Mathematical Models for Light Distribution in Rice Canopy

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Abstract

In this paper, the regression effects, accuracies and figures of three mathematical models for simulating light vertical distribution in rice canopy were compared. The model $Sz = S_0 e^{a+bLAI(z)}$ was the best. The quantitative relationships between a, b and $LAI(z)$ accumulated to a height above the ground, solar altitudes were discussed. A universal available model for light distribution in crop canopy was proposed, and the error in utilizing the model was analysed. Meanwhile, daily changes in light intensity in representative layers of the canopy were described by harmonic analysis.

Key words: Rice, Canopy, Light distribution, Simulation, Model

Studying the relationship between light—distribution characteristics in crop canopy and canopy structure is of great practical significance for utilizing solar radiation energy effectively and increasing rice yields. This article aims at find a simple and useful mathematical model for light distribution within a crop canopy.

1. Materials and method of the experiment

August 15, 1990. It was a clear sky and the rice was in the primary stage of ripening. Light intensities within different layers were measured using a GE—1 illuminometer. (made at the Yinchuan Electrometer Factory, China) within a high—yield canopy (Shennong 91—418, per unit area yield 10.35t/ha, crop height about 100cm, leaf area index $LAI=6.83$) and a general canopy (per unit area yield 9.21t/ha, crop height about 90cm, $LAI=4.44$). The experiment began at sunrise and ended at sunset. (distance between layers is 10cm) Particular time was taken around the exact hours. Meanwhile, the weight of green leaves, LAI and the specific leaf weights etc. were measured by the stratifying clip method so as to understand the vertical structure of the canopy.

The vertical distributions of light and daily changes within the rice canopy are analysed using a mathematical simulation method and was verified with experimental materials.

2. The vertical distribution characteristics of light within rice canopy.

1) The choice of mathematical model

The relationship between relative light intensities and the LAI (z) of different layers was simulated in a computer by linear, quadratic and exponential function, mean values of F for 14 times obtained from the three models for high—yield canopy (5:00—18:00) are 38.2, 84.6 respectively. The average values of the residue standard errors are 15.76, 8.61, 7.54 respectively. Although the test results of F with the three models all reach the significance level of 0.01, the regression effects and fit accuracies of the exponential model are the best no matter for high—yield canopy or for general canopy. This can be seen from the compar-

Fig. 1 The relationship between the relative light intensity and LAI

isons of the regression effects and accuracies of the three models. (for the exponential model; F the highest and Se the lowest). The quadratic curve model is the second—best and the linear model the worst. Seen from a computer diagram, the exponential curve also best coincides with the dots from practical observation. (see Fig. 1)

The exponential model employed in this article is

$$S_z/S_0 = e^{a+bLAI(z)} \quad \dots(1)$$

Where: S_z , S_0 are the light intensities (klux) Zcm and 0 cm from the surface of the canopy. For the purpose of easy calculation, the relative sunshine hours of different layers are expanded 100 times; for example, the practical value 7.5% for 7.5. $LAI(z)$ is the leaf area index which is accumulated to Zcm below the upper canopy surface. a and b are parameters to be determined.

It is generally considered that the simple theoretical model $S_z = S_0 e^{-bLAI(z)}$ ¹⁾ is suitable for the light distributions within crop canopy, where b is the light—decreasing coefficient. Such a model assumes that the vertical distribution of light within the canopy is only influenced by the leaf area and that the distribution of leaf blades is homogeneous. But in fact, apart from the leaf area, many factors such as stem, colour of leaves, thickness of leaves and the incident angles of light all affect the reflection, absorption and transparent capacity of light. So there must be a parameter a in the equation when the relationship between the relative light intensity and $LAI(z)$ is simulated using exponential model, i. e. :

$$S_z = S_0 e^{a+bLAI(z)}$$

From the time and space distributions of errors in the models, (tables omitted) below the active surface, (the height of 2/3 crop height) errors of 63 percent of the simulated values are within the range of $\pm 1\%$, errors of 96 percent of the simulated values are within the range of $\pm 5\%$, the average range of errors is $\pm 3\%$. When it is before 8 : 00 or after 16 : 00, errors are relatively greater above the active surface. Thus this model is more applicable for layers under the active surface ($LAI(z) > 3$), and also where the light intensity is comparatively weak, sometimes photosynthesis is restricted, therefore requiring further study it. It is thus seen that expression (1) is a simple and useful mathematical model for simulating the vertical distributions of light within rice canopy.

