FROM COLLINEARITY THROUGH SIMILARITY TO EQUILATERAL TRIANGLES: TRANSITION OF GEOMETRICAL RELATIONSHIPS IN THE OPEN PLANS OF MIES VAN DER ROHE'S PROJECTS IN THE 1940'S

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Regarding geometrical relationships closely connected with Mies' "successful relationship" in the open plans from analysis of his transitional, from his European to American, projects in the 1940's, the Museum for a Small City, the Farnsworth House, the 1947 Exhibition and the 50 by 50 Feet House, it is found that Mies was increasing the geometrical relationships one by one, from collinearity through similarity to equilateral triangles, in response to the appearance of the new type of open plan.

Keywords: Mies van der Rohe, Farnsworth House, geometrical method, collinear, similarity, equilateral triangles
ミース・ファン・デル・ローハ、ファーンスワース邸、幾何学的方法、
共線性、相似性、正三角形

1. Introduction
"We shall emphasize the organic principle of order as a means of achieving the successful relationship of the parts to each other and the whole" said Mies regarding his architectural principle. Then also in his famous open plan, in which its interior components, freestanding walls and so on, are arranged free from structural restrictions, we ought to be able to see "the successful relationship." In my previous study "the successful relationship" in the open plans of Mies' masterpieces in his mature American period, IIT's Crown Hall of 1950-56, the New National Gallery of 1962-68 and so on, was considered and three geometrical relationships closely connected with "the successful relationship", collinearity, similarity, and equilateral triangles in the relationships among interior components, were found in each of the floor plans. 

By the way it is generally said that "Mies' major projects of his mature European years suggest a freer, more subjective approach to space and massing than is evident in severity and compactness of his American designs."1 We can definitely find the transition in the projects with the open plan. For instance in the Barcelona Pavilion of 1929, the Tugendhat House of 1930 and the Model House in Berlin Building Exposition of 1931 in his European years both the outline of the building and the arrangement of interior components are informal and asymmetrical. On the other hand in IIT's Crown Hall and the New National Gallery and so on in his mature American years both are quite compact and strictly symmetrical.

Investigating the transition, there can be found some composite projects in the 1940's.4 Although they have a compact, rectangular or square, outline of the building like his mature American projects, its interior components are arranged in a manner of a free and asymmetrical configuration like his mature European ones. Taking it into consideration that after the 1940's the arrangement of interior components almost became symmetrical,5 it can be said that the composite

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open plans in the 1940's might be transitional.4

Needless to say, each of Mies' projects must have achieved "the successful relationship" in its own manner.

How did Mies achieve "the successful relationship" in the different types of open plan having a complicated outline and an asymmetrical interior arrangement, a compact outline and an asymmetrical arrangement, and a compact outline and a symmetrical arrangement?

It is certain that means of achieving "the successful relationship" relate to various factors and the problem is never limited to only a geometrical aspect in floor plans. However to take it into consideration that in my previous study geometrical relationships closely connected with "the successful relationship were found in open plans, it seems to be reasonable to suppose that the question I have to ask here can be also considered from the viewpoint of geometrical relationship. How did the three geometrical relationships relating to "the successful relationship" concretely connect with the different types of open plan?

Although limited to two-dimensional forms in floor plans, this problem relates to elucidating one aspect of Mies' formal design method for achieving "the successful relationship". It is worth considering for developing a better understanding of Mies' formal methods.

Regarding his open plans with a complicated outline and an asymmetrical interior arrangement in his mature European years it has been already pointed by researchers that collinearity, some ends and corners of different components in the plan being in the same line, can be found.5 However similarity and equilateral triangles have not been referred. Analysing the open plans of mature European projects, it is certainly difficult to find them out.6 Then it follows from it that similarity and equilateral triangles are likely to have emerged in the process of the transition from European forms to American ones.

How did similarity and equilateral triangles concretely relate to the composite open plans in the 1940's? In this paper to answer the question I consider the composite open plans in turn, first the Museum for a Small City of 1942, second the Farnsworth House of 1945-51, third the installation of the 1947 retrospective exhibition of Mies van der Rohe's work at the MoMA, and last the 50 by 50 Feet House of 1950-51.

