TIMOSHENKO BEAM THEORIES FOR FOLDED PLATE
EFFECT OF MULTISPAN GABLE FRAME STRUCTURES

Part I A case of asymmetric deformations

Youichi MINAKAWA*

皆川 洋一

Multispan gable frame structures with suitable beams and braces on the roof show a significant cubic folded plate effect that reduces the displacements and the stress resultants of each gable frame. The basic equations of the structures are derived by introducing an orthotropic plate theory to the roof plate. Then, restrictions on the displacements of the roof plate enable to transform the gable frame structures with columns and end walls into Timoshenko beam theories. A case where the gable frame structures behave under asymmetric displacements with respect to the roof ridge is shown to demonstrate the procedure that gives precise solutions.

Keywords : gable frame, folded plate effect, cubic effect, Timoshenko beam, asymmetric deformation

1. Introduction

Gable frame structures are used as warehouses, gymnasiums, and factories. Cubic analyses of the structures show that there is a significant cubic folded plate effect on the displacements and the stress resultants. The cubic analyses require much data, and computing facilities and time. So they are usually analyzed as a plane frame^{12,14)} under a certain loading mode. Although the treatment may make it easy to analyze the structures, it never allows to get the cubic effect. On the other hand, there are papers that reported the folded plates effects of the structures including the gable frame. Not only approximate solutions¹⁾⁻³⁾ but also exact solutions⁵⁾⁻⁸⁾ of the folded plates have been presented. They are developed to analyze folded plate structures^{4,10,11)} composed of isotropic and homogeneous plates. However, gable frame structures are usually composed of shaped steel members. The roof plates of these structures may show orthotropic characteristics. Supposing that the roof plates of the gable frame structure have suitable bracing on the roof, it is possible to deal with the plate as orthotropic plates. At first, adopting an orthotropic plate theory to the gable frames we derive the basic equations of the cubic frame. Then, introducing some assumptions and imposing restrictions on the displacements of the roof plate we can transform the whole gable frame structures with columns and walls into beam theories including the effect of transverse shear deformations so-called Timoshenko beams¹⁵⁾. Here we describe a case where the gable frame structures taking a symmetric shape with respect to the roof ridge behave under asymmetric displacements with respect to the ridge. We derive some functionals that govern the whole cubic gable frame structures. There are two unknowns in the functionals expressed with beam theories that allow us to reduce a work load of the analysis of the multispan gable frame structures. Adopting Ritz's method to the functionals and applying them to typical gable frame structures, we show the numerical results and compare them with those obtained by cubic frame analyses.

Notations h_c : height of column, $\tan\phi$: slope of roof, a : width of roof plate, s_x : pitch of gable frame, s_y : pitch of beam, L : length of longitudinal direction, L_y : length of transverse direction, $\gamma_a = L/L_y$: shape ratio, $n_x = L/s_x$, $n_y = 2a/s_y$, E : Young's Modulus of Steel, A_y : sectional area of gable frame, I_y : moment of inertia of gable frame for strong axis, I_c : moment of inertia of column, A_x : sectional area of beam, I_{x1} : moment of inertia of beam for strong axis, I_{x2} : moment of inertia of beam for weak axis, A_{br} : sectional area of roof brace, C_1 : inplane stiffness for longitudinal direction, C_2 : inplane stiffness for transverse direction, C_{12} : inplane shear stiffness, D_1 : bending stiffness for M_x , D_2 : bending stiffness for M_y , $D_c = EI_c/s_x$, d_H : horizontal displacement of roof plate, θ_z : rotation around the inverse vertical direction at ridge

* Prof., Department of Architecture and Architectural Engineering,
Faculty of Engineering, Kagoshima Univ., Dr. Eng.

鹿児島大学工学部建築学科 教授・工博

2. An Orthotropic Plate Model of Multispan Gable Frame Structures under Asymmetric Deformations

We treat a gable frame structure that has a symmetric shape with respect to its roof ridge. The gable frame structure is composed of multiple gable frames, beams and struts between the gable frames, end walls as diaphragms, and walls at the end of the longitudinal direction. An isometric view of the structure and introduced Cartesian coordinates ($o-x, y, z$) into a roof plate are shown in Fig.1. Considering a half of the gable frames with respect to the roof ridge and applying an orthotropic plate bending theory to the roof plate, we can get the potential energy expression of asymmetric deformations with respect to the ridge, where components of force are shown in Fig.2.

