An Experimental-numerical Hybrid Method for Evaluating Thermal Strains from Measured Displacement Fields

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Abstract: An experimental-numerical hybrid method is proposed for obtaining reliable and accurate strains induced by temperature change. Strains obtained from measured displacement distributions are suffered from the measurement errors. Therefore, the measured displacements are used as the input data for determining boundary condition of a finite element model. Nodal forces by the temperature change at all nodes in the finite element model are identified from the measured displacements by the proposed method. Simultaneously, the reliable displacements and the strains are obtained. Effectiveness is validated by applying the proposed method to the displacement fields in dissimilar materials under thermal load obtained by digital image correlation. Results show that the nodal forces for a local finite element model can be obtained by the proposed method and subsequent strain analysis can be performed. It is expected that the proposed method can be powerful tool for stress and strain analysis of electronic devices under thermal load.

Keywords: Thermal Strain, Hybrid Method, Digital Image Correlation, Finite Element Method, Inverse Analysis

1. Introduction

In recent years, electronic packages have been miniaturized and highly integrated for weight saving and the improvement of the performance. Damages such as wire and solder debonding occur by thermal strains caused by the difference of thermal expansions of materials in a package. Therefore, the improvement of the structural reliability of packages is required. Thermal strains and stresses are usually evaluated through a measurement using strain gauges and a simulation using finite element analysis. However, with the miniaturization and the complication of structures, it is difficult to evaluate actual thermal deformations. Therefore, in order to ensure the structural reliability of electronic packages, the development of a quantitative evaluation technique of thermal stresses and strains is required. Because of the heterogeneity and the complication of electronic packages, a full-field and noncontact measurement method is suitable for measuring thermal strains [1].

Optical methods such as speckle interferometry, holographic interferometry, and digital image correlation are known as the full-field and noncontact methods for measuring surface deformation in solid mechanics [2-4]. Generally, these experimental techniques provide only the information of surface displacements. In other words, stresses and strains cannot be obtained directly by these methods. Therefore, it is required to differentiate displacement distributions to obtain stresses and strains. The derivatives of displacements at a point can be estimated from the values of displacements at the neighboring points. The use of a finite difference method, however, has the disadvantage that the errors in the measured values cause the greater errors in their derivatives. For this reason, various techniques have been proposed for the numerical differentiation in the calculation of in-plane strains from the displacements [5-10]. However, it is still difficult to eliminate the influence of the measurement errors on the computation of strains. Another difficulty of the numerical differentiation is the data treatment near boundaries and discontinuities. It is difficult to evaluate the strains at the boundary accurately because the measured values do not exist beyond the boundary. In addition, the measured values at the boundary frequently contain relatively large errors compared with those inside the boundary. Furthermore, the difficulty exits in the evaluation of the strains near the crack tip because the small local region for computing strains sometimes overlap crack faces that have opposite displacements.

On the other hand, the stress and strain distributions can be obtained by numerical methods such as a finite element method provided that an appropriate model is used and appropriate boundary conditions are given. However, because actual boundary conditions are not always known, reliable results are not always obtained by numerical methods. In order to obtain the accurate and reliable stresses and strains, the concept of hybrid method has been introduced, and various experimental-numerical hybrid methods have been proposed. For example, Weathers et al. [11], Morton et al. [12], Tsai and Morton [13], and Jayarama et al. [14] used the measured displacements obtained by moiré or speckle interferometry as the boundary condition for solving the finite element equation. Nishioka and coworkers [15] developed an intelligent hybrid method that can eliminate measurement errors. Fujikawa and Takashi [16] improved Nishioka's method for obtaining the smooth stresses near the boundaries. In these methods [11-16], the measured displacements at the boundaries of the analysis region are used as the boundary condition. In other words, the measured values inside the analysis region are not sufficiently utilized for obtaining stresses and strain even if full-field displacement distributions are obtained by optical methods. In order to obtain reliable and accurate stresses and strains and to take advantage of optical methods, one author [17] has proposed an experimental-numerical hybrid method that utilizes measured values inside the analysis region. In this method, tractions along the analysis region are inversely determined from the measured displacements inside that region using the method of least-squares. Then,
stresses and strains are obtained using finite element direct analysis by applying the computed tractions.

In the present study, an experimental-numerical hybrid method for thermal strain evaluation is proposed for obtaining reliable and accurate strain distributions from measured displacements. Displacement fields subjected to thermal load are measured using an optical method, digital image correlation. Nodal forces of a finite element model are inversely determined from the measured displacements using the method of least-squares. Then, strain distributions are obtained using finite element direct analysis by applying the identified nodal forces. Effectiveness is demonstrated by applying the proposed method to the displacement fields of a dissimilar materials specimen and the displacement fields of an electronic device. Results show that the nodal forces of a finite element model can be determined from the measured displacements and then strains can be obtained by the proposed method.

