PREDICTION OF SOUND FIELDS IN ROOMS WITH MEMBRANE MATERIALS

—Development of a limp membrane element in acoustical FEM analysis
and its application—

膜材を有する室内の音場予測に関する研究
—FEM 音場解析における無張力膜要素の開発と適用—

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In order to predict sound fields in rooms with membrane materials, a new method using limp membrane elements in acoustical FEM analysis is developed, which is based on a limp membrane model. This method brings about a simple and easy treatment for membranes and a reduction in computational storage and time. The method is validated through its application to an ideal sound field solvable theoretically; and subsequently, it is applied to two kinds of analyses on a model room. The results computed are compared to those experimented, where good agreement can be seen. Moreover, the superiority of the method over the ordinary method assuming local reaction is confirmed in prediction accuracy.

keywords: *Numerical analysis, FEM, Membrane, Natural frequency, Absorbent structure*
数値解析, 有限要素法, 膜, 固有周波数, 吸音構造体

1. INTRODUCTION

Membranes are utilised as building materials in various forms, such as membrane structures and interior finishings, because they tend to be thin, lightweight, flexible and sometimes light transmissible. Naturally, they affect the acoustic conditions in the rooms to some extent, and also, there are some cases deliberately making use of the acoustic properties of membranes. For example, membranes are often used as coverings for porous absorbing materials which are not suitable for facings. Permeable membranes like fibrous cloth are also used as absorbing materials themselves, while impermeable and relatively heavyweight membranes are occasionally used as stage reflectors in auditoria. From the viewpoint of architectural acoustic designing, it is necessary to understand and predict the effects of these various types of membrane on the sound fields.

Acoustic properties of single membranes^{1) 2)}, double membranes³⁾ and layered absorbent structures including membranes²⁾, were investigated theoretically in the infinite extent models. However, in order to predict arbitrary sound fields with membranes, a numerical approach is indispensable. On numerical analysis of sound fields coupled to membrane vibration fields, some approaches^{4) 5)} were examined — although large amounts of computational storage and

time are required in applying them to sound fields in rooms. On the other hand, in most of the cases where membranes are used as building materials, the tension affects the sound fields slightly, while the density and permeability affect them substantially²⁾. If the effects of the tension are assumed to be disregarded, the membrane vibration system can be described simply. Then there is no need to apply a numerical method to the membrane vibration system, thus fairly reducing computational storage and time.

In this paper, based on the limp membrane vibration model where a membrane has some surface density and permeability but no tension, a new type of element for membranes in the Finite Element Method (FEM) for acoustical analysis, called a limp membrane element, is proposed. This element is simply linked to the ordinary air- or absorption element through only the nodal sound pressures, because of inevitably ensuring equality of the normal velocity on the membrane surface. This feature brings about a simple and easy treatment for membranes and a further reduction in computational storage and time, in addition to the reduction by disregarding the tension. At first, in order to validate the present method using the limp membrane elements, it is applied to an ideal sound field in an acoustic tube, and the result is compared with the theoretical solution.

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Next, as its practical application to sound fields in rooms, two kinds of analyses are carried out: firstly of natural frequencies of a room with an impermeable membrane; and secondly of transmission characteristics in a room with a layered absorbent structure including a membrane. These results are compared with those experimented on a model room, and in addition, with those by the ordinary method in which the absorbent structure is assumed to be locally reacting.

2. THEORETICAL DESCRIPTION

2.1. Limp membrane vibration model

In a limp membrane vibration model, it is assumed that a membrane has no thickness, no tension (the thickness and the tension equal zero), the surface density σ , and the flow resistance R . Fig.1 illustrates that the limp membrane is vibrated by the sound pressures on both the sides of the membrane. In this case, the steady state vibration equation of the membrane is expressed as

$$p_a - p_b = j\omega\sigma v_p, \quad (1)$$

where p_a and p_b are the sound pressures on the two sides of the membrane, v_p is the vibration velocity of the membrane, and the time factor is $\exp(j\omega t)$. As for the permeability of the membrane, according to the porous blanket model proposed by A. D. Pierce⁶⁾, the flow resistance is defined as