$$\begin{aligned} \Pi_1^a(u, v, w, N_y, R_1, R_2) = & 2 \left[\int_0^{L/2} \left[\int_{-a/2}^{a/2} \left\{ \frac{1}{2} [C_{11} u_{,xx}^2 + 2N_{y,y} v_{,y} - N_y^2 / C_{22}] \right. \right. \right. \\ & + C_{12} (u_{,y} + v_{,x})^2 + D_{11} w_{,xx}^2 + D_{22} w_{,yy}^2 \} + q_{ag} (w \cos \phi + v \sin \phi) \cos \phi \\ & + q_{an} w \} dy + R_1 (w \cos \phi + v \sin \phi) |_{y=a/2} + R_2 (w \cos \phi + v \sin \phi) |_{y=-a/2} \\ & \left. \left. + M_{ac} w_{,y} |_{y=a/2} - H_{ac} (v \cos \phi - w \sin \phi) |_{y=a/2} \right] dx \right] \quad (1) \end{aligned}$$

where the twisting moment of the plate and the effect of Poisson's ratio are neglected. The symbols R_1 and R_2 are Lagrange multipliers corresponding to constraint of displacements. The effect of the columns of the gable frame are expressed with the stress resultants M_{ac} and H_{ac} depending upon the boundary conditions of the bottom of column and loading condition at eaves. The plate may be expressed as a model shown in Fig.3. In order to introduce inextensional in the y direction of the orthotropic plate, we adopt Hellinger-Reissner type potential energy expression for N_y . The variation of Eq.(1) is given by

$$\begin{aligned} \delta \Pi_1^a(u, v, w, N_y) = & 2 \left[\int_{-a/2}^{a/2} [N_x \delta u + N_{xy} \delta v + M_x \delta w_{,x} - V_x \delta w]_0^{L/2} dy + \int_0^{L/2} \{ [N_{xy} \delta u + (N_y + R_1 \sin \phi) \delta v + M_y \delta w_{,y} \right. \\ & + (-V_y + R_1 \cos \phi) \delta w]_{y=a/2} + [-N_{xy} \delta u + (-N_y + R_2 \sin \phi - H_{ac} \cos \phi) \delta v + (-M_y + M_{ac}) \delta w_{,y} \\ & \left. + (V_y + R_2 \cos \phi + H_{ac} \sin \phi) \delta w]_{y=-a/2} \} dx - \int_0^{L/2} \int_{-a/2}^{a/2} \{ L_u \delta u + L_v \delta v - L_w \delta w - \delta N_y (v_{,y} - N_y / C_{22}) \} dy dx \right] \quad (2) \end{aligned}$$

where the following symbols are introduced

$$L_u = C_{11} u_{,xx} + C_{12} (u_{,y} + v_{,x})_{,y}, \quad L_v = C_{12} (u_{,y} + v_{,x})_{,x} + N_{y,y} - q_{ag} \sin \phi \cos \phi \quad (2-1,2)$$

$$L_w = D_{11} w_{,xxx} + D_{22} w_{,yyy} + q_{an} + q_{ag} \cos^2 \phi, \quad N_x = C_{11} u_{,x}, \quad N_{xy} = C_{12} (u_{,y} + v_{,x}) \quad (2-3,4,5)$$

$$M_x = D_{11} w_{,xx}, \quad M_y = D_{22} w_{,yy}, \quad V_x = D_{11} w_{,xxx}, \quad V_y = D_{22} w_{,yyy} \quad (2-6,7)$$

There are two subsidiary conditions for Eq.(2)

$$w \cos \phi + v \sin \phi = 0 \text{ at } y = \pm a/2 \quad (3)$$

The boundary conditions at $y=a/2$ are given by

$$u = 0, \quad M_y = 0, \quad N_y + R_1 \sin \phi = 0, \quad -V_y + R_1 \cos \phi = 0 \text{ at } y = a/2 \quad (4-1,2,3,4)$$

Expressions Eq.(4-3,4) are the same with the following equations

Fig.1 Overall View of Gable Frames

Fig.2 Components of Force

Fig.3 Analyzed Roof Plate

$$R_1 = -N_y \sin \phi + V_y \cos \phi, N_y \cos \phi + V_y \sin \phi = 0 \text{ at } y = a/2 \quad (5-1,2)$$

The expressions show that R_1 is a vertical reaction and the horizontal component of stress resultant is zero at the edge. The boundary conditions at $y=-a/2$ are given by

$$N_{xy} = 0, -M_y + M_{ac} = 0, -N_y + R_2 \sin \phi - H_{ac} \cos \phi = 0, V_y + R_2 \cos \phi + H_{ac} \sin \phi = 0 \quad (6-1,2,3,4)$$

Expressions Eq.(6-3,4) are also expressed by

$$R_2 = N_y \sin \phi - V_y \cos \phi, H_{ac} = -N_y \cos \phi - V_y \sin \phi \quad (7-1,2)$$

The above equation show that R_2 is a vertical reaction and H_{ac} is a horizontal force at the edge.

3. A Timoshenko Beam Theory of the Multispan Gable Frame Structures

3-1 Assumptions of Beam Theories

There are some assumptions to get a beam theory including the effect of transverse shear of the gable frame structures.

1) The twisting moment is neglected in Eq.(1). Furthermore, assume the following equation

$$D_t = 0 \quad (8)$$

2) The extension of the columns of the gable frame is neglected. Introducing the multiplier R_2 in Eq.(1), we have already considered the assumption.