2) Discussions on parameters a, b of the exponential model

Parameter a, b are as shown in Table 1.

Table 1 Parameter a, b in exponential model

Time	high—yield canopy		general canopy	
	a	b	a	b
5	4.5808	—0.5958	4.3813	—0.6763
6	4.2715	—0.4949	4.2403	—0.7444
7	4.1711	—0.5160	4.3025	—0.7681
8	4.3279	—0.5489	4.4619	—0.7769
9	4.5758	—0.5269	4.6347	—0.6879
10	4.6585	—0.4994	4.5861	—0.5996
11	4.6079	—0.3326	4.5318	—0.5227
12	4.5670	—0.3187	4.5746	—0.4871
13	4.6080	—0.4165	4.3237	—0.5352
14	4.5585	—0.4710	4.5233	—0.6522
15	4.6265	—0.5392	4.3788	—0.6931
16	4.4926	—0.5299	4.0376	—0.6060
17	4.1350	—0.4684	3.9530	—0.5270
18	4.1832	—0.2980	4.2282	—0.3988

Table 1 shows that the values of a and b vary with observational times. This in essence is the change of solar altitude. The incident angle of light follows it and finally results in the change of the vertical distribu-

tions of light energy. The formula of solar altitude could be expressed as :

$$\sin h = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos t \quad \dots\dots\dots(2)$$

Parameter a has a linear relationship with the sine of solar altitude, which is

$$a = a_1 + a_2 \sinh \quad \dots\dots\dots(3)$$

When LAI = 6.83, $a_1 = 4.0499$, $a_2 = 0.6739$

$$(n = 13 \quad R = 0.89^{**})$$

When LAI = 4.44, $a_1 = 4.0591$, $a_2 = 0.5261$

$$(n = 12 \quad R = 0.64^{**})$$

The average value is $a_1 = 4.0545$, $a_2 = 0.60$

$$(n = 26 \quad R = 0.75^{**})$$

Through tracing and analyzing the diagram, it can be found that the up—or—down trend of the curve of b which varies daily runs opposite to that of $\sin 2h$, their peaks and valley are going against each other. (diagram omitted) When a canopy is very thick, from 8 : 00 to 16 : 00, the correlation coefficient can reach the significant level of 0.05. Then the expression is

$$b = b_1 + b_2 \sin 2h \quad \dots\dots\dots(4)$$

Where $b_1 = 0.3810$, $b_2 = 0.9200$

$$(n = 9, \quad R = 0.6734^{**})$$

When LAI is small or in the early morning, evening, solar altitude varies rapidly and negative correlation becomes weak with no significance.

3) The establishment of a universal available model

Solar altitude reflects the space distributional factors (geographical latitude φ) and time distributional factors, (annual variation δ and daily changes t) so if a general applicable model is established which is available anywhere at anytime, the factor of solar altitudes must be introduced—i. e. : to establish a universal available model. The exact computational technique for solar altitude can be found in reference²⁾. Put (3), (4) into (1):

$$S_z = S_0 e^{a_1 + a_2 \sinh + b_1 \text{LAI}(z) + b_2 \sin 2h \text{LAI}(z)} \quad \dots\dots\dots(5)$$

Where a_2 and b_1 are respectively the responses of the coefficients of $\sinh$ and $\text{LAI}(z)$; b_2 could be considered as the interaction term of $\sinh$ and $\text{LAI}(z)$; a_1 is the affection of other factors. Their values are: $a_1 = 4.0545$, $a_2 = 0.6$, $b_1 = 0.381$, $b_2 = -0.92$. Then, the expression (5) could be considered as the universal available model of light distribution within a crop canopy.