2. Collinearity and Similarity in the Plan of the Museum for a Small City of 1942

The Museum for a Small City7 is a glazed pavilion with a flat roof supported mostly on a grid of columns (Fig.1). The exhibit space (Fig.2:ABCD) contains a court, an auditorium, and service facilities.8 The arrangement of interior components is informal and asymmetrical. For instance the inner court (EFGH) is situated off center. However the outline of the exhibit space (ABCD) is completely rectangular. It can be said that this is the first of all Mies' glazed buildings with a rectangular outline.9

Besides this project was designed for an issue of a magazine.10 Then its floor plan (Fig.1) was for presentation. From this it is appropriate to suppose that the floor plan must be desirable one and have achieved "the successful relationship".

Although at first glance the interior components seem to be freely placed and to have different proportions, from close analysis of the floor plan (Fig.1)11 first there can be found some collinearities (Fig.2:e.g. ALE, BDK, CMG, ILFJ). These collinearities can not be coincidences at the same time.

It is reasonable to suppose that the main components of the building, the exhibit space, the side stone walls and so on, were intentionally connected with each other by the collinearities. It can be said that for instance latent diagonal BDK achieves the relationship between the exhibit space and the pool, and the reverse diagonal ILFJ of the whole achieves the relationship between both the side stone walls, the inner court, and the auditorium.

Next let us investigate similarity. Since both outline ABCD and outline EFGH are clear rectangles, we can explicitly find similarity between them (Fig.3).12 Interestingly outline abcd of the relatively small component, the print department area, is similar to large rectangle APJD enclosing the roofed area and the outer court. Further there can be found some approximate similarities (Fig.3:e.g.efgh ∞ BNOC, iklj ∞ BPJC, TUVK ∞ IQRS).13 Such many similarities can not be also coincidences at the same time. That is, it is reasonable to think that the rectangular outlines of the small area and the large area including it were derived from similarity and such many combinations were complicatedly superimposed. It can be said that the similarities achieve the strict relationship of the components of the building to each other (Fig.3).

In the Museum for a Small City the similarities,
Fig. 1 Plan of the Museum for a Small City of 1942

Fig. 2 Collinearity in the Plan of the Museum for a Small City

Fig. 3 Similarity in the Plan of the Museum for a Small City

Fig. 4 Plan of the Farnsworth House of 1945-51

Fig. 5 Collinearity in the Plan of the Farnsworth House

Fig. 6 Similarity in the Plan of the Farnsworth House
almost never seen before in Mies' open plans, were definitely found. Similarity is likely to be derived from the appearance of the simple rectangular outline of the building (ABCD). To prove it let us analyse the next composite open plan, the Farnsworth House's floor plan.

3. Collinearity and Similarity in the Plan of the Farnsworth House of 1945-51

The Farnsworth House is Mies' first executed clear-span structure. In the floor plan (Fig.4) there can be found some collinearities (Fig.5: e.g. AHC, DIBF). For instance the two diagonals, AC and BD, achieve the strict relationship between the whole interior (ABCD), the dinning table (I) and the storage wall (H).

Outline ABCD of the interior is just similar to rectangle JKLM (Fig.6) enclosing the whole H-core. Long side NO of the chaise longue on the terrace is just parallel to diagonal DF. It shows that diagonal DF was certainly concieved and further outline ABCD is similar to latent rectangle EFGD enclosing the house and the terrace.17

It is interesting that H-shaped core JKLM, interior ABCD including the core, and rectangle EFGD including the interior are connected with each other by similarities like circles with the same center.18

In both the Museum for a Small City and the Farnsworth House there can be found strict collinearities and similarities behind the free arrangement of the components. Taking it into consideration that each of them has a simple rectangular outline of the interior space, never seen before, it is reasonable to suppose that similarity closely relates to the rectangular outline.

Although similarities can be found like this, it is still difficult to find out equilateral triangles. How does an equilateral triangle relate to the plan? To investigate it let us analyse the next composite open plan, the floor plan of the 1947 exhibition.

4. Collinearity and an Equilateral Triangle in the Plan of the 1947 Exhibition

The installation of the 1947 exhibition was designed by the architect himself (Fig.7).19 Within the existing gallery space Mies designed a configuration of four freestanding walls arranged in a pinwheel fasion.20 To one side of each of these walls he attached a large photomural, edge to edge and floor to ceiling. Around them the other components of the exhibition, smaller photographs, models, furniture, and a mock-up of the structural detail, were arranged.