3) The inplane stiffness C_2 is large enough to assume that the roof plate is inextensional in the y direction. Then we assume the displacement v is an only function of parameter x .

$$v(x, y) = d_H(x) \cos \phi \quad (9-1)$$

4) The displacement u is proportional to the horizontal distance from the ridge. Using the rotation θ_z around the inverse vertical direction at the ridge, it is expressed by

$$u(x, y) = -\theta_z(x)(y - a/2) \cos \phi \quad (9-2)$$

3-2 A Derivation of a Beam Theory of the Multispan Gable Frame Structures

Considering the deformation expressed by Eq.(9-1) and the subsidiary condition Eq.(3), w and δw are given by

$$w = -d_H \sin \phi, \delta w = -\delta d_H \sin \phi \text{ at } y = \pm a/2 \quad (10)$$

Introducing these, Eq.(8) and Eq.(9) into Eq.(2), and integrating with respect to y , we have the expression

$$\begin{aligned} \delta \Pi_b(\theta_z, d_H, w, N_y) = & 2 \left[\frac{1}{2} [M_z^a \delta \theta_{z,x} - Q_z^a \delta d_H]_0^{L/2} + \int_0^{L/2} \{ [N_y \cos \phi + V_y \sin \phi] \delta d_H + M_y \delta w_{,y} \}_{y=a/2} \right. \\ & + \{ - (N_y \cos \phi + V_y \sin \phi - H_{ac}) \delta d_H + (-M_y + M_{ac}) \delta w_{,y} \}_{y=-a/2} - \frac{1}{2} (M_{z,x}^a + Q_z^a) \delta \theta_z - \frac{1}{2} Q_{z,x}^a \delta d_H \\ & \left. - \int_{-a/2}^{a/2} \{ (N_{y,y} - q_{ag} \sin \phi \cos \phi) \delta d_H \cos \phi - (D_2 w_{,yyyy} + q_{an} + q_{ag} \cos^2 \phi) \delta w - \delta N_y (v_{,y} - N_y / C_2) \} dy \right] dx \quad (11) \end{aligned}$$

$$\text{where } M_z^a = 2(C_1 a^3 / 3) \theta_{z,x} \cos^2 \phi, Q_z^a = 2C_{12} a (d_{H,x} - \theta_z) \cos^2 \phi \quad (11-1,2)$$

The subsidiary conditions for the variation are given by Eq.(3) and Eq.(9). Here we can define solutions $w(y), M_y(y), N_y(y)$ that satisfy the following equilibrium equations of the roof plate corresponding to inplane y direction and the normal direction, and the boundary conditions at $y = a/2$ and $y = -a/2$.

$$N_{y,y} - q_{ag} \sin \phi \cos \phi = 0, D_2 w_{,yyyy} + q_{an} + q_{ag} \cos^2 \phi = 0 \text{ in } y \in [-a/2, a/2] \quad (12-1,2)$$

$$N_y \cos \phi + V_y \sin \phi = 0, M_y = 0, w = -d_H \sin \phi \text{ at } y = a/2 \quad (13-1,2,3)$$

$$w = -d_H \sin \phi, -M_y + M_{ac} = 0 \text{ at } y = -a/2 \quad \text{where } M_{ac} = M_{cl}/s_x = (3D_c/h_c)(w_{,y} + d_H/h_c) - q_{ac} h_c^2 / 8 \quad (14-1,2)$$

The bending moment M_{ac} corresponds to pinned support at the bottom of column shown in Appendix I, where q_{ac} shown in Fig.2 is equivalent to q_c in Fig.a1. The five unknowns that appear when integrating Eq.(12) are defined by the five boundary conditions expressed in Eqs.(13) and (14). On the other hand, the last term in Eq.(11) permits the displacement v given by

Eq.(9-1) that means inextensional in the y direction

$$M_{ac} = \{3D_c d_H / h_c^2 - (q_{an} + q_{ag} \cos^2 \phi) a^2 / 8 - q_{ac} h_c^2 / 8\} / (1 + \gamma) \quad (15-1)$$

$$N_y \cos \phi + V_y \sin \phi = q_{an} a (1/2 - y/a) \sin \phi \quad (15-2)$$

$$N_y = \tan \phi [q_{ag} \{y(1 + \gamma) - \alpha \gamma / 8\} \cos^2 \phi + q_{an} a (4 + 3\gamma) / 8 - q_{ac} h_c^2 / (8a) - 3D_c d_H / (2ah_c^2)] / (1 + \gamma) \quad (15-3)$$

$$w = \{[5 - 4y^2/a^2 - 2\gamma(2y/a - 1)(y/a - 1)](q_{an} + q_{ag} \cos^2 \phi) a^2 + (2y/a - 3)q_{ac} h_c^2\} a^2 (y^2/a^2 - 1/4) / \{96D_2(1 + \gamma)\} \\ + \{(y/a + 1/2)(y/a - 1/2)(3 - 2y/a)(a/h_c)\gamma / \{4(1 + \gamma)\} - \sin \phi\} d_H \\ \text{where } \gamma = a D_c / (h_c D_2)$$

