Experimental Study on the Influence of Frequency Ratio on Thrust and Vortex Structure of a Pitching Wing in a Periodic Flow

Yoshitaka ISODA¹, Yohsuke TANAKA² and Shigeru MURATA²
¹ Graduate School of Science and Technology, Kyoto Institute of Technology, Kyoto 606-8585, Japan
² Faculty of Mechanical Engineering, Kyoto Institute of Technology, Kyoto 606-8585, Japan

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Abstract: The purpose of this study is to investigate the influence of the frequency ratio on the thrust and the vortex street in a periodic flow. The periodic flow is the model of an unsteady freestream velocity, and the frequency ratio is defined as the frequency of pitching wing divided by that of the periodic flow. The thrust and the vortex street are measured by a load cell and particle image velocimetry (PIV), respectively. In this study, the pitching wing is modeled as a NACA 0012 airfoil pitching sinusoidally at a Reynolds number of 4000. We compare a steady with the periodic flow to find the difference in the thrust and the vortex street at frequency ratios of 1, 2, and 3. Furthermore, FFT analysis of the thrust is conducted to investigate the difference in more detail. It is found that the thrust and vortex street are more affected by periodic flow at the low-frequency ratio than at the high-frequency ratio.

Keywords: Pitching wing, Unsteady freestream velocity, Thrust, Frequency ratio

1. Introduction

In Japan, many port facilities such as quays and piers have been aging because they had been built during Japan’s period of high economic growth from the 1950s to 1970s [1]. Underwater robots have been used for maintenance and inspection of port facilities [1]. However, the flow in the harbor is an unsteady flow induced by the water wave [2]. The unsteady flow makes it difficult to control the position of robots. It is necessary to control the position stably in order to perform maintenance and inspection of port facilities.

We focus on a fish-like robot because fish has high maneuverability in the presence of disturbances [3–5]. Fish moves by generating a reverse Kármán vortex street shedding from the trailing edge of the caudal fin [6]. The rotation of all vortices is opposite of conventional Kármán vortex street. This leads to inducing the thrust instead of the drag. The caudal fin of fish is modeled as an oscillating NACA 0012 airfoil [7]. In previous studies, it is found that the thrust is affected by the vortex shedding in a steady flow [8]. Therefore, controlling the vortex shedding is one of the strategies to control the position of robot fish in the presence of disturbances.

It is important to study the thrust and the vortex street in an unsteady flow because that leads to the development of highly maneuverable robot fish. In a previous experimental study, Koochesfahani et al. investigate the influence of the frequency of a pitching wing on the thrust and the vortex structure in a steady flow [9, 10]. They find that as frequency increases, the vorticity and the thrust increase. In a theoretical study, the force induced by Kármán vortex street depends on the vortex circulation and the vortex spacing [11]. Thus, the thrust is affected by the vortex street structure in a steady flow.

In this study, we investigate the influence of the frequency ratio on the thrust and the vortex street in a periodic flow. The frequency ratio is defined as the frequency of the pitching wing divided by that of the periodic flow. The periodic flow is the model of unsteady flow in the harbor and changes sinusoidally in the mainstream direction. We compare the steady with the periodic flow, and the difference in the thrust and the vortex street is found. Furthermore, an FFT analysis is conducted to investigate the details of the difference between the steady and periodic flow.

2. Modeling

2.1 Periodic flow

Unsteady flow is modeled as the velocity oscillating sinusoidally in some previous studies [12, 13]. We model the unsteady flow in a harbor as a periodic flow, which changes sinusoidally in the freestream direction. The periodic flow is defined as

\[ u(t) = U_0 [1 + N_s \sin(2\pi f_p t)] \] (1)

where \( U_0 \) is the mean velocity, \( N_s \) is the amplitude ratio, and \( f_p \) is the frequency. The first term in Eq. (1) is a steady flow component induced by high and low tides. The second term is an unsteady component varying periodically induced by a water wave. The mean velocity of a periodic flow is the same as the freestream velocity in a steady flow. At low amplitude ratio, \( N_s \leq 1.0 \), the freestream velocity oscillates only in the forward direction, while the velocity oscillates in both forward and backward directions at high amplitude ratio, \( N_s > 1.0 \).