The exhibition projects may constitute a separate genre. However since Mies was "responsible for the nature of the display, its plan, the appearance of the room," in this installation he must have achieved "the successful relationship".

From analysis of the floor plan there can be also found many collinearities (Fig.8: KJI, KOC, DLM, HON, EGF). All eight photomurals and the whole gallery space are strictly connected with each other by collinearities.

At first glance equilateral triangles in the relationships among the components can not be found. For instance we can not find any equilateral triangle whose line of symmetry is, relating to the drawing (Fig.9), vertical middle line PQ between both the side walls.23 Strictly speaking the four existing columns are not symmetrical with respect to PQ. They stand a little toward the left. Then it is appropriate to suppose that the central axis of the space is middle line PQ' between the right and the left two columns rather than middle line PQ between both the side walls. The left end of the entrance is just in PQ'. It also indicates the latent middle line PQ'24.

Then taking PQ' between columns (Fig.9) into account and making a triangle by connecting upper end Q', right end K of photomural KL and crossing S of the extention line of KL and PQ', and rotating this right triangle Q'KS on PQ' to the left side, interestingly the resultant large triangle Q'KR just equals to an equilateral triangle.

Photomural KL and the four existing columns are strictly connected with each other by an equilateral triangle. However equilateral triangle Q'KR may be a coincidence.

Besides although the Museum for a Small City and the Farnsworth House had a rectangular outline, a central axis in the plan was not indicated. Then it is likely that a central axis, equivalent to a line of symmetry between columns, indicated by four columns and so on closely connects with a latent equilateral triangle whose line of symmetry is the central axis of the space. To prove it let us investigate the next open plan, the plan of the 50 by 50 Feet House.

5. Collinearity, Similarity and Equilateral Triangles in the Plan of the 50 by 50 Feet House of 1950-51

The 50 by 50 Feet House contains only one square space of 50 by 50 feet.23 In the published version (Fig.10) the interior is clear-span space and
Fig. 7 Plan of the 1947 Retrospective Exhibition of Mies van der Rohe

Fig. 8 Collinearity in the Plan of the 1947 Exhibition

Fig. 9 An Equilateral Triangle in the Plan of the 1947 Exhibition

Fig. 10 Plan of the Fifty by Fifty Feet House of 1950-51

Fig. 11 Collinearity and Similarity in the Plan of the Fifty by Fifty Feet House

Fig. 12 Equilateral Triangles in the Plan of the Fifty by Fifty Feet House
articulated into a living area, a dining area and sleeping areas by a H-shaped service core and so on. It shows close connection with the Farnsworth House (Fig.4). However the roof is supported by four columns, each positioned exactly in the center of each of the four floor-to-ceiling glass walls. Narrow terraces circle the house and a square terrace emerges asymmetrically from it and shares the floor plane.

There can be found some collinearities (Fig.11: e.g. AMR, ANC, BQP, COE). The square outline of the house, the core, the storage wall, the tables, and the terrace are strictly connected with each other by collinearities. It is interesting that the square table in the living area, whose lower-right corner (O) is just set in diagonal CE, plays an important role for strengthening latent diagonal CE connecting the house (ABCD) with the terrace (EFGH).

It is easy to find similarity (Fig.11:ABCD ∝ LJJK ∝ EFGH). Further the rectangle enclosing the H-core is also approximate square and almost similar to the house and the terrace. The H-core, the house and the terrace are similar to each other like circles with the same center.

Besides although the arrangement of the interior components is not symmetrical, the upper and the lower column (Fig.12: T,S) positioned in the center of each of the walls indicate vertical central axis TS of the square room. Further the square table situated just in line TS strengthens it.

Then connecting midpoint T, lower-right corner M of the storage wall by the dinning table" and crossing Z of the extention line of MW and TS, and rotating this right triangle TMZ on TS to the left side, interestingly resultant large triangle TMU just equals to an equilateral triangle. Further point U is just in the left side of the bed. ∆ TMU is an equilateral triangle whose line of symmetry is the central axis of the room (Fig.12)." Besides rotating right triangle SWZ on TS to the left side, Interestingly resultant small upside-down triangle SWV just equals to an equilateral triangle.

