Investigation of Wind-induced Behavior and Aerodynamic Stability of a Long-span Flat Roof Based on Wind Tunnel Experiment and FSI Simulation with Free-vibration Technique

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Abstract: The present paper discusses the wind-induced vibration of a long-span flat roof based on a wind tunnel experiment and a fluid-structure interaction (FSI) simulation with the free-vibration technique. The experiment and simulation were carried out in a smooth uniform flow and a turbulent boundary layer to evaluate the effect of turbulence on the dynamic response. The variation of mean and standard deviation of the roof displacement with wind velocity was obtained. In the experiment, the similarity between the practical membrane structure at full scale and the model used in the wind tunnel experiment was not satisfied because of the limitation of experimental conditions, such as the model material. Subsequently, an FSI simulation that couples a computational fluid dynamics (CFD) analysis for fluid with a finite element method analysis for structure was carried out to examine the practical behavior of a long-span flat roof in strong winds. In addition, the modes and natural frequencies of vibration at each wind velocity are detected by using a proper orthogonal decomposition analysis. The results indicate that the aerodynamically unstable vibration does not occur under the condition proposed by Matsumoto (1983). It is found that the mean deformation is one of the most important factors for assessing the vibration mechanism of long-span flat roofs.

Keywords: Long-span flat roof, Dynamic response, Wind tunnel experiment, CFD, FSI, Aerodynamic stability

1. Introduction

In recent years, many long-span structures have been constructed all over the world. In many cases, the roofs are made of membrane. Because such roofs are light and flexible, the wind-induced dynamic response is one of the most important phenomena in the structural design. Furthermore, the dynamic response of the roof is affected not only by approach flow and separated flow turbulence, but also by unsteady aerodynamic forces induced by the roof vibration. Therefore, the wind forces and wind-induced responses of long-span roofs become very complicated.

In previous studies, many researchers have investigated the aerodynamic stability and wind-induced vibrations of long-span roofs. Two methods were used generally for investigating these subjects; i.e., ‘free-vibration technique’ and ‘forced-vibration technique’. The free-vibration test focuses on the relationship between roof displacement and wind velocity. Uematsu and Uchiyama [1] investigated the wind-induced dynamic behavior of a long-span suspended roof in a smooth uniform flow. Matsumoto [2] investigated the aerodynamic stability of a long-span suspended roof and proposed a criterion for evaluating the critical wind velocity causing an aerodynamically unstable vibration. Miyake et al. [3] investigated the aeroelastic response of a suspension roof in a smooth uniform flow and a turbulent boundary layer. In the forced-vibration test, on the other hand, focus is on the aerodynamic forces, i.e., the aerodynamic stiffness and damping denoted as the in-phase and out-of-phase components of the aerodynamic force with respect to the roof’s displacement, respectively. Daw and Davenport [4] investigated the aerodynamic characteristics of a semi-circular roof. Ohkuma and Marukawa [5] investigated the basic characteristics of aerodynamic forces and the aerodynamic stability of a long-span flat roof based on the variation of aerodynamic damping with wind velocity. Ding et al. [6] investigated the aerodynamic stability of a cylindrical roof based on a wind tunnel experiment and a computational fluid dynamics (CFD) simulation. Li et al. [7] used a CFD simulation to examine the results of the previous wind tunnel experiments conducted by Ohkuma and Marukawa [5] and Ding et al. [6]. Note that the vibration mode used in these forced-vibration tests is the first anti-symmetric mode.

As mentioned above, the stability of long-span roofs has been discussed by many researchers. However, the mechanism of wind-induced vibration has not been made clear yet. Furthermore, the effects of mean deformation and vibration of the roof on the wind pressures have not been investigated in detail.

In the present study, the wind-induced behavior and aerodynamic stability of a long-span flat roof are investigated based on a wind tunnel experiment and a fluid-structure interaction (FSI) simulation with the free-vibration technique. In the wind tunnel experiment, the displacement of a roof model is measured by three laser displacement sensors installed at the locations of the node and anti-nodes of the first anti-symmetric mode in a smooth uniform flow and a turbulent boundary layer. The basic characteristics of roof vibrations are first presented. Subsequently, the FSI simulation is performed to get further understanding of the dynamic behavior of the roof. Focus is on the roof vibration as well as on the distribution of wind pressure coefficients on the vibrating roof. Finally, the aerodynamic stability of the flat roof is discussed.