$$R = \frac{p_a - p_b}{v_f - v_p}, \quad (2)$$

where v_f is the particle velocity on and inside the membrane. At this point, the effective surface density of the membrane μ (complex quantity) is formally defined as

$$\mu = \frac{\sigma}{1 + j\omega\sigma / R}. \quad (3)$$

Eliminating v_p by comparing Eq.(1) with Eq.(2), and substituting Eq.(3), the expression of the relation between the sound pressures and the particle velocity on the membrane can be obtained as

$$p_a - p_b = j\omega\mu v_f. \quad (4)$$

Fig.1 Illustration of a limp membrane vibrated by sound in the air.

2.2. Application of the Galerkin method

Let us consider a closed sound field including a membrane and a porous material, as shown in Fig.2. The air density ρ_0 and the sound speed c_0 in the air (Ω_0), and the effective density ρ_e , the effective sound speed c_e and the flow resistivity R_f in the porous material (Ω_a) are given. The sound field has three types of boundary: rigid boundary (Γ_0), impedance boundary (Γ_1) and vibrating boundary

(Γ_2); and the normal acoustic impedance ratio z_n on Γ_1 , and the normal vibration velocity v_n on Γ_2 are given as boundary conditions. Applying the Galerkin method to the sound field, excluding the porous material, gives the following integral equation:

$$\begin{aligned} \iiint_{\Omega_0} (\nabla N_i \cdot \nabla p) dV - \frac{\omega^2}{c_0^2} \iiint_{\Omega_0} N_i p dV + j\frac{\omega}{c_0} \iint_{\Gamma_1} N_i \frac{p}{z_n} dS \\ = j\omega\rho_0 \iint_{\Gamma_2 + M_a + M_b} N_i v_n dS, \end{aligned} \quad (5)$$

where N_i is the interpolation function of the i -th node, p is the sound pressure, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$, M_a and M_b denote the two sides of the membrane. Using the interpolation functions, the sound pressure at an arbitrary point is approximately represented as follows

$$p(x, y, z) = \sum_{i=1}^N N_i(x, y, z) p_i, \quad (6)$$

where p_i is the sound pressure of the i -th node, and N is the total number of the nodes. In Eq.(5), the contribution of the membrane surfaces I_i is expressed as

$$I_i = j\omega\rho_0 \iint_{M_a + M_b} N_i v_n dS. \quad (7)$$

Here, as shown in Fig.3, two nodes — one on each side of the membrane, are arranged so as to have the same coordinates and make a pair, although they have respective values of sound pressure. Considering the normal directions of both the sides m_a and m_b , substituting Eq.(4) into Eq.(7) yields the following as

$$I_i = - \iint_{M_a} \frac{\rho_0}{\mu} N_i (p_a - p_b) dS + \iint_{M_b} \frac{\rho_0}{\mu} N_i (p_a - p_b) dS. \quad (8)$$

Fig.2 Illustration of a sound field with a membrane and a porous material.

Fig.3 Illustration of a limp membrane element.

Representing both the sides of the membrane by M , the general expression of Eq.(8) can be obtained as

$$I_i = - \iint_M \frac{\rho_0}{\mu} N_i (p - \hat{p}) dS, \quad (9)$$

where $\hat{p}$ is the sound pressure on the opposite side of the membrane. Consequently, substituting Eq.(6) into Eq.(9) gives the discrete expression not including the particle velocity as

$$I_i = - \iint_M \frac{\rho_0}{\mu} N_i \sum_{j=1}^N (N_j - \hat{N}_j) p_j dS, \quad (10)$$

where $\hat{N}_j$ is the interpolation function of the node corresponding to the j -th node. Considering a section of the membrane to be a type of element which consists of pairs of nodes, I_i can be regarded as the total contribution of such elements. In the following, this new type of element is called a limp membrane element.