When calculating the relative light intensities among different layers within the LiZa rice canopy (leaf type and crop height similar to Shennong91) with expression (5), the standard error for calculating values of 40 experimental points is 4.39%; taking 5.5 percent of the experimental points, the error is within the range of $\pm 3\%$; If taking 75 percent of the points, the error is within $\pm 5\%$. (table omitted) It should be noted that there are 24 points below the level 2/3 of the crop height, and the standard error is just $\pm 2.20\%$, moreover, it is just $\pm 1\%$ for ten points. However when it is above the active surface level, the errors are relatively greater because the distribution of leaf blades is sparse and inhomogeneous.

3. The daily changes of the light intensity within the rice canopy

As to the daily changes, although the quadratic curve and perpendicular polynomial could be used for simulation, the application of harmonic analysis³⁾ is clearer and more perceptible. Because the light intensity at horizontal surface is proportional to $\sinh$, the daily changes of light intensity could be considered as a combination of some sine—waves and its expression is:

$$Y_i(t) = A_0 + \sum A_n \sin(n\omega t_i + \theta_n) \quad \dots\dots\dots(6)$$

Where ① A_0 is the mean value of these observational series and its unit is klux. $Y_i(t)$ is the estimated value for the same layer tested at different times and i is the number of observation times. ② A_n is the half of the daily amplitude of light intensity wave for different periods. ③ ω is frequency ④ t_i stands for the time of

No. i observation. ϕ_n is the primary phase of the n -th wave. r represents fit degree.

Simulated formulae for different typical layers are shown as follows:

Top of the canopy:

$$Y_i(t) = 60.14 + 63.74\sin(\omega t_i + 272.3^\circ) \quad \dots\dots(7)$$

Fit goodness of the basic wave (period length 16 hours) is $r_1 = 0.992$.

Active surface level

$$Y_i(t) = 12.25 + 17.69\sin(\omega t_i + 277.1^\circ) + 80.9\sin(2\omega t_i + 283.1^\circ) \quad \dots\dots(8)$$

Period length is also 16 hours, fit degree $r_1 = 0.767$; If 8 hours are added to the 16 hours period length, the fit degree can reach $r_2 = 0.927$

Bottom of the canopy (80cm below the canopy surface):

$$Y_i(t) = 2.9 + 4.17\sin(\omega t_i + 274.5^\circ) + 2.34\sin(2\omega t_i + 289.8^\circ) + 12.7\sin(4\omega t_i + 314.0^\circ) \quad \dots\dots(9)$$

Fit degree of one periodical wave is only 0.583 for two periodical waves $r_2 = 0.766$, for three waves the integrated $r_3 = 0.82$

Expression (7), (8) and (9) show: Average daily light intensity decreases down the canopy surface (60.14 → 12.25 → 2.9 klux); Daily amplitude of the basic wave ($2 \times A_1$) becomes smaller ($2 \times 63.72 \rightarrow 2 \times 17.69 \rightarrow 2 \times 4.17$) and the fit degree also decreases gradually (0.992 → 0.767 → 0.583). Between the top level and the active surface, the primary phase of the basic wave becomes stagnant. The situation under the active surface is complicated and not in accordance with the rules described above. This can be explained like this: The top surface of the canopy and upper layers are more affected by the daily solar radiation variation rule of the sun than the lower layers which is affected to a great extent by the vertical structure of the canopy.

Fig. 2 The daily changes of the light intensity within different typical layers

Comparing the errors between the simulated curves and the practical observation points. (see Fig. 2)

4. Conclusion

1) When simulations of light distributional characteristics within a crop canopy are studied, it is more suitable to choose a period in which the canopy grows at its fastest speed, (LAI greatest) i. e., from 8 : 00 to 16 : 00 during the day.

2) $S_z/S_0 = e^{a+bLAI(z)}$ is the best form for the simulations of the vertical distribution of light energy within a canopy since it has the best simulation effect and a higher accuracy.

3) Further study of the relationship between parameters a, b and solar altitude, and the internal structure of a canopy makes it possible to establish a universal available model for the light distribution within crop fields which would be applicable anywhere at anytime. This article gives a trial and the error of the simulation for 75% points within a range of $\pm 5\%$.

4) It is a simple method and has an obvious physical significance to study the daily change characteristics of light intensity or radiation intensity within the typical layers in a crop canopy using the harmonic analysis.

The simulation of light distribution within a crop canopy is a complicated problem, the above conclusion still needs further studying and modification so as to make it perfect.

References

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