Center X of the round table on the square terrace is just in the line drawn from the point equivalent to upper-left corner G of the terrace 60 degrees to the horizontal joint line (Fig.12: ∠ GXY=60°). Further each of the two chairs on the terrace is placed just 60 degrees to the horizontal joint line. It is certain that measure of 60 degrees was intentionally used. It also backs up that an equilateral triangle whose interior angles are 60 degrees was also conceived and further the two equilateral triangles, ∆ TMU and ∇ SWV, were not coincidences.

The square outline, the storage wall and so on are also connected with each other by latent equilateral triangles. For instance it can be said that equilateral triangle TMU achieves the strict relationship of the lower-right storage (MW) wall and the lower-left bed (U) to each other and whole square room ABCD.

In each of the floor plans of the 1947 Mies' Exhibition and the 50 by 50 Feet House there can be found an equilateral triangle whose line of symmetry is the central axis of the space. Taking it into account that in each of the plans a central axis of the room is indicated, it is reasonable to suppose that the equilateral triangle closely relates to the central axis of the space.

6. Conclusion: From Collinearity through Similarity to Equilateral Triangles

While collinearity was constantly found, explicit similarity first emerged in the early 1940's corresponding to the appearance of a simple rectangular outline in the Museum for a Small City of 1942 and next was also found in the Farnsworth House of 1946 with a simple rectangular outline, then an equilateral triangle first emerged in the late 1940's corresponding to the appearance of a central axis of the room in the 1947 Exhibition, and both similarity and equilateral triangles were found in the 50 by 50 Feet House of 1950 with both a simple rectangular outline and a central axis.

From this it is concluded that similarity closely related to a simple rectangular outline of a floor plan and an equilateral triangle did to a central axis of it.

Further it is appropriate to think that in his formal transition from Europe to America Mies was increasing geometrical relationships closely connected with "the successful relationship" one by one, from collinearity through similarity to equilateral triangles, in response to an appearance of a new type open plan.

As the result after the 1940's in his mature American projects with a compact outline and a symmetrical arrangement of its interior components all the three geometrical relationships can be almost found at the same time. That is, the interior components were being gradually more closely connected with each other by the latent geometrical relationships. "Severity and compactness of his American designs" come from not only a compact outline and a symmetrical arrangement, but also the latent geometrical relationships of the components.

In each of the composite plans we can see Mies' ingenious concrete formal methods connecting the components with each other by the latent geometrical
relationships. For instance in the Museum for a Small City the central inner court (Fig.3:EFGH) is connected with not only the main exhibit space (ABCD) by similarity, but also even the far side stone walls (IJ) by collinearity, in the Farnsworth House the H-core (Fig.6:JKLM), the interior (ABCD) including the core and the whole house (EFGD) including the two are connected with each other by similarity like circles with the same center, in the 1947 exhibition the front large photomural (Fig.9:KL) is related to not only the other ones by collinearity, but also all the four existing columns by an equilateral triangle, and in the 50 by 50 Feet House the small table (Fig.11:O) in the living area plays a double role for connecting the house with the terrace by collinearity (Fig.11:COE) like the inner court in the Museum for a Small City (Fig.2) and for connecting storage wall with the bed by an equilateral triangle (Fig.12:△ TMU).

Needless to say, this study did not consider what the causes of Mies' transition of form, from his European forms to his American ones, were. It is nothing but a study elucidating how the geometrical relationships connected with "the successful relationship" related to the composite open plans in the 1940's. However following also Mies' words, "the important question to ask is "how," the I should report this conclusion for developing a better understanding of Mies' formal methods.