Supposing following horizontal force H_{ac} and Eq.(14-2) at $y = -a/2$, we have

$$M_{ac} = M_{ct} / s_x, \quad H_{ac} = Q_{ct} / s_x + H_a = -M_{ad} / h_c + q_{ac} h_c / 2 + H_a \quad (16)$$

where M_{ct} and Q_{ct} are the bending moment and the shear force carried by the top of the column, respectively. They are defined in Appendix I. Introducing Eqs.(12), (13), (14), and (16) into Eq.(11), we can get the following expression

$$\delta \Pi_b^a(\theta_z, d_H) = 2 \left[\frac{1}{2} [M_z^a \delta \theta_{z,x} - Q_z^a \delta d_H]_0^{L/2} + \int_0^{L/2} \left\{ -\frac{1}{2} (M_{z,x}^a + Q_z^a) \delta \theta_z + \left(-\frac{1}{2} Q_{z,x}^a + \frac{1}{2} k_{ac} \right) \delta d_H \right. \right. \\ \left. \left. - (C_{an} + C_{ag} + C_{ac} + H_a) \delta d_H \right\} dx \right] \quad (17)$$

where $k_{ac} = 6D_c / \{(1 + \gamma)h_c^3\}$, $C_{ag} = \gamma q_{ag} a^2 \cos^2 \phi / \{8h_c(1 + \gamma)\}$,

$$C_{an} = q_{an} a \{(1 + \gamma) \sin \phi + \gamma a / (8h_c)\} / (1 + \gamma), \quad C_{ac} = (5 + 4\gamma) q_{ac} h_c / \{8(1 + \gamma)\} \quad (18-1, 2, 3, 4)$$

Expression (18) is defined under the condition that the bottom of the column is the pinned support. It is possible to derive the equivalent expressions under the condition where the bottom is assumed by the fixed end or others.

3-2 Some Functionals of the Timoshenko Beam Theory of the Multispan Gable Frame Structures

A functional of which variation is expressed by Eq.(17) is defined by

$$\Pi_b^a(\theta_z, d_H) = 2 \left[\int_0^{L/2} \left[\frac{1}{4} \{ (EI)_z^a \theta_{z,x}^2 + (GA)_z^a (d_{H,x} - \theta_z)^2 + k_{ac} d_H^2 \} - (C_{an} + C_{ag} + C_{ac} + H_a) d_H \right] dx \right] \quad (19)$$

where $(EI)_z^a$ is flexural rigidity of the whole roof with respect to vertical direction at the ridge, $(GA)_z^a$ is a shear rigidity of the whole roof in the horizontal direction, which are defined in Appendix II. There are two unknown functions d_H and θ_z in the functional. The expression is a potential energy of the case that the gable frame structures show asymmetric deformations under the force q_{an} , q_{ag} , q_{ac} , and H_a . In order to derive the potential energy expression of transformed beam theory we don't assume that $(EI)_z^a = \text{constant}$, and $(GA)_z^a = \text{constant}$ in the whole length. Then Eq.(19) is valid for the case where $(EI)_z^a$ and $(GA)_z^a$ are functions with respect to x . While deriving the third and fourth terms in Eq.(19), we suppose that each gable frame is distributed along the x directions. If we assume that each gable frame is installed at $x = x_j$, and equivalent span s_{xj} , we have

$$\Pi_b^a(\theta_z, d_H) = 2 \left[\int_0^{L/2} \left[\frac{1}{4} \{ (EI)_z^a \theta_{z,x}^2 + (GA)_z^a (d_{H,x} - \theta_z)^2 \} dx + \sum_j \left\{ \frac{1}{2} s_{xj} k_{ac} d_H^2 - (C_{an} + C_{ag} + C_{ac} + H_a) d_H \right\} \right] \right] \quad (20)$$

where each gable frame can take independent values of k_{ac} , C_{an} , C_{ag} , C_{ac} , and H_a .

Since the functional of Eqs.(19) and (20) have simple expressions, we can introduce various effects on the potential energy that are neglected at the start. Here we show the way how to deal with the deformation of the end walls and the stiffness of the longitudinal direction including the weak stiffness of the columns. Supposing the stiffness of the whole end wall $2K_{ew}$, those of the longitudinal end wall K_{Lw} and the longitudinal stiffness along the columns k_2 for unit length of the direction, and introducing them into Eq.(19), we have the following expression corresponding to a continuous model of columns

$$\begin{aligned} \Pi_{bC}^a(\theta_z, d_H) = & 2 \int_0^{L/2} \left[\frac{1}{4} \{ (EI)_z^a \theta_{z,x}^2 + (GA)_z^a (d_{H,x} - \theta_z)^2 + k_{ac} d_H^2 + 2k_2 \theta_z^2 a^2 \cos^2 \phi \} \right. \\ & \left. - (C_{an} + C_{ag} + C_{ac} + H_a) d_H \right] dx + K_{ew} d_H^2 |_{x=L/2} + K_{Lw} \theta_z^2 |_{x=L/2} a^2 \cos^2 \phi \end{aligned} \quad (21)$$