2. Experimental Apparatus and Procedure

2.1 Wind tunnel flow

The wind tunnel experiment was carried out in an Eiffel-type boundary-layer wind tunnel at Tohoku Institute of Technology, which has a working section 1.0 m high, 1.0 m wide, and 9.0 m long. A smooth uniform flow and a
turbulent boundary layer were used to understand the effect of approach flow turbulence on the dynamic response of the roof; these flows are respectively called ‘smooth flow’ (Smooth) and ‘turbulent flow’ (TBL), hereafter. Figure 1(a) shows the vertical profiles of mean wind velocity $U$ (circles) and turbulence intensity $I_U$ (triangles) in the longitudinal direction at the location of model center without model, in which $Z$ represents the height above the wind tunnel floor. The solid lines represent the specifications in the AIJ Recommendations for Loads on Buildings [8]. The terrain category is assumed ‘III’ (suburban exposure). The mean wind velocity profile is represented by a power law and the exponent $\alpha$ is specified as 0.2 for suburban exposure. The mean wind velocity $U_{ref}$ at a reference height ($Z_{ref} = 600$ mm) is varied from approximately 3 to 13.5 m/s. In the turbulent flow case, the turbulence intensity at $Z > 150$ mm is smaller than that specified in the AIJ Recommendations [8]. However, the values at $Z = 20 – 100$ mm agree well with the specification. Figure 1(b) shows the power spectral density of fluctuating wind velocity at $Z = 100$ mm, where $L_\alpha$ represents the integral scale of turbulence in the longitudinal direction, which agrees well with the Karman type spectrum.

In ordinary wind tunnel experiments of building aerodynamics, the boundary layer thickness is seldom used as a characteristic length when discussing the similarity of wind tunnel experiment. For a low-rise building, the similarity of $L_\alpha$ in the wind tunnel experiment is less important, while the turbulence intensity $I_{\text{ref}}$ at the roof height $H$ affects the wind pressures on the roof significantly (e.g. Tieleman et al. [9, 10]). In the present study, the values of $I_{\text{ref}}$ (where $H = 100$ mm, as will be shown later) agrees well with the specified value in the AIJ Recommendations, as Fig. 1(a) shows.

2.2 Building model and displacement measurement
The span $L$ and eaves height $H$ of a prototype building are assumed 120 m and 20 m, respectively. These values are determined based on a survey of practical membrane structures constructed in Japan [11]. The geometric scale of the wind tunnel model is assumed 1/200. Figure 2 shows the experimental setup. The span, height and width of the model are 600, 200 and 300 mm, respectively. In the smooth flow case, the model was set on a flat plate placed at a height of 250 mm above the wind tunnel floor, as shown in Fig. 2(a), because a boundary layer of approximately 150 mm thickness developed along the wind tunnel floor. A pair of side walls, 1200 mm wide and 800 mm high, were used to make the flow around the model two-dimensional. The leading edge of the side wall was sharpened so that the flow would not separate at the edge. The side walls were placed approximately 2 mm apart from the side edges of the roof model not to interfere with the vibration of roof. The model roof was made of a rubber sheet of 0.5 mm thickness. The density per unit area and the Young’s modulus of the sheet are approximately 4.0 kg/m² and $3.8 \times 10^5$ N/m², respectively. The displacements of the model roof were measured by three laser displacement
sensors installed at the location of the node and anti-nodes of the first anti-symmetric mode (Points A to C) along the centerline, as shown in Fig. 3(a). The sampling frequency was 100 Hz.