2.2 Pitching wing

A pitching wing is modeled as a NACA 0012 airfoil pivoted by about 1/4 chord point. The pitching motion of the airfoil is described by sinusoidal motion as

\[ \theta(t) = \theta_0 \sin(2\pi f_w t) \] (2)

where \( \theta_0 \) is the pitching amplitude and \( f_w \) is the pitching frequency.
2.3 Non-dimensional parameters
The thrust of the pitching wing in a periodic flow is governed by three non-dimensional parameters. The parameters are the Reynolds number, $Re$, the Strouhal number of a periodic flow, $St_p$, and the Strouhal number of the pitching wing, $St_w$, defined as

$$Re = \frac{c U_0}{\nu}$$

$$St_p = \frac{f_p c}{U_0}$$

$$St_w = \frac{f_w c}{U_0}$$

where $c$ is the chord length, and $\rho$ and $\nu$ are the fluid density and kinematic viscosity, respectively. Additionally, we introduce the non-dimensional parameter of frequency ratio, $N_f$, combined with Eqs. (4) and (5) defined as following:

$$N_f = \frac{St_w}{St_p} = \frac{f_w}{f_p}$$

$N_f$ is the ratio of the frequency of the periodic flow to the pitching frequency. For a given pitching frequency, the ratio increases with decreasing frequency of the periodic flow, and the steady flow can be seen as a special case of the periodic flow at $N_f = \infty$. Generally, the ratio $N_f$ is more than 1.0 because the significant wave period in harbors is in the range of 4.0 to 10.0 seconds [14].

3. Experimental Setup
The experiment is conducted using devices shown in Figs. 1 and 2. This is composed of a water tunnel, a load cell, an optical device for PIV, and a pitching wing.

The water tunnel generates steady and periodic flow defined in Eq. (1). The water tunnel has a test section 1000 mm long, 300 mm wide, and 400 mm high. The periodic flow is generated using two pumps (MX-401, Iwaki) driven by an inverter (VF-nC3, Toshiba) connected to a function generator (SG-4115, Iwatsu). Table 1 describes water tunnel conditions. The amplitude ratio is set to 0.42 in order to consider the case where the direction of a vortex advection is the positive $x$-direction. A motion of the pitching wing is developed using a NACA 0012 airfoil driven by a stepper motor (AZ46AA-PS10-0, Oriental Motor) shown in Fig. 2(b). The stepper motor is supported by the load cell. Table 2 shows the pitching wing conditions.

The optical device for PIV is used to measure the vortex street structure of the pitching wing. A green laser generator (DPGL-2W, Japan Laser) is used for flow visualization, and two full mirrors reflect a laser sheet to visualize flow behind the airfoil. A high-speed camera (Fastcam Mini UX-100, Photron) records particle images through a full mirror under the conditions listed in Table 3.

A load cell is used to measure the thrust acting on the pitching wing. Strain gauges (KSPB-2-120-E4, Kyowa Electronic Instruments) attached to the load cell measure the bending strain by the thrust. The thrust is estimated using the bending strain. It has been checked that the wall has little effect on the measured force in the previous study [15].

### Table 1 Water tunnel conditions

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean velocity</td>
<td>$U_0$</td>
<td>[mm/s]</td>
<td>38</td>
</tr>
<tr>
<td>Amplitude ratio</td>
<td>$N_s$</td>
<td>[-]</td>
<td>0.42</td>
</tr>
<tr>
<td>Frequency</td>
<td>$f_p$</td>
<td>[Hz]</td>
<td>0.25</td>
</tr>
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</table>

### Table 2 Pitching wing conditions

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>Chord length</td>
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<td>[m]</td>
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</tr>
<tr>
<td>Span</td>
<td>$b$</td>
<td>[m]</td>
<td>0.38</td>
</tr>
<tr>
<td>Frequency</td>
<td>$f_a$</td>
<td>[Hz]</td>
<td>0.25, 0.50, 0.75</td>
</tr>
<tr>
<td>Pitching amplitude</td>
<td>$\theta_0$</td>
<td>[deg]</td>
<td>6</td>
</tr>
<tr>
<td>Volume of airfoil</td>
<td>$V_c$</td>
<td>[m$^3$]</td>
<td>$3.11 \times 10^4$</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>$Re = \frac{c U_0}{\nu}$</td>
<td>[-]</td>
<td>4000</td>
</tr>
<tr>
<td>Strouhal number of $f_p$</td>
<td>$St_p = f_p c/U_0$</td>
<td>[-]</td>
<td>0.66</td>
</tr>
<tr>
<td>Strouhal number of $f_w$</td>
<td>$St_w = f_w c/U_0$</td>
<td>[-]</td>
<td>0.66, 1.32, 1.97</td>
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</table>