We denote C-model in which Eq.(21) is used. Timoshenko beam theory cannot evaluate the shear deformation caused by the spring k_2 and K_{Lw} . If we adopt a higher order theory of a beam¹³⁾, it is possible to evaluate the shear deformation. Then, we neglect the deformation here. Adopting the discrete springs of the gable frame and discrete springs K_{2j} at $x=x_j$ of the stiffness of the longitudinal direction, we have the following potential energy of a discrete model of columns and other stiffness in the longitudinal direction

$$\begin{aligned} \Pi_{bD}^a(\theta_z, d_H) = & 2 \int_0^{L/2} \left[\frac{1}{4} \{ (EI)_z^a \theta_{z,x}^2 + (GA)_z^a (d_{H,x} - \theta_z)^2 \} dx + K_{ew} d_H^2 + K_{Lw} \theta_z^2 a^2 \cos^2 \phi \right. \\ & \left. + \sum_j [s_{xj} \{ \frac{1}{2} k_{ac} d_H^2 (C_{an} + C_{ag} + C_{ac} + H_a) d_H \} + K_{2j} (\theta_z(x_j))^2 a^2 \cos^2 \phi] \right] \end{aligned} \quad (22)$$

We denote D-model where Eq.(22) is used. Expressions (21) and (22) permit us to examine the effect of not only the deformations of end walls but also the longitudinal stiffness installed orthogonal to the displacement of the roof.

Fig.4-1 T_1 with $n_x=4$ Fig.4-2 Bracing Type T_1 with $n_x=24$ Fig.5-1 T_2 with $n_x=4$ Fig.5-2 Bracing Type T_2 with $n_x=24$

4. Analyses of the GableFrame Structures

4-1 Analyzed Model and Force

The force components (unit tonf, cm) per unit length of longitudinal direction of the gable frame are assumed by

$$q_{an} = 0.65 p_r \sin \phi, \quad q_{ac} = 0.3 p, \quad q_{ag} = 0, \quad H_a = 0.15 p h_c \quad (23-1,2,3)$$

$$\text{where } p = 60 \sqrt{h_c / 2} / 10^8 \text{ tonf/cm}^2, \quad p_r = 60 \sqrt{h_c + (a/2) \tan \phi} / 10^8 \text{ tonf/cm}^2 \quad (23-4,5)$$

The forces are asymmetrical component of the wind load of a gable frame adopted in Japan, where part of force on windward and leeward walls is assumed to transfer to the strut setting between the gable frames. We adopt two types of roof bracing T_1 and T_2 . The number of spans of the gable frame structure n_x is a parameter of the analyses and will take several numbers 2, 4, 8, 12, 16, 20, and 24, where every s_{xj} is the same length s_x . The roof plans of the types T_1 and T_2 for the cases $n_x = 4$ and 24 are shown in Fig.4 and Fig.5. Depending upon $n_x = 4$ and 24, they have 41 and 181 nodes in whole structures. Since a stress-strain relation in Eq.(II-3) in Appendix II is adopted in the present analyses, it is certain that the bracing type T_1 is more appropriate model to the procedures than the bracing type T_2 which is one of typical examples that has small number of bracing members on the roof. Supposing gable frame structures composed of steel members of which Young's Modulus $E=2100 \text{ tonf/cm}^2$, we use the following values except the sectional area of the roof braces. Set each gable frame with $h_c=600 \text{ cm}$, $\tan \phi=0.5$, $a=1,341.64 \text{ cm}$, $s_x=600 \text{ cm}$ and each intermediate gable frame has $I_y=33,500 \text{ cm}^4$, $A_y=96.76 \text{ cm}^2$. The end gable frames at the both ends in longitudinal direction has $I_y=16,750 \text{ cm}^4$, $A_y=48.38 \text{ cm}^2$, which are the half values of the intermediate frame. There are two braces with sectional area 8.727 cm^2 at each corner of the roof corresponding to both longitudinal and transverse directions as shown in Fig.1. These braces are evaluated as two springs K_{ew} and K_{Lw} at the corner for analytical models. The struts at the ends have sectional area 32.68 cm^2 , where the flexural rigidity of the struts is neglected. Each beam has sectional area $A_x=23.18 \text{ cm}^2$, moment inertia $I_{x1}=1580 \text{ cm}^4$ for strong axis and $I_{x2}=114 \text{ cm}^4$ for weak axis. One plate of the analytical models has three beams installed at equal interval $s_y=670.82 \text{ cm}$. One of them setting at eaves has strong axis being horizontal direction. Two of them are set to have strong axis being parallel to the plate. Giving

negligible small values for the other constants of members, we can construct two standard gable frame structures T_1 (with sectional area of each roof brace 3.7635cm^2) and T_2 (with sectional area of each brace 15.04cm^2) for cubic frame analyses where number of span is a parameter and every joint is rigid-joint. When T_1 and T_2 have these sectional areas, it is evaluated that they have the same $(EI)_z^a$ and $(GA)_z^a$ in Eqs.(21) and (22). We neglect the moment of inertia for weak axis of each gable frame which have few effect on the deformation of the structures. For analytical modes we evaluate that $K_{cw} = K_{Lw} = 10.15\text{tonf/cm}$ corresponding to the stiffness of walls.