2. An Experimental-numerical Hybrid Method for Thermal Strain Analysis
It can be considered that the reasonably accurate stress and strain distributions are obtained by a finite element method when appropriate boundary conditions are given, provided that an appropriate finite element model is used and material properties are known. In thermal deformation problems, the boundary condition means the temperature distribution and the constraint. From the temperature distribution, nodal forces are determined and a finite element equation can be solved. In the proposed method, therefore, the nodal forces inside the analysis region are inversely determined from the measured displacements. Simultaneously, the reliable displacements and the strains are obtained. The stresses can also be evaluated if the temperature distribution is known.

Consider that a two-dimensional linearly elastic body is thermally loaded and in-plane displacements inside the analysis region are obtained using an optical method. The material properties, that is, the elastic modulus, Poisson's ratio, and the coefficient of thermal expansion are known. In a finite element method, the nodal force \( f_i \) by temperature change is obtained as

\[
f_i = \int_{V_i} \mathbf{B}^T \mathbf{D} \mathbf{e} \, dV,
\]

where \( \mathbf{B} \) represents the strain-displacement matrix, \( \mathbf{D} \) expresses stress-strain matrix, \( \mathbf{e} \) is the thermal expansion, and \( V \) is the volume of an object. The thermal expansion is related to the temperature change \( \Delta T \) as

\[
\mathbf{e} = \begin{bmatrix} \mathbf{e}_{x} \\ \mathbf{e}_{y} \\ \mathbf{e}_{xy} \end{bmatrix} = \begin{bmatrix} \alpha \Delta T \\ \alpha \Delta T \\ 0 \end{bmatrix},
\]

where \( \mathbf{e}_{x}, \mathbf{e}_{y}, \) and \( \mathbf{e}_{xy} \) are the components of thermal expansion and \( \alpha \) represent the coefficient of thermal expansion. As shown in Eqs. (1) and (2), the nodal forces at all nodes of a finite element model are determined from temperature change. Therefore, thermal strain analysis can be performed by determining the nodal forces. Figure 1 schematically shows a two-dimensional finite element model of the analysis region. The nodal forces at all nodal points except the nodes at which the fixed boundary condition is applied are determined from the measured displacements in this study. The displacements of some nodes are fixed so that the rigid body motion is not allowed. Then, a unit force along one direction of the coordinate system is applied to a node of the model. That is, the finite element analysis is performed under the boundary condition of the unit force at a point. The analysis is repeated by changing the direction of the unit force and the node at which the unit force is applied. The displacement components at a point \((x_i, y_i)\) for the applied unit force \( P_i = 1 \) \((j = 1~N)\) are represented as \( u'_i \) and \( v'_i \). Here, \( i = 1~M \) is the data index, \( j \) is the index of the applied force, \( M \) is the number of the data points, and \( N \) is the number of the forces to be determined at the nodes of the model. The displacement components \( u'_i \) and \( v'_i \) under the unit force can be considered as compliances that connect the force applied at a point and the displacements at another point. The displacement components \( u_i \) and \( v_i \) at the point \((x_i, y_i)\) under the actual nodal forces \( F_j \) \((j = 1~N)\) can be expressed using the principle of superposition as

\[
\begin{align*}
  u_i &= u'_i F_j \\
  v_i &= v'_i F_j 
\end{align*}
\]

where the summation convention is used. That is, for example,

\[
\begin{align*}
  u_i &= u'_i F_j \\
  v_i &= v'_i F_j \\
  &= \sum_{j=1}^{N} u'_i F_j \\
  &= u'_1 F_1 + u'_2 F_2 + \cdots + u'_M F_M 
\end{align*}
\]

In Eq. (3), \( u'_i \) and \( v'_i \) can be considered as compliances obtained by a finite element method, and \( u_i \) and \( v_i \) express the displacements obtained by an optical method. Equation (3) expresses linear equations in the unknown coefficients \( F_j \). For numerous data points, that is, if the number \( M \) of the data points is greater than the number \( N \) of the nodal forces, an overdetermined set of simultaneous equations is obtained. In this case, the nodal forces \( F_j \) at a point in the model can be estimated using linear least-squares as

\[
\mathbf{F} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{U},
\]

where \( \mathbf{F}, \mathbf{A}, \) and \( \mathbf{U} \) are the nodal force vector, the compliance matrix, and the measured displacements, respectively. They are expressed as
After determining the nodal forces, the strain components can be obtained from the strains under the unit force and the nodal forces. That is, the strain components \((\varepsilon_{ij})_0, (\varepsilon_{ij}), (\gamma_{ij}),\) at a point \((x_i, y_j)\) can be obtained as

\[
\begin{align*}
(\varepsilon_{ij}) & = (\varepsilon_{ij})_0 F_j \\
(\varepsilon_{ij}) & = (\varepsilon_{ij})_0 F_j \\
(\gamma_{ij}) & = (\gamma_{ij})_0 F_j
\end{align*}
\]

\[(i = 1 \sim M, j = 1 \sim N),\]

where \((\varepsilon_{ij})_0, (\varepsilon_{ij}), (\gamma_{ij}),\) express the strain components at a point \((x_i, y_j)\) for the applied unit force \(F_j = 1 (i = 1 \sim N).\)