Notes
1. Mies's inaugural address as director of architecture at Armour Institute of Technology in 1938 (Johnson, Philip, MIES VAN DER ROHE, New York, 1947, pp.191-195).
4. In the 1940's there were a series of IIT's projects (Master Plan of 1940-41, Metal Research Building of 1942-43, Library and Administrating Building of 1944, Engineering Research Building of 1944-46, Alumni Memorial Hall of 1945-46, Perlstain Hall of 1945-46, Mechanical Engineering of 1948-53, Chapel of 1949-52, Museum for a Small City of 1942, Concert Hall of 1942, Caner Drive-In Restaurant of 1945-46, Farnsworth House of 1945-51, Promontry Apartments of 1946-49, 1947 Exhibition and the 50 by 50 Feet House of 1930-51. 5. The interior of the Bacardi Company of 1957 was asymmetrical and it seems to be exceptional. 6. Regarding the Canter Drive-In restaurant of 1945-46 although most of the plans in the design process had symmetrical arrangement of interior components, there were also some plans with asymmetrical arrangement (Schulze, op. cit., An Illustrated Catalogue of the Mies., vol.13, pp.273-4, pp.282-284).
8. We can certainly find collinearities in each of the plans of the Tugendhat House of 1930 (Drexler, Arthur, ed., An Illustrated Catalogue of the Mies van der Rohe Drawings in the Museum of Modern Art, Part I, vol.2, New York, 1986, p.358) and the Model House in Berlin Building Exposition of 1931 (Ibid., vol.3, p.160). However explicit similarity and equilateral triangles can not be found there. 9. In February of 1941 Mies was asked by Architectural Forum to submit a design. He accepted it and offered this project which he shared with his student, George Danforth (Schulze, op. cit., An Illustrated Catalogue of the Mies., vol.13, p.68).
10. Another court is situated at one of the short ends of the building and embraced by a stone wall. A pool is placed near another short end. These components and devices have their antecedents in his European years, especially of the Barcelona Pavilion and the Brussels Pavilion of 1934. However one major feature anticipates the clear-span structures of his American period, IIT's Crown Hall, the New National Gallery and so on. 11. Strictly speaking, there were such projects, the Adam Department Store of 1928 and the Bank Building in Stuttgart of 1928. However the details of the plans can not be known. 12. See note 9. 13. Except the plan printed in Architectural Forum (Schulze, op. cit., An Illustrated Catalogue of the Mies., p.74) there is another one reproduced later (Blaser, Werner, MIES VAN DER ROHE : less is more, New York, 1986, p.151). The two are approximately similar to each other. 14. ABCD is a rectangle of 3 × 4 proportions. Interestingly it closely relates to the Pythagorean triangle. 15. Outline efgb is the open passage is approximately similar to outline BNOC of the roofed passage. Rectangular outline ijk of the offices is also approximately similar to outline BPN enclosing the outer court and the passage area. Further outline TVUK of the pool is similar to rectangle IQRS enclosing the upper, the right side and the lower stone wall. 16. Except this floor plan printed in many books on Mies there is another plan of 1946 exhibited in the 1947 exhibition (Schulze, op. cit., An Illustrated Catalogue of the Mies., vol.13, p.93).
17. The dimensions of JKL'M's floor area are 24'-7" × 14'-6" (GA Detail Mies van der Rohe Farnsworth House, A.D.A.EDITA Tokyo, 1976, pp.42-43) and its ratio is 1:1.97. Ones of ABCD's are 55'-3" × 28'-6" (Ibid., pp.12-13) and its ratio is 1:1.93. Ones of EFGD's are 99'-3" × 52'-1/2" (Ibid.) and its ratio is 1:1.91. The similarities are also proved by dimensions. 18. In my previous study it is found that the interior space (ABCD) is devided into some small and large Golden Rectangles by the H-shaped core. That is, in the interior space there can be also found similarities (Sano Junichi, "On the Golden Ratio in the Plan of Farnsworth House by Mies van der Rohe" Journal of Architecture, Planning and Environmental Engineering, AIJ, No.466, 1994). 19. Mies' first exhibition design in America was assembled in 1939 at the Princeton University Gallery, the Saint Louis Art Museum and the Smith College Museum of Art. A later and more modest exhibition of his work was staged in 1946 at the Renaissance Society. The next was this exhibition. (Schulze, op. cit., An Illustrated Catalogue of the Mies., p.20) 20. Although the existing gallery is approximately 70 by 70 feet, strictly speaking, it is not square and part of the surrounding walls a little projects. The arrangement of the photomurals evokes Mies' European
われわれの研究でミース言うところの「部分相互、及び部分と全体との骨組み関係を達成する手段としての有機的秩序…、個々の要素に、それに相応しい場所を割り当てる秩序に」密接に係わると考えられる、主要部分相互、及び全体との関にみられる潜在的な三つの「幾何学的関係」、共線性、相似性、正三角形の各の1950年代以降の傑作、IIT・クライヴェル高校で造立されたギャラリーや建物分析からも踏出した。しかしその作品はバーゼルナ、バスピリオやダーグハット等のヨーロッパ時代の作品にみられる複雑な輪郭と骨組みから、先の簡潔な輪郭と厳格なスケルトン構成をもつアメリカ時代の作品と大きく変化している。そして「幾何学的関係」によって言えば、骨組み関係と自立的な構成のヨーロッパ時代の平面には、共線性はみられぬものの、他に相似性と正三角形は見受けがたい。