4-2 Analysis of the Timoshenko Beam Theory of the Multispan Gable Frame Structures

Depending upon the potential energy expressions in the previous section, we can get two differential equations with the unknown θ_z and d_H . The coefficients of equations have constants in arbitrary intervals of x . So we can define the analytical solution of the basic equations. However, it may be reasonable to seek the analytical solution at the case that these coefficients are constants being independent of x . Examining solutions where these coefficients have variable values basing on x , we may adopt the Ritz's method or the Galerkin's method where the unknowns are assumed with Fourier series. Considering the deformation of the end walls, we can assume the solutions of these unknowns in the following Fourier series

$$d_H = d_{H0} + \sum_{k=1,3}^{2N_k-1} d_{Hk} \cos \frac{k\pi x}{L}, \quad \theta_z = \sum_{k=1,3}^{2N_k-1} \theta_{zk} \sin \frac{k\pi x}{L} \quad (24)$$

Applying the expressions to the potential energy expressions (21) or (22), we can analyze the gable frame structure.

4-3 Convergence of the Presented Solutions

Setting $n_x=8$ and adopting D-model, we examine convergence of the solution with respect to the number of Fourier series. Horizontal displacement at the center $d_H(0)$ and the shear stress resultant of the roof plate at the end $Q_z^a(L/2)$ vs number of the Fourier series are shown in Fig.6, where N_k takes from 1 to 21. It seems that the solution may converge well. Then in the following description of this paper we use the solution where $N_k=11$.

Fig.6 Convergence of Disp. d_H & Shear Force Q_z^a

4-4 Comparison of the Analytical Solutions with Cubic Frame Analyses

Setting $n_x=2, 4, 8, 12, 16, 20$, and 24 and adopting the sectional area for each roof brace of T_1 and T_2 , we establish the gable frame structures with multiple span. A cubic frame analyzing procedure enables us to analyze the structures. Applying C-model and D-model to them, we can analyze the same structures. We show the displacement $d_H(0)$ obtained by the presented methods and the cubic frame analyses of T_1 and T_2 in Table1, where the values in the parentheses are the ratio with respect to the value obtained by D-model. The table shows two points. One is that the displacements of T_1 and T_2 have near equal values for each longitudinal span. The other is that D-model allows us to get quite precise displacements of T_1 in the longitudinal span L from $1,200\text{cm}(\gamma_a=0.5)$ to $14,400\text{cm}(\gamma_a=6)$, of which maximum difference is within 3.5%. The horizontal displacements at the center of the eaves are shown in Fig.7, where the solid line and the dashed line correspond to those obtained by D-model and C-model, respectively, and $\bigcirc$ and $\triangle$ indicate those of T_1 and T_2 obtained by cubic frame analyses, respectively. We show the bending moment of the center gable frame at the eaves, in other words, at the top of the column vs the number of span in Fig.8, where the lines and notations correspond to the same analyzing procedures as those in Fig.7. In these figures the displacement and the moment obtained by a plane analysis are also illustrated, which show the

Table 1. Horizontal Displacement d_H at the Center of Gable Frame Structures (unit; cm)

Number of Span	2	4	8	12	16	20	24
Span $L(\text{cm})$	1,200	2,400	4,800	7,200	9,600	12,000	14,400
Present D-model	0.494(1)	1.208(1)	3.193(1)	5.640(1)	8.070(1)	10.10(1)	11.59(1)
Present C-model	0.488(0.988)	1.176(0.974)	3.069(0.961)	5.409(0.959)	7.768(0.963)	9.77(0.968)	11.27(0.972)
Cubic Frame T_1	0.477(0.966)	1.215(1.006)	3.194(1.000)	5.597(0.992)	7.965(0.988)	9.94(0.984)	11.42(0.985)
Cubic Frame T_2		1.263(1.046)	3.170(0.993)	5.804(1.029)	8.032(0.995)	10.22(1.012)	11.50(0.992)

Fig.7 Horizontal Disp. d_H vs Number of Span

Fig.8 Moment at Top of Column vs No. of Span

significant cubic folded plate effect.

4-5 Some Effective Parameters on the Gable Frame Structures

In order to examine the effect of increasing the sectional area of each roof brace or the area of beam upon the horizontal displacement, we construct two cases. One has twice as large sectional area of roof brace as the standard gable frame structure, which is denoted as Case DA_{br} . The other has twice as large the area of beam as the standard frame, which is denoted as Case DA_b . Horizontal displacement ratio of these two cases to the displacement of the standard structure are depicted in Fig.9, where a dotted line with $\blacktriangle$ and a dot-dash line with $\blacksquare$ correspond to Case DA_{br} and Case DA_b , respectively. Both curves have minimum ratios with respect to the longitudinal length (n_x). However, the Case DA_{br} allows to decrease the ratio more effective than the Case DA_b in the region $\gamma_s \in \{0.5, 6.0\}$. The roof plans of $\gamma_s=1$ and 6 are already shown in Figs.4 and 5. The region may include ordinary gable frame structures. Then, increasing the sectional area of the roof brace may be more effective than increasing the sectional area of beam in order to reduce the horizontal displacement at the mid-span. It shows that the shear deformation is more dominant than the bending deformation in the roof plate in this region. However, the deformation of end walls is larger than that of the roof in the region of small shape ratio γ_s . Then in order to reduce the horizontal displacement of the structures with small shape ratio, it is important to install end walls (diaphragms) with fairly rigid stiffness.