The measurements in the smooth and turbulent flows were carried out three times under the same condition. Before the measurements in each flow, the model was reconstructed, resulting in a slight change in the natural frequencies of the model. The natural frequencies of each model were measured in still air three times by using the free-vibration records obtained after tapping the roof at Point A or B. Figure 4(a) shows an example of the time history of displacement at Point B, from which the critical damping ratio of vibration was estimated at approximately 2%. Figure 4(b) shows examples of the power spectral densities of roof vibrations at Points A (#1 and #2) and B (#3 and #4). The spectrum has peaks at the first and second natural frequencies, which correspond to the first symmetric and anti-symmetric vibration modes, respectively. Table 1 shows the natural frequencies of the wind tunnel models, which were measured three times. The first and second natural frequencies ranged from 5.3 to 7.2 Hz and from 8.9 to 13.0 Hz, respectively.

2.3 Mean and RMS displacements

Figure 5 shows the variation of mean ($\bar{w}$) and standard deviation ($\sigma_w$) of the displacement with increasing wind velocity $U_{Hi}$ at the eaves height $H$. Here, the displacement is normalized by the span $L$. The results plotted in this figure are the averaged values of three runs to show the general relationship between roof displacement and wind velocity. The values of $\bar{w}$ and $\sigma_w$ gradually increase with increasing $U_{Hi}$. The magnitude of $\bar{w}$ at Point A is larger than that at Point C because of larger suction induced by the flow separation at the leading edge. The reduced mean displacement $\bar{w}/L$ in the smooth flow is generally larger than that in the turbulent flow. This is because relatively large suction caused by the flow separation at the leading edge act on the roof over a larger area in the smooth flow than in the turbulent flow [12, 13]. By contrast, the magnitude of $\sigma_w/L$ in the smooth flow is generally smaller than that in the turbulent flow. This is because larger dynamic responses are induced by higher turbulence of the approach flow and the separated flow in the turbulent flow case. The magnitude of $\sigma_w/L$ is almost constant in the smooth flow when $U_{Hi} > 7 \text{ m/s}$. It is thought that the upward deformation of the roof changes the flow around the roof, which weakens the separation vortices shed from the leading edge. The magnitude of $\sigma_w/L$ is the largest at Point A among the three points when the wind velocity is relatively small. However, the magnitude of

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**Figure 5** Displacement vs. wind velocity

(a) Point A  (b) Point B  (c) Point C

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**Figure 6** Displacement vs. non-dimensional wind velocity

(a) Mean                  (b) Standard deviation

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**Figure 7** Power spectral density of the roof’s displacement in the smooth flow

(a) Point A  (b) Point B  (c) Point C

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**Figure 8** Power spectral density of the roof’s displacement in the turbulent flow

(a) Point A  (b) Point B  (c) Point C
$\sigma_w/L$ is the largest at Point B when $U_{\text{ref}} > 5.5$ m/s in the smooth flow and when $U_{\text{ref}} > 9$ m/s in the turbulent flow. When the wind velocity is relatively small, the displacement is mainly affected by the separated flow. At higher wind velocities, on the other hand, the distribution of wind pressures on the roof is affected by the mean deformation of the roof, which affects the values of $\sigma_w/L$ significantly. The relation between wind pressure and mean wind velocity deviation of displacement with increasing non-dimensional $\sigma_w/L$ significantly. The relation between wind pressure and mean deformation can be investigated in detail by using an FSI simulation, which will be carried out in the next section.

Figure 6 shows the variation of the mean and standard deviation of displacement with increasing non-dimensional wind velocity $U_{\text{ref}}f_1/L$, where $f_1$ is the natural frequency of the first vibration mode. Note that the results of all runs are plotted in the figure to understand the dispersion of the data. It can be seen that the value of $\sigma_w/L$ decreases with an increase in $U_{\text{ref}}f_1/L$ when $U_{\text{ref}}f_1/L > 2.5$ in some cases. This is probably because the displacement of the roof exceeded the measurement limit of the sensor. According to Matsumoto [2], who investigated the aerodynamic stability of a suspended roof, an aerodynamically unstable vibration may occur when $U_{\text{ref}}f_1/L > 1.1$, where $f_1$ is the natural frequency of the first anti-symmetric mode. In the present experiment, however, such an unstable vibration did not occur. The vibration amplitude gradually increased with increasing non-dimensional wind velocity, as shown in Fig. 6.