3. ACOUSTICAL FEM ANALYSIS

3.1. Global matrix equation

To deal with porous materials in acoustical FEM analysis, the absorption element proposed by A. Craggs⁷⁾ is available. In the finite element subdivision of the sound field as shown in Fig.2, three types of element: air-, absorption- and limp membrane element, are used. Discretising Eq.(5) by Eq.(6), considering the absorption elements, and substituting Eq.(10) for the contribution of the membrane surfaces, the global matrix equation can be obtained as follows

$$([K] + j\omega[C] - \omega^2[M] + \rho_0[L])\{p\} = j\omega\rho_0[Q]\{v_n\}, \quad (11)$$

with

$[K]$: elastance matrix:

$$K_{ij} = \iiint_{\Omega_0} (\nabla N_i \cdot \nabla N_j) dV + \Lambda \iiint_{\Omega_a} (\nabla N_i \cdot \nabla N_j) dV,$$

$[M]$: inertance matrix:

$$M_{ij} = \frac{1}{c_0^2} \iiint_{\Omega_0} N_i N_j dV + \frac{\Lambda}{c_a^2} \iiint_{\Omega_a} N_i N_j dV,$$

$[C]$: damping matrix: $C_{ij} = \frac{1}{c_0} \iint_{\Gamma_1} \frac{1}{2} N_i N_j dS,$

$[Q]$: conformation matrix: $Q_{ij} = \iint_{\Gamma_2} N_i N_j dS,$

$[L]$: limp membrane matrix: $L_{ij} = \iint_M \frac{1}{\mu} N_i (N_j - \hat{N}_j) dS,$

where $\{p\}$ is the nodal sound pressure vector (only unknown), $\{v_n\}$ is the nodal vibration velocity vector, and $\Lambda = j\omega\rho_0/(j\omega\rho_a + R_f)$. The limp membrane matrix can be divided into the real part and the imaginary part as follows

$$[L] = [L_m] + j\omega[L_c], \quad (12)$$

with $[L_m]$: $L_{mij} = \iint_M \frac{1}{\sigma} N_i (N_j - \hat{N}_j) dS,$

$$[L_c]$$
 : $L_{cij} = \iint_M \frac{1}{R} N_i (N_j - \hat{N}_j) dS.$

The real part is related to the surface density; and the imaginary part is related to the flow resistance, which acts upon the global system as a damping term. In a porous material, only the flow

resistivity is assumed to be given as a known condition, and the effective density and the effective sound speed are given by the following equations:

$$\rho_e(\omega, R_f) = \frac{\gamma(\omega, R_f)Z(\omega, R_f) - R_f}{j\omega}, \quad (13)$$

$$c_e(\omega, R_f) = \frac{j\omega}{\gamma(\omega, R_f)}, \quad (14)$$

where γ is the propagation constant and Z is the characteristic impedance, calculated by using Miki's empirical equations⁸⁾.

In the ordinary way, the matrix equations for acoustic systems and membrane vibration systems are composed respectively, and are related to one another through the nodal sound pressures and normal velocities on the interfaces. On the other hand, using the limp membrane elements, the above global matrices are composed by simply assembling all the elemental matrices, because a procedure for conforming the normal velocity is needless on the interface. In addition, the computational storage is reduced due to excluding the nodal unknowns of the normal velocity on the interface. These are brought about by the fact that equality of the normal velocity on the interface is inevitably satisfied, and the limp membrane element is simply linked to the other type of element through only the nodal sound pressures. Moreover, in the finite element subdivision, all the elements and the nodes can be numbered independently of the elemental types. This enables the composition of the global matrices of efficient band shape, which results in a further reduction in computational storage.

3.2. Treatment of the limp membrane element

The information as to the positions of membrane surfaces and pairs of nodes in a limp membrane element is previously given by fixing the arrangement of the node numbers in the element — and this means that the limp membrane element has directivity. As an example, Fig.4 shows a hexahedral element having eight nodes in the local coordinate system; however, the two nodes in the pair have the same coordinates in the global coordinate system. The interpolation functions of the nodes are formally of the same type as used in the air- and absorption element, which is convenient for programming. The components of the limp membrane matrix are calculated by the numerical integration on the membrane surfaces.

Fig.4 Hexahedral limp membrane element having eight nodes in the local coordinate system.