The stress distributions can be calculated if the temperature distribution is known using the relation among the stresses, the strains, and the temperature. In the proposed method, the nodal forces are determined so that the displacement distributions obtained by finite element method can become identical with those obtained by measurement. In addition, least-squares method is used for reducing the influence of the measurement errors. Therefore, it is considered that the reliable and accurate results can be obtained by the proposed method.

3. Experimental Verification of the Proposed Method
A simple problem is analyzed to verify the proposed method.
unit force at a point on the boundary, the displacements at
some nodes must be fixed to prevent the rigid body motion.
In this study, the x and y components $u_x$ and $u_y$ of
the displacement at the point A and the y directional
displacement $u_y$ at the point B are assumed not to displace
though these points are displaced actually. This assumption
is valid because the rigid body translation and the rotation of
the analysis region do not affect the strain distribution. In
order to input the data into the algorithm by the proposed
method, the rigid body translation and the rotation are
excluded from the measured displacements so that the $x$ and
$y$ directional displacements at the point A and the $y$
displacement at the point B become zero. Then, the
displacements at all nodes relative to those at the point A
and B are obtained. The nodal forces at the other nodes are
obtained by the proposed method. The number of data
points is $M = 1686$. On the other hand, the number of the
nodes of the model is 389 and thus the number of the nodal
forces is 775 because the three displacement components at the points A and B are fixed.

The nodal forces are obtained by the proposed method
and the strains are then computed. Figures 7 and 8 show the
displacement distributions and the strains obtained using the
hybrid method. As shown in Fig. 7, the smooth displacement
distributions are obtained. These distributions are almost identical to the measured displacement
distribution in Fig. 5. That is, the nodal forces are
successfully obtained using the proposed procedure and then
the appropriate displacements are computed form the nodal
forces as shown. It is observed, however, that the strains in
Fig. 8 do not show the smooth distributions and is still
affected by the measurement error. In other words, the
proposed hybrid method cannot eliminate the measurement
error for calculating strains effectively. In the proposed
method, the nodal forces at all nodes of the model are
determined from the measured displacements. Therefore,
the nodal forces obtained by the proposed method give the
displacements that include the measurement errors.

In order to eliminate the effect of the measurement errors,
the nodal forces along the boundary and the interface are
determined by the proposed method assuming the
temperature change is uniformly distributed. In this case,
the number of the nodal forces determined is $N = 235$
Figure 9 shows the strain distributions obtained using the
hybrid method. Smooth strains distributions are obtained.
The values of the uniformly distributed strains apart from
the interface agree with the values calculated from the
coefficients of the thermal expansion and the temperature
change. It is also observed that the strains are concentrated
at the interface because of the difference of the coefficients
of the thermal expansion. The strain distributions show
fairly good agreement with those obtained using finite
element direct analysis in Fig. 10. As a result, the nodal
forces of the finite element model can be determined from
the measured displacements and subsequent strain analysis
can be performed by the proposed method.
4. Strain Analysis of Electronic Device

A cross-section of an electronic device is observed and the thermal strains on the cross-section are evaluated by the proposed method. The specimen is sliced from an actual device. The details of the specimen cannot be described here and thus they are omitted. Figure 11 shows the reference image of the specimen's cross-section. In this image, the length of 1 mm corresponds about 141 pixels. The random pattern is created by applying boron nitride and iron oxide powder. Figures 12 and 13 show the displacement and strain distributions at the temperature of \( T = 473 \text{ K} \) (\( \Delta T = 175 \text{ K} \)) obtained by the proposed method. High strain concentrations are observed at the bonding wires between the upper and lower parts. It is expected that the proposed method can be applied to the detailed study of the thermal strains in an electronic assemblies.

5. Conclusions

In this study, an experimental-numerical hybrid method for analyzing thermal strains is proposed. Nodal forces of a finite element model induced by temperature change are inversely determined from measured displacements inside the analysis region. Then, the strain distributions are obtained. Effectiveness of the proposed method is demonstrated by analyzing the strains of a bimaterial specimen and an electronic device. Results show that the nodal forces of the finite element model can be determined from the measured displacements and then the individual strains can be obtained by the proposed method.

References