ところで彼の1950年代以降の簡潔な輪郭と厳格なスケルトン構成をもつ作品の登場は先だつて、1940年代には軽量な輪郭をもつ一方、内部構成はヨーロッパ時代の自立的な構成をみる過渡期でもあると言える混成的作品が数多く見られる。1942年の小市のための美術館（Fig.1）、1945-51年のファーンズワース邸（Fig.4）、1947年のミース邸（Fig.7）、1950-51年の50 × 50フィート住宅（Fig.10）などである。当然これら混成の平面においても「骨組み関係」は達成されている筈である。すなわち「骨組み関係」が達成のための三つの「幾何学的関係」は混成平面とどのように関係しているでしょうか。これは異なるタイプの平面に対して「幾何学的関係」がどのように関係しているかという問題であり、相似性や正三角形の平面の経験を持つ問題もある。そしてもちろなくミースの造形手法の解明につながっている。

そこで1940年以降のこれら混成的平面を次に分析することによって、相似性や正三角形が平面のどのような条件と関連を達成してきたかについて考察し、上記問題の解明を試みた。

ミースは混成的平面全てに見られたが（Fig.2,5,8,11）、相似性は主要部が複雑な矩形の輪郭をなす小市のための美術館（Fig.1）において初めて見出される（Fig.3）、次いで一層に主要部が正三角形の輪郭をもつファーンズワース邸でも確認できた（Fig.4）、正三角形は1947年ミース邸の、室内的中心軸が暗い部分を過ぎてみられた（Fig.9）、最後に正方形形平面でかつその全体が暗い部分を露出する50 × 50フィート住宅（Fig.10）においては相似性、及び正三角形の表現が確認された（Fig.11,12）。

この分析結果から、まず、相似性は簡雑的な輪郭に、正三角形は平面の中の軸に密接に関係していると言える。そしてヨーロッパ時代からアメリカ時代への形態の変化過程を示されると1940年代のダ・ハ・ニオ作品における新たなタイプの平面の出現に対して、ミースが「骨組み関係」達成のための「幾何学的関係」を、共線性から、相似性、そして正三角形と順次増やしてゆく過程が見出された。

またここには、個々の混成平面における諸要素をこれら「幾何学的関係」によって関係づけ「骨組み関係」を達成しているミースの巧みな具体的事例を示すことができた。例えば、小市のための美術館の屋内コーナーは共線性によって両側の壁面を関係づけられているばかりか（Fig.2）、相似性によって展示空間全体と関係づけられている（Fig.3）。ファーンズワース邸の平面図と室内全体、さらに建物とテラスを囲む骨組みは同心円の如き相似関係で関係づけられている（Fig.6）。1947年ミース邸の正面の独立展示パネルは共線性によって他のパネルと関係づけられているとともに、正三角形によって4本の柱とも関係づけられている（Fig.8,9）。「50 × 50フィート住宅のリビングのリタルクールは、共線性によって住宅とテラスを結びつけると同時に正三角形によって収納庫を“lift”を、またそれらを平面全体に関係づけ二役を果たす巧みな位置に置かれている（Fig.11,12）。

本研究は、言うまでもなくヨーロッパ時代からアメリカ時代への形態の変化の原因は「何か」という問題を突き詰めたものではなく、1940年代の過渡的な混成平面が「幾何学的関係」と「いかに関連していたか」と考えたしたにすぎない。しかしミースの言葉「問うべきは『いかに』である」に従い、彼の造形手法解明の一助のため、この結論を報告する。