4. APPLICATION AND DISCUSSION

4.1. Analysis of an ideal sound field in an acoustic tube

In order to confirm the accuracy of the present method, it is examined through its application to the ideal sound field in an acoustic tube, as shown in Fig. 5. The walls of the acoustic tube are infinitely rigid; a limp membrane and a porous material are placed inside; and one end is vibrating in the x -direction. In this one dimensional sound field, the radiation impedance ratio on the vibrating surface z_r is theoretically obtained as

$$z_r = \frac{A^+ E_0^+ - A^- E_0^-}{A^+ E_0^+ + A^- E_0^-} \quad (15)$$

where $A^\pm = (\rho_0 c_0 \pm j\omega\mu \pm Z)E_a^+ - (\rho_0 c_0 \pm j\omega\mu \mp Z)E_a^-$,

$E_0^\pm = \exp(\pm j\omega L_0/c_0)$, $E_a^\pm = \exp(\pm j\omega L_a/c_0)$, L_0 and L_a are the x -directional lengths of the air and the porous material, respectively.

In the computation, the physical condition is given as shown in Table 1, and the size of every element is $d = 0.01$ m (the number of elements: 30 air-, 20 absorption- and 1 limp membrane element). Fig. 6 shows the radiation impedance ratios computed by the present numerical method and by the theoretical solution. At low frequencies, good agreement between both the results can be seen; therefore the validity of the present method is confirmed.

Next, to examine the effects of the membrane on the sound field in the acoustic tube, the distributions of the sound pressure are numerically computed in three cases (Case1: with only the porous material, Case2: with only the membrane, and Case3: with both the membrane and the porous material). Fig. 7 shows the distributions of the sound pressure normalised by the velocity on the vibrating surface at the frequency 1 kHz. As an effect of the membrane, the discontinuity of the distribution occurs on the membrane surface in Case2 and Case3. Moreover, in comparing the results between Case1 and Case3, it can be clearly seen that the distribution in the air changes by setting the membrane.

4.2. Analysis of natural frequencies of a room with a membrane

It is reported that setting a membrane material in a room causes a change in the natural frequencies of the room^(9) 10). Furthermore, making use of this nature, the adjustment of natural frequency distribution by membranes is also proposed¹¹⁾. In such cases, it is useful to predict the natural frequencies of the rooms with membranes; and an eigenvalue analysis using the limp membrane elements is expected to be effective in the prediction. In this section, the present method is applied to the sound field in a model room with an impermeable membrane, and the results computed are compared with those experimented.

The model room used in the experiment is shown in Fig. 8. The walls are made of acrylic boards (the thickness 10 mm), and the loudspeaker for the sound source and the microphone for the receiver are placed in two corners of the room, respectively. Table 2 shows the properties of the membranes used in the experiment. One of the types of impermeable membrane (M1-M4) is set at one of the

Fig. 5 Geometry of the acoustic tube with a membrane and a porous material.

Table 1 Physical condition in the acoustic tube.

Membrane	$\sigma = 0.25 \text{ kg/m}^2$, $R = 1000 \text{ N}\cdot\text{s/m}^3$
Porous material	$R_f = 10000 \text{ N}\cdot\text{s/m}^4$, $L_a = 0.20 \text{ m}$
Air	$L_0 = 0.30 \text{ m}$

Fig. 6 Radiation impedance ratios computed by the present numerical method and by the theoretical solution.

Fig. 7 Distributions of normalised sound pressure computed in three cases (Case1: with only the porous material, Case2: with only the membrane, Case3: with both of them).

Table 2 Properties of membranes used in the experiment.

Type	Surface density σ [kg/m ²]	Flow resistance R [N·s/m ³]
M1	0.028	∞
M2	0.055	∞
M3	0.195	∞
M4	0.390	∞
M5	0.327	454

positions (A-E). In a preliminary experiment, it is confirmed that the effect of the tension of the membrane is negligible. Through the measuring system shown in Fig.8, the sound pressure at the receiving point and the vibrating velocity on the loudspeaker surface are observed; and the natural frequencies are identified by the frequency response of the sound pressure to the vibrating velocity.

