.6 MPa, Tensile stress = 5.0 MPa)

**Fig. 8** Relationship between measured transformation plastic deflection and cooling temperature (Bending stress = 28.4 MPa, Tensile stress = 0.0, 2.5, 5.0 and 10.0 MPa)

**Fig. 9** Relationship between measured transformation plastic deflection and cooling temperature (Bending stress = 34.2 MPa, Tensile stress = 0.0, 2.5, 5.0 and 10.0 MPa)

**Table 2** Pearlite transformation plasticity coefficients considering only bending deformation

<table>
<thead>
<tr>
<th>Applied bending stress, $\sigma_{\text{bend}}$ [MPa]</th>
<th>Applied tensile stress, $\sigma_{\text{tens}}$ [MPa]</th>
<th>TP coefficient, $K \times 10^{-5}$ [MPa$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.6</td>
<td>0.0</td>
<td>8.75</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>3.67</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td>0.61</td>
</tr>
<tr>
<td>28.4</td>
<td>0.0</td>
<td>8.59</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>3.78</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>1.99</td>
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<tr>
<td></td>
<td>10.0</td>
<td>0.90</td>
</tr>
<tr>
<td>34.2</td>
<td>0.0</td>
<td>8.10</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>3.71</td>
</tr>
<tr>
<td></td>
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<td>2.07</td>
</tr>
<tr>
<td></td>
<td>10.0</td>
<td>0.86</td>
</tr>
</tbody>
</table>

**Fig. 10** Relationship between transformation plasticity coefficient and applied tensile stress, Bending stress = 22.6 MPa