To investigate the dynamic response of the roof in more detail, the power spectral density of the roof displacement is examined. Figures 7 and 8 show the power spectral densities of the roof displacements at Points A to C in the smooth and turbulent flows. The spectral densities at Points A and B have peaks at almost the same frequency. By contrast, the spectrum at Point C has a peak at a somewhat higher frequency. The spectral peak frequency slightly increases with increasing wind velocity in all cases. It does not coincide with the natural frequency of the roof. It is thought that the natural frequency changed with the deformation of the roof. Furthermore, the flow field around the roof was also changed by the deformation. We will investigate these effects in more detail in a future study. The experimental results obtained here imply that aerodynamically unstable vibration does not occur in such a case as tested in the present study.

### 3. Fluid-Structure Interaction Simulation

#### 3.1 Computational conditions

To investigate the dynamic response of long-span roofs in more detail, an FSI simulation is carried out. This simulation is based on a CFD analysis for flow and a finite element method (FEM) analysis for structure, which are coupled with each other at each time. The coupling simulation is carried out by using a commercial CFD code, ANSYS Fluent (Ver. 17.2). In our previous studies [12, 13], the requirement for mesh division was made clear. Furthermore, the forced-vibration technique was used to simulate the unsteady aerodynamic forces acting on the vibrating roof. The geometric scale was assumed 1/400. The same mesh and computational conditions are used in the present study. That is, the simulation is carried out at an experimental scale. Table 2 summarizes the computational conditions for CFD analysis. A spring-based smoothing technique by using Hooke’s law is applied to the dynamic mesh. The interior nodes of the mesh move and absorb the movement of the boundary. Table 3 shows the computational conditions for the FEM analysis. Some of the parameters used in the analysis are determined based on the similarity with typical long-span membrane roofs constructed in Japan. The flexural rigidity is assumed zero because the roof material is membrane.

Figure 9(a) shows the wind velocity profiles of the smooth and turbulent flows at the location of the model center without model. The inflow turbulence is generated by the spectral synthesizer installed in ANSYS Fluent. The inflow is tuned so that the velocity profile at the model center agrees with that for Terrain Category III (suburban exposure) specified in the AIJ Recommendations [8]. The mean wind velocity at the eaves height is varied from 6 to 15 m/s with an increment of 3 m/s. Fig. 9(b) shows the power spectral density of fluctuating wind velocity at the eaves height, compared with the Karman type spectrum.

![Table 2 Computational conditions for CFD analysis](image)

<table>
<thead>
<tr>
<th>Computational domain</th>
<th>9.5 ( L(x) \times 0.7 \ L(y) \times 2.5 \ L(z) )</th>
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<tr>
<td>Inflow conditions</td>
<td>Smooth and turbulent flows</td>
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<td>Upper and side walls</td>
<td>Slip condition</td>
</tr>
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<td>Floor and model surface</td>
<td>No-slip condition</td>
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<td>Second order central differential scheme</td>
</tr>
<tr>
<td>Temporal discretization</td>
<td>Second order backward time differential scheme</td>
</tr>
<tr>
<td>Pressure-velocity coupling</td>
<td>PISO algorithm</td>
</tr>
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<td>Non-dimensional time step ( t' (= tU_{\text{ref}}/L) )</td>
<td>4.0×10^{-3}</td>
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![Table 3 Computational conditions for FEM analysis](image)

<table>
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<th>Number of mesh elements</th>
<th>2,400 (x × y × z: 600×400×1)</th>
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<td>Weight per unit area</td>
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<tr>
<td>Young’s modulus</td>
<td>6.56×10^{10} N/m²</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.32</td>
</tr>
<tr>
<td>Damping parameter</td>
<td>1% (Rayleigh damping)</td>
</tr>
</tbody>
</table>

![Fig. 9 Wind velocity profile and power spectral density](image)
3.2 Comparison with the experimental results

To verify the FSI simulation method used here, the above-mentioned wind tunnel experiment was first simulated, and the computed results were compared with the experimental ones. Figure 10 shows the $w_{\text{mean}}$ and $\sigma_w$ values at Points A to C plotted against the non-dimensional wind velocity. Note that the computational conditions and the model properties are determined based on the wind tunnel experiment. In the figures, the results of the FSI simulations with and without considering the flexural rigidity of the roof are plotted by the black and gray symbols, respectively. The simulation results for $\sigma_w/L$ in the smooth flow are in fairly good agreement with the experimental ones. Regarding the results in the turbulent flow, the agreement is good at Point A. However, the values at Points B and C obtained from the FSI simulation are somewhat larger than those obtained from the wind tunnel experiment. This may be due to the difference in the inflow condition between simulation and experiment. As a result, it can be concluded that the present FSI analysis captures the dynamic behavior of the roof appropriately. Furthermore, it is found that the flexural rigidity of the roof does not affect the dynamic response significantly.