Assuming that the walls of the room are infinitely rigid, the general eigenvalue problem is obtained from Eq.(11) as follows

$$\{([K] + \rho_0[L]) - (2\pi f_n)^2[M]\}\{p_n\} = \{0\}, \quad (16)$$

where f_n is the natural frequency and $\{p_n\}$ is the nodal eigen vector of sound pressure. Numerically solving Eq.(16), the natural frequencies of the room are calculated. Fig.9 shows the distributions of the natural frequencies measured and computed in two cases: empty and with the membrane (position A, type M3). It can be seen that, on the whole, the distribution shifts to low frequencies by setting the membrane; and fair agreement is shown between the results measured and computed. Next, concerning the effects of the surface density and the position of the membrane on the low natural frequencies, the results measured and computed are shown in Fig.10. In each normal mode, the ratio of the drop in the natural frequency becomes higher as the surface density increases, and also as the position becomes closer to where the normal particle velocity on the membrane is higher. With respect to this tendency, good agreement can be seen between both the results — although the natural frequencies computed are a little lower than those measured. In view of these findings, the eigenvalue analysis using the limp membrane elements will be useful for prediction of the natural frequencies of the rooms with membranes.

4.3. Analysis of transmission characteristics in a room with a layered absorbent structure

Porous materials are usually covered with membranes for surface protection, and air layers are sometimes provided behind them to raise the absorptivity at low frequencies. When predicting the sound field in a room with such a layered absorbent structure, the method — in which the normal acoustic impedance on the surface of the structure is given as the boundary condition on the assumption of local reaction — is often used^{(12) (13)}. However, in the case of a relatively thick structure, this application may cause a decrease in prediction accuracy; and moreover, it is necessary for the application that the normal acoustic impedance is previously known in some way. On the other hand, the present method using three types of element is expected to yield advancement in prediction accuracy, and this method needs only the properties of membranes and porous materials, respectively. In this section, the present method is applied to the sound field in a model room with a layered absorbent structure. Next, the results computed are compared with those experimented, and in addition, with those computed by the ordinary method assuming local reaction.

In the experiment, the model room and the measuring system

Fig.8 Geometry of the model room with an impermeable membrane and diagram of the measuring system.

Fig.9 Distributions of natural frequencies measured and computed in two cases: empty and with the membrane (position A, type M3).

Fig.10 Effects of the changes in the surface density and the position of a membrane on the low natural frequencies, measured and computed in four normal modes.

Fig. 11 Geometry of the model room with a layered absorbent structure.

are the same as used in the previous section, whilst the absorbent structure, the loudspeaker and the microphone, are arranged as shown in Fig. 11. Basically, the absorbent structure is composed of the air layer, the glass wool layer (the density 32 kg/m^3) and a membrane (Table 2); and it is altered by the type of membrane and whether the glass wool is present or not. In such conditions, the frequency responses of the sound pressure to the vibrating velocity are observed.

In the computation, the flow resistivity of the glass wool is assumed to be $R_f = 9000 \text{ N} \cdot \text{s/m}^4$,¹²⁾ and the normal acoustic impedance ratio of the walls is given as the following empirical equation:¹³⁾

$$z_n = 1 / (4.1 \times 10^{-3} + 1.5 \times 10^{-9} f^2 + j 3.3 f^{-1}), \quad (17)$$

which is obtained by a preliminary experiment in the empty room. Fig. 12 shows the frequency responses measured and computed in eight cases (the left four cases — without glass wool, the right four cases — with glass wool). In comparing each case, it can be seen that the surface density and the flow resistance of the membrane considerably affect the transmission characteristics in the room. Especially, in the case of the permeable membrane (type M5) without glass wool, the damping of some modes is remarkable. In every case, good agreement between the results measured and computed can be seen; thus it is considered that this method will be useful for predicting the effects of such layered absorbent structures on the sound fields in rooms.