### Table 3 Two-dimensional PIV conditions

<table>
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<tr>
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<th>Unit</th>
<th>Value</th>
</tr>
</thead>
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<td>Frame rate</td>
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</tr>
<tr>
<td>Spatial resolution</td>
<td>[mm/pixel]</td>
<td>0.318</td>
</tr>
<tr>
<td>Image size</td>
<td>[pixel]</td>
<td>1280x800</td>
</tr>
<tr>
<td>Interrogation window</td>
<td>[pixel]</td>
<td>32x32</td>
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<tr>
<td>Over lap</td>
<td>[%]</td>
<td>50</td>
</tr>
<tr>
<td>Grid space</td>
<td>[mm]</td>
<td>5</td>
</tr>
</tbody>
</table>

Fig. 1 Water tunnel for observing pitching wing in steady and periodic flow

(a) Top view (b) Side view

Fig. 2 Measurement device for thrust force, optical device for PIV, and stepper motor for NACA 0012 airfoil pitching sinusoidally
4. Result and Discussion

4.1 Vortex street structure

The vortex wake is visualized by the $Q$-criterion to investigate the difference in the vortex wake structure between steady and periodic flow qualitatively for changing frequency ratio. The $Q$ is the second invariant of velocity tensor [16]. Positive values of $Q$ are used to identify a flow dominated by vorticity, while the negative values of $Q$ are associated with a flow in which the strain rate is large [17].

Figures 3 and 4 show the vortex street in the wake of the pitching wing in steady and periodic flow, respectively. The $Q$-criterion of the second vortex from the trailing edge in a periodic flow is higher than that in a steady flow in Figs. 3(a) and 4(a). Furthermore, vortex positions in the periodic flow differ from those in the steady flow. In contrast, there are no noticeable differences in the $Q$-criterion and the positions of the three vortices closest to the trailing edge of the NACA 0012 airfoil in Figs. 3(c) and 4(c). To sum up, the vortex street structure is more affected by a periodic flow at the low-frequency ratio than at the high-frequency ratio as shown in Figs. 3 and 4.

4.2 Thrust

The pitching frequency is changed from 0.25 to 0.75 in order to investigate the influence of the frequency ratio on the thrust in a periodic flow.

Figure 5 shows the freestream velocity, $u$, the added mass force, $F_{am}$, and the thrust, $F_T$. The added mass force is the additional force required to accelerate the surrounding fluid when the airfoil accelerates. The added mass is calculated by Eq. (7) including the effect of freestream oscillation [18].

$$F_{am} = - \left( \rho M b + \rho V_w \right) \frac{du}{dt} \tag{7}$$

$M$ denotes the added mass coefficient of NACA 0012 airfoil, and $b$ and $V_w$ denote the airfoil span and volume, respectively. We use the added mass coefficient of $M = 1.038 \times 10^{-4}$ m$^2$ in the case of $c = 0.1$ m [19]. The second term in Eq. (7) comes from the variable pressure gradient caused by the periodic flow.

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![Fig. 3 Vortex street visualized by Q-criterion in steady flow for changing pitching frequency](image1)

![Fig. 4 Vortex street visualized by Q-criterion in periodic flow for changing pitching frequency](image2)
The mean thrust and amplitude increase with increasing pitching frequency under conditions of both steady and periodic flow. The added mass force \( F_{am} \) is the drag in the range of freestream acceleration and the thrust in the range of freestream deceleration. This is because there are negative and positive pressure gradient comes from the flow oscillation around the pitching wing. Due to the added mass force \( F_{am} \) in the periodic flow, the thrust \( F_T \) is large in the range of freestream deceleration, while it is small in the range of freestream acceleration at all frequency ratios.