3.3 Distribution of wind pressure coefficients on the roof

Figure 11 shows the distributions of the mean and RMS fluctuating wind pressure coefficients, $C_{\text{p,mean}}$ and $C_{\text{p,}'},$ on the roof at various wind velocities in the smooth flow, in which ‘$s$’ represents the horizontal distance from the leading edge in the windward direction and ‘$s_{\max}$’ the maximum value of $s,$ or the span of the roof ($L$). The distributions on a rigid (undeformed) flat roof are also plotted in the figure for a comparative purpose. The distributions of $C_{\text{p,mean}}$ and $C_{\text{p,}'}$ on the rigid roof are significantly different in both magnitude and shape from those on the flexible roof. Larger suction act on the flexible roof near the windward edge. Furthermore, the magnitude of negative $C_{\text{p,mean}}$ values rapidly decreases leeward in a range of $0 < s/s_{\max} < 0.5.$ As the wind velocity increases, the magnitude of $C_{\text{p,mean}}$ decreases in the windward area, while increases in the central and leeward areas. The magnitude of $C_{\text{p,}'}$ on the flexible roof is fairly small compared with that on the rigid roof. This may be due to the effect of upward mean deformation of the roof on the strength of vortices shed from the leading edge, as will be mentioned later.

Figure 12 shows the distributions of $C_{\text{p,mean}}$ and $C_{\text{p,}'}$ on the roof in the turbulent flow. Again, the results for the rigid roof is also shown in the figure. The $C_{\text{p,mean}}$ values in the windward area on the flexible roof is generally smaller than that on the rigid roof. The $C_{\text{p,}'}$ value on the flexible roof is generally smaller than that on the rigid roof. The $C_{\text{p,}'}$ value in the turbulent flow is generally larger than that in the smooth flow. This feature indicates that the turbulence of approach flow significantly affects the fluctuating wind pressures on the roof.

3.4 Roof's displacement

Figures 13 and 14 show the distributions of $w_{\text{mean}}$ (positive upward) and $\sigma_w$ of the roof in the smooth and turbulent flows. These values are normalized by $L.$ The magnitude of

![Fig. 10 Plots of $\sigma_w/L$ vs. non-dimensional wind velocity $U_{ref}/L$ : comparison between FSI simulation and experiment](image-url)
$w_{\text{mean}}$ generally increases as the wind velocity increases. The mean displacements are generally larger in the smooth flow than in the turbulent flow, except when $U_{\text{ref}} = 15 \text{ m/s}$. This result may be related to the size of the separation bubble. As shown in Figs. 13 and 14, the deformed shape is similar to a cylindrical roof. The flow separation at the leading edge may be affected by the roof deformation significantly. The upward deformation of the roof accelerates the reattachment of the separated flow to the roof surface. As a result, the magnitude of $C_p'$ becomes smaller. The $\sigma_c$ value becomes the maximum at $s/s_{\text{max}} = 0.25$ and 0.6 and the minimum at $s/s_{\text{max}} = 0.5$ in the smooth flow. These points roughly correspond to the location of the anti-nodes and node of the first anti-symmetric mode of vibration, respectively. In the turbulent flow, on the other hand, the maximum $\sigma_c$ value is observed at $s/s_{\text{max}} = 0.5$, which corresponds to the anti-node of the first symmetric mode of vibration. The magnitude of $\sigma_c/L$ in the turbulent flow is much larger than that in the smooth flow. It is interesting to note that the shape of distribution of $\sigma_c/L$ is minutely dependent on the wind velocity.