Next, the ordinary method assuming local reaction is applied to the sound field in the same room. The normal acoustic impedance ratio on the surface of the absorbent structure z_a is theoretically given as

$$z_a = \frac{j\omega\mu}{\rho_0 c_0} + \frac{Z}{\rho_0 c_0} \frac{B^+ F_a^+ - B^- F_a^-}{B^+ F_a^+ + B^- F_a^-},$$

where $B^\pm = (Z \pm \rho_0 c_0) F_0^\pm - (Z \mp \rho_0 c_0) F_0^\mp$,

$F_0^\pm = \exp(\pm j\omega d_0/c_0)$, $F_a^\pm = \exp(\pm j\omega d_a/c_a)$, d_0 and d_a are the depths

Fig. 12 Frequency responses of relative sound pressure level measured and computed in eight cases: empty, with only a membrane (M2, M4, M5), with only the glass wool (GW) and with a membrane and the glass wool (M2+GW, M4+GW, M5+GW).

Fig. 13 Frequency responses of relative sound pressure level measured and computed by the method assuming local reaction, in four cases: with only a membrane (M4, M5) and with a membrane and the glass wool (M4+GW, M5+GW).

of the air layer and the glass wool layer, respectively. Fig. 13 shows the frequency responses measured and computed by the method assuming local reaction in four cases (the left two cases — without glass wool, the right two cases — with glass wool). In comparing the four cases between Fig. 12 and Fig. 13, the results obtained using the present method are in better agreement with those measured,

than those obtained by the ordinary method assuming local reaction. Especially in the cases with only a membrane, it can be seen that applying the latter method causes a remarkable decrease in prediction accuracy. Therefore, it is concluded that the present method is superior in prediction accuracy to the ordinary method assuming local reaction when dealing with layered absorbent structures including membrane.

5. CONCLUSION

A method using a new limp membrane element in acoustical FEM analysis was developed, which is applicable to most of the cases where membranes are used as building materials in rooms. This element is easy to handle because it is simply linked to the ordinary air- or absorption element through only the nodal sound pressures. In this method, no need to apply a numerical method to the membrane vibration system results in fairly reducing computational storage and time. Moreover, this method makes it possible to exclude the nodal unknowns of the normal velocity on the membrane surfaces and to compose the global matrices of efficient band shape; and both of which bring about a further reduction in computational storage and time. The accuracy of this method was confirmed through its application to the ideal sound field in an acoustic tube which is solvable theoretically. In practical application to the sound field in a model room, good agreement was seen between the results computed and measured; and in addition, concerning the treatment for layered absorbent structures including membranes, the superiority in prediction accuracy of this method over the ordinary method assuming local reaction was clarified. The present method will be useful for prediction of sound fields including membranes: especially, of natural frequencies of rooms with membranes, and of acoustic characteristics in rooms with layered absorbent structures including membranes.

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和文要約

1. はじめに

現在、建材としての膜材の利用形態は多岐に渡り、その音響的利用例も少なくない。膜材の音響特性に関して無限大膜モデルに基づく理論解析¹⁾²⁾³⁾により吸音率・透過損失の予測が行われる一方、任意音場の波動音響学的予測を目的とした音響-膜振動連成数値解析手法⁴⁾⁵⁾が検討されている。本論文では、膜材が建材として使用される場合の大半において音場に及ぼす膜の張力の影響が微小であることに着目し、張力 0 を仮定した無張力膜モデルに基づき、FEM 音場解析における無張力膜要素を提案する。無張力膜要素を用いた解析では膜振動場に対する数値解析手法の適用が不要なことから、張力を考慮した厳密な連成解析に比べて計算に要する記憶容量・時間が大幅に低減する。さらに、無張力膜要素は従来の空気要素・吸音要素と節点音圧未知量のみを介して接続されることから、その取扱いが簡便であると同時に、更なる記憶容量・時間の低減が図られる。本方法は音響管内の理想音場における数値解と理論解との比較によりその妥当性が検討された後、室内音場への適用例として、非通気性膜を有する室内の固有周波数解析、膜を含む多層吸音構造体を有する室内の伝送特性解析を行い、模型実験との比較からその有効性を検討する。さらに、吸音構造体表面において局所作用を仮定する従来の方法との比較検討についても行う。