Fig.9 Horizontal Disp. Ratios vs Number of Span

5 Conclusion

1. Introducing an orthotropic plate theory into a roof plate of multispan gable frame structures that have a symmetric shape with respect to the ridge, we derived basic equations of a continuum of the structures at the case that they behave under asymmetric force with respect to the ridge, where the effect of existing columns is considered.
2. Imposing restrictions on the displacements of the plate in the asymmetric deformations, we transformed the roof plate into a beam including the effect of transverse shear deformation, so-called Timoshenko beam theory.
3. Applying the Ritz's method to the transformed Timoshenko beam theory, we analyzed some examples. The results show that the theory allows us to obtain quite precise solutions of the displacements and stress resultants of the each gable frame.
4. Examining various sectional area of beam and bracing of the roof plate, it is certain that increasing the area of bracing members may generally have more effective than increasing the area of beams to strengthen the folded plate effect of the roof plate with a shape ratio smaller than 5 or 6.

Acknowledgment

The author would like to express his thanks to persons in charge in Nippon Steel Corporation in Japan who asked him to evaluate the cubic effect of the multispan gable frame structures. This is a part of work that the author studied in Aerospace & Ocean Dept., Virginia Polytechnic and State University (Virginia Tech), U.S.A. where he had visited professors R.Haftka and R.Kapania as a visiting scholar. The author thanks from the bottom of his heart to both professors and other persons in U.S.A. who showed a good deal of kindness and generosity to the stranger.

Appendix I

In order to get simple analyses we adopt the following model where all columns are represented by boundary conditions. Suppose that columns are inextensional. The relations between the end stress resultants and the rotations of the member in Fig.a1 are expressed by

$$M_{ab} = \frac{2EI}{h_c} (2\theta_a + \theta_b - 3R) + s_x q_c h_c^2 / 12 \quad (\text{I-1})$$

$$M_{ba} = \frac{2EI}{h_c} (2\theta_b + \theta_a - 3R) - s_x q_c h_c^2 / 12 \quad (\text{I-2})$$

$$Q_{ba} = - (M_{ab} + M_{ba}) / h_c + s_x q_c h_c / 2 \quad (\text{I-3})$$

where EI is flexural rigidity of a column.

Supposing that $M_{ab} = 0$, and eliminating the rotation θ_a in Eq.(I-1,2), we have the moment expression for pinned supports at bottom as shown in Fig.a2.

$$M_{ba} = \frac{3EI}{h_c} (\theta_b - R) - s_x q_c h_c^2 / 8 \quad (\text{I-4})$$

Introducing the following parameters used in this paper

$$EI = s_x D_c, \quad \theta_b = w_{,y}, \quad R = -d_H / h_c, \quad M_{ba} = M_{ct}, \quad Q_{ba} = Q_{ct} \quad (\text{I-5})$$

we have the expressions of a moment and a shear force of every column at the top of columns

$$M_{ct} = s_x \{ 3(D_c / h_c)(w_{,y} + d_H / h_c) - q_c h_c^2 / 8 \}, \quad Q_{ct} = -M_{ct} / h_c + s_x q_c h_c / 2 = s_x \{ -3(D_c / h_c^2)(w_{,y} + d_H / h_c) + 5q_c h_c / 8 \} \quad (\text{I-6,7})$$

where $w_{,y}$ and d_H are a rotation and a sway displacement at the top of column respectively, and h_c is a height of the column.

Suppose that the bottom of the columns is the fixed ends. Introducing $\theta_a = 0$ in Eq.(I-1,3), then we have the moment and the shear force are expressed by the following

$$M_{ct} = s_x \{ 2(D_c / h_c)(2w_{,y} + 3d_H / h_c) - q_c h_c^2 / 12 \} \quad (\text{I-8})$$

$$Q_{ct} = s_x \{ -6(D_c / h_c)(w_{,y} + 2d_H / h_c) + q_c h_c / 2 \} \quad (\text{I-9})$$

Appendix II

II-1 Equivalent Stiffness

Suppose that each gable frame with sectional area A_y and moment of inertia I_y is installed in every distance s_x , each beam with sectional area A_x and I_x in x direction is installed in every distance s_y , and there are cross braces as shown in Fig.a3. At first we define the equivalent inplane constitutional law⁹⁾. Taking into account the truss in a rectangular unit consisted of two beams with $A_x/2$ and two gable frame with $A_y/2$, and two braces where each brace has the sectional area $A_{br}/2$, the strain energy of the unit area U is given by

$$U = \frac{1}{2} \{ \epsilon \}' \begin{bmatrix} C_1 & C_{12} & 0 \\ C_{12} & C_2 & 0 \\ 0 & 0 & C_{12} \end{bmatrix} \{ \epsilon \}, \quad \text{where } \{ \epsilon \} = \{ \epsilon_x \quad \epsilon_y \quad \gamma_{xy} \} \quad (\text{II-1})$$

$$C_1 = EA_x(1 + c^3 A_{br}/A_x)/s_y, \quad C_2 = EA_y(1 + s^3 A_{br}/A_y)/s_x, \quad C_{12} = EcsA_{br}/\sqrt{s_x^2 + s_y^2}, \quad c = s_x/\sqrt{s_x^2 + s_y^2}, \quad s = s_y/\sqrt{s_x^2 + s_y^2} \quad (\text{II-2})$$