According to the above-mentioned results, it can be said that the dynamic behavior of the roof significantly depends on the turbulence of approach flow. The size of separation bubble and separation vortex in the turbulent flow is smaller than those in the smooth flow [5, 13]. We will investigate the effect of approach flow and separated flow turbulence on the dynamic behavior of flexible roof and the coupling of the flow with the vibrating roof in our future study.

**3.5 Vibration modes**

The proper orthogonal decomposition (POD) technique [14] is applied to the time history of the roof displacements to detect the predominant components of vibration. This technique, which is often applied to the pressure fields, can decompose the fluctuating mode of a parameter based on the standard deviation. In the present analysis, the roof vibration is expanded into 300 modes. Figures 15 and 16 respectively show the first three modes at several wind velocities ranging from 6 to 15 m/s in the smooth and turbulent flows. Table 4 shows the contribution of each eigenvalue to the sum of the total 300 eigenvalues.

![Fig. 15 First three vibration modes of the long-span flat roof in the smooth flow](image)

![Fig. 16 First three vibration modes of the long-span flat roof in the turbulent flow](image)

Table 4 Contribution of each eigenvalue to the sum of the first 300 eigenvalues

<table>
<thead>
<tr>
<th>$U_{\text{ref}}$</th>
<th>6 m/s</th>
<th>9 m/s</th>
<th>12 m/s</th>
<th>15 m/s</th>
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</thead>
<tbody>
<tr>
<td>Smooth flow</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>First mode</td>
<td>0.48</td>
<td>0.61</td>
<td>0.63</td>
<td>0.79</td>
</tr>
<tr>
<td>Second mode</td>
<td>0.40</td>
<td>0.22</td>
<td>0.22</td>
<td>0.12</td>
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<tr>
<td>Third mode</td>
<td>0.10</td>
<td>0.11</td>
<td>0.09</td>
<td>0.06</td>
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<tr>
<td>Turbulent flow</td>
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<td></td>
</tr>
<tr>
<td>First mode</td>
<td>0.99</td>
<td>0.99</td>
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<tr>
<td>Second mode</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
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<tr>
<td>Third mode</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
is smaller in size in the turbulent flow than in the smooth flow [5, 13], the effect of the flow separation on the roof vibration becomes smaller in the turbulent flow. As a result, the vibration in the anti-symmetric mode is hard to occur. It seems that the dominant vibration mode causes a large difference in the RMS value of roof’s displacement between smooth and turbulent flows. Therefore, it is thought that the mean deformation as well as the approach flow turbulence, which may be related to the size of the separation bubble, affect the roof vibration significantly.

4. Concluding Remarks
The wind-induced response and aerodynamic stability of a long-span flat roof has been investigated based on a wind tunnel experiment and an FSI simulation. The results indicate that no aerodynamically unstable vibration occurs in both the experiment and the simulation, because the mean deformation affects not only the flow around the roof but also the flow separation at the leading edge. This feature implies that the mean deformation of the roof should be considered when estimating the dynamic response of long-span structures. The size of the separation bubble and the vortices intermittently shed from the leading edge of the roof depend on the turbulence of approach flow significantly, which may be one of the most important factors for investigating the dynamic response and aerodynamic stability of the roof. In our future study, we will make a comparison between the experimental and numerical results to understand the phenomenon in more detail. Furthermore, we will investigate the mechanism of aerodynamic stability of the roof focusing on the interaction between the separation bubble and the roof vibration by getting more data from the FSI simulation.

Nomenclatures
α  power law exponent
C_p  wind pressure coefficient
C_p RMS fluctuating wind pressure coefficient
C_pmean  mean wind pressure coefficient
f  frequency
f_1  first natural frequency
H  eaves height
I_u  turbulence intensity for along wind direction
I_uH  turbulence intensity at the eaves height H
L  length of the roof
L_s  integral scale of turbulence
U_H  mean wind velocity at the eaves height H [m/s]
\text{s}  a coordinate along the undeformed roof (rigid roof) in the windward direction with its origin at the leading edge
s_{max}  maximum value of ‘s‘ (= L)
S_0  power spectral density of roof’s displacement
w  roof’s displacement
\sigma_{w}  RMS value of roof’s displacement
\phi_M  mode amplitude

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References