2. 無張力膜要素の理論的導出

厚さ 0・張力 0 および面密度・流れ抵抗を有する無張力膜モデルを想定し、膜振動方程式、流れ抵抗に関する定義式および新たに定義した膜の実効面密度により膜面上音圧と粒子速度の関係を定式化する。次に、膜と多孔質材を含む室内音場を想定し、Galerkin 法の適用により積分方程式を導出した後、前述した関係式の代入および変形により膜面の寄与を膜面上粒子速度を含まない形式で表す。ここで、膜面上の節点を膜両面を対を成すように同一座標に配置し、その節点对から構成される膜部分を一種の有限要素（以下、無張力膜要素）とみなすことにより、膜両面の寄与を無張力膜要素の寄与として取り扱うものとする。

3. FEM による解析方法

空気要素・吸音要素・無張力膜要素の 3 種類の要素を用いて解析領域の要素分割を行い、全体マトリクス方程式を構成する。ここで、全体マトリクス方程式は膜に関する無張力膜マトリクスが従来のマトリクス方程式に加わる形式で表され、その無張力膜マトリクスは面密度に関する実数部と流れ抵抗に関する虚数部から成る。無張力膜要素の適用により膜面上粒子速度に関する適合手続きが不要なことから、各全体マトリクスは各要素マトリクスの単純な重ね合わせにより組み立てられる他、膜面上粒子速度に関する節点未知量の排除および効率的なバンドマトリクスの生成により記憶容量・時間が低減される。無張力膜要素の取扱いに関しては、要素内の膜面位置・節点对に関する情報を予め要素内節点番号の配列に付与し、各節点の内挿関数は空気要素・吸音要素と同様の内挿関数を形式的に用いることができる。

4. 解析方法の適用と考察

剛壁からなる音響管内に膜と多孔質材を設置した 1 次元の理想音場を想定し、端部振動面の比放射インピーダンスに関して数値解と理論解の比較を行った。両者は低周波数域でほぼ完全に一致したことから、本方法の妥当性が確認された。さらに、膜が音響管内音場に及ぼす影響を調べた結果、膜面における音圧分布の不連続性、膜の設置による音圧分布の変化が確認された。

次に、室内音場への第一の適用例として、非通気性膜を設置した模型室内の固有周波数解析を行い、模型実験との比較検討を行った。膜の設置による室内固有周波数低下の割合に関しては計算値の方が実測値よりやや高い割合を示すものの、膜が室内固有周波数に及ぼす影響の全般的傾向という点では実測結果と計算結果の良好な対応が見られた。従って、膜を有する室内の固有周波数予測に本方法は有効であることが認められた。

室内音場への第二の適用例として、膜・グラスウール・空気層から成る多層吸音構造体を有する室内の伝送特性解析を行い、模型実験との比較検討を行った。種々の吸音構造体条件における受音点音圧の周波数応答に関して実測結果と計算結果は良好な対応を示したことから、吸音構造体を有する室内の音場予測に本方法は有効であることが認められた。さらに、吸音構造体表面において局所作用を仮定する従来の方法との比較検討を行った結果、局所作用を仮定する方法によると厚い吸音構造体では大きく予測精度が低下し、今回提案する方法が計算精度においてかなり優位であることが示された。

5. まとめ

室内音場の FEM 解析における膜材の取扱い方法として、膜の張力 0 を仮定し、面密度・通気性を考慮する無張力膜モデルに基づく新種の無張力膜要素を提案した。FEM 音場解析における無張力膜要素の具体的な取扱い方法が述べられ、取扱いの簡便さ、適用による数値計算の効率化が示された。理論解が得られる音響管内音場への適用から本方法の妥当性が確認され、模型室内音場への適用から膜を有する室内の固有周波数予測、膜を含む多層吸音構造体を有する室内の音場予測における有効性が認められた。さらに、吸音構造体の取扱いに関しては局所作用を仮定する方法による予測精度の低下を指摘し、本方法の優位性が明らかにされた。

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