We calculate the cross-correlation coefficient of the thrust between steady and periodic flow to investigate the difference between steady and periodic flow. Figure 6 shows the relationship between the frequency ratio and the cross-correlation coefficient of thrust. The cross-correlation coefficient of 1.0 means that the difference in thrust is little. The cross-correlation coefficient tends to increase with increasing frequency ratio. This indicates that the thrust is more affected by a periodic flow at the low-frequency ratio than at the high-frequency ratio. The quantitative analysis of the vortex street in Figs. 3 and 4 should be investigated to reveal the relationship between the cross-correlation coefficient of thrust and the vortex street in future work.

![Fig. 5 Freestream velocity, \( u \), added mass force, \( F_{am} \), and thrust, \( F_T \) in steady and periodic flow for changing frequency ratio. Velocity is constant in steady flow. In contrast, velocity changes sinusoidally in periodic flow.](image)

![Fig. 6 Cross-correlation coefficient of thrust between steady and periodic flow.](image)

We conduct an FFT analysis to reveal the difference between a steady and periodic flow in more detail. Figure 7 shows the thrust spectrum in a steady on the left and periodic on the right. There is only one peak in a steady flow. The frequency of the peak is the same as the frequency of the vortex shedding, \( 2f_w \).

In contrast, there is an additional peak at the frequency of 0.25 Hz in a periodic flow. This comes from the added mass force \( F_{am} \) in Fig. 5 because the frequency of the added mass is also 0.25 Hz. The height of the peak at frequency \( 2f_w \) is smaller than that at the frequency \( f_p \) at the frequency ratio of \( N_f = 1 \). Conversely, the height of the peak at frequency \( 2f_w \) is larger than that at the frequency \( f_p \) at the frequency ratio of \( N_f = 3 \). Therefore, as the frequency ratio increases, the force by the vortex shedding is more dominant. Hence the cross-correlation coefficient increases with increasing frequency ratio.

In periodic flow, the peak of the vortex shedding at the frequency of \( 2f_w \) is almost the same as that in a steady flow. However, there are side-band peaks at frequencies of \( 2f_w \pm f_p \) at all frequency ratios due to non-linear interaction between the pitching wing and the periodic flow. The peak of the vortex shedding tends to dominate over side-band peaks with increasing frequency ratio. Therefore, side-band peaks have little effect on the maneuverability of fish-like robots at the high-frequency ratio.
Fig. 7 Thrust spectrum in steady and periodic flow. Frequency of peak in steady is the same as frequency of vortex shedding.

5. Conclusions
The purpose of this study is to investigate the influence of the frequency ratio between a pitching wing and a periodic flow on the thrust and the vortex street. First, as the frequency ratio increases, the difference in the vortex street becomes smaller. Second, it is found that the influence of a periodic flow on thrust is relatively small at the high-frequency ratio. This is explained using an FFT spectrum, and the peak of vortex shedding is larger than that of the periodic flow at the high-frequency ratio. Thus, underwater robots can maneuver as intended at the high-frequency ratio.

Nomenclature
\( b \) span [m]
\( c \) chord length [m]
\( F_T \) thrust [N]
\( F_{am} \) added mass force [N]
\( f_p \) frequency of periodic flow [Hz]
\( f_w \) frequency of pitching wing [Hz]
\( M \) added mass coefficient \([m^2]\)
\( N_a \) amplitude ratio [-]
\( N_f \) frequency ratio [-]
\( Q \) Q-criterion \([1/s^2]\)
\( Re \) Reynolds number [-]
\( St_p \) Strouhal number of periodic flow [-]
\( St_w \) Strouhal number of pitching wing [-]
\( T \) time period [s]
\( u \) freestream velocity \([mm/s]\)
\( U_0 \) mean velocity \([mm/s]\)
\( V_w \) volume of airfoil \([m^3]\)
\( \theta \) pitching angle [rad]
\( \theta_0 \) pitching amplitude [rad]
\( \nu \) kinematic viscosity \([m^2/s]\)
\( \rho \) density \([kg/m^3]\)

Subscripts
\( 0 \) mean
\( a \) amplitude
\( am \) added mass
\( p \) periodic flow
\( w \) wing

References