When the truss has only one brace with A_{br} in Fig.a4, the constitutional law doesn't take the expression in Eq.(II-1). However, neglecting non-diagonal terms, we adopt the following constitutional law in the unit area for the analysis.

$$U \approx \frac{1}{2} \{ \epsilon \}' \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_{12} \end{bmatrix} \{ \epsilon \} \quad (\text{II-3})$$

Fig. a1 Moment & Rotation

Fig. a2 Pinned Support at the Bottom

Fig.a3 Cross Brace at an Unit

Fig.a4 Single Brace at an Unit

These diagonal terms have the same values in both truss units. Neglecting moment of inertia of the roof brace, we get

$$D_1 = EI_x / s_y, D_2 = EI_y / s_x \quad (\text{II-4})$$

The stiffness of each column with moment inertia I_c is obtained in a same way as D_2

$$D_c = EI_c / s_x \quad (\text{II-5})$$

II-2 Equivalent stiffness for a transformed beam

Assume that there are suitable bracing members on the roof so that the whole roof can behave like a beam. The flexural rigidity of the whole roof is evaluated from Eq.(11-1)

$$(EI)_z^a = (2/3)C_1 a^3 \cos^2 \phi = (2/3)\{EA_x(1+c^3 A_{br}/A_x)/s_y\} a^3 \cos^2 \phi \quad (\text{II-6a})$$

Evaluating the flexural rigidity under the model where a member with sectional area A_x is installed in every distance s_y , we have

$$(EI)_z^a = 2 \sum_j^{a/s_y} s_y C_1 (j s_y)^2 \cos^2 \phi = (2/3)\{EA_x(1+c^3 A_{br}/A_x)/s_y\} a^3 (1+s_y/a)(1+s_y/(2a)) \cos^2 \phi \quad (\text{II-6b})$$

Considering a case that s_y/a is not near zero, we adopt the expression (II-6b) in the present analyses. Adding up shear stiffness of the rectangular unit with roof brace in the two plates at the section $x=\text{constant}$, we have

$$(GA)_z^a = 2 \sum_{m=1}^{a/s_y} \{EA_{br} s_x s_y^2 / (s_x^2 + s_y^2)^{3/2}\} \cos^2 \phi \quad (\text{II-7})$$

If there is no bracing member in a unit m in Eq.(II-7), the sectional area A_{br} is zero for the unit at the section.

References

- 1) A.C.Scordelis: A MATRIX FORMULATION OF THE FOLDED PLATE EQUATIONS, Proc. ASCE, Vol.86, NO.ST10, 1960, pp.1-22
- 2) D.Yitzhaki, M.Reiss: ANALYSIS OF FOLDED PLATES, Proc. ASCE, Vol.88, No.ST5, 1962, pp.107-141
- 3) J.L.Meek: MATRIX DERIVATION OF THE FOLDED PLATE EQUATIONS, Proc. ASCE, Vol.89, No.ST3, 1963, pp.77-105
- 4) Report of the Task Committee: PHASE I REPORT ON FOLDED PLATE CONSTRUCTION, Proc. ASCE, Vol.89, No.ST6, 1963, pp.365-406
- 5) A.DeFries-Skene, A.C.Scordelis: DIRECT STIFFNESS SOLUTION FOR FOLDED PLATES, Proc. ASCE, Vol.90, No.ST4, 1964, pp.15-47
- 6) P.E.Mast: NEW METHOD FOR EXACT ANALYSIS OF FOLDED PLATES, Proc. ASCE, Vol.93, No.ST2, 1967, pp.453-476
- 7) E.Goldberg, M.J.Gutzwiller, R.H.Lee: ANALYTICAL AND MODEL STUDIES OF CONTINUOUS FOLDED PLATES: Proc. ASCE, Vol.94, No.EM5, 1968, pp.1127-1158
- 8) Y.K.Cheung: FOLDED PLATE STRUCTURE BY FINITE STRIP METHOD, Proc. ASCE, Vol.95, NO.ST, 1969, pp.2963-2979
- 9) 日置興一郎, 坂寿二: 立体トラス平板の解析, 日本建築学会論文報告集, 第157号, 昭和44年, pp.33-39
- 10) J.S.B.Iffland: Folded Plate Structures, Proc. ASCE Vol.105, ST1, 1979, pp.111-123
- 11) A.A.Wahab, A.M.Mousa: Folded Plate with Intermediate Flexible Supports, Proc. ASCE Vol.105, ST10, 1979, pp.111-123
- 12) V.B.Watwood: Gable Frame Design Considerations, J. Structural Engineering, Vol.111, No.7, 1985, pp.1543-1558
- 13) 皆川洋一: 軸方向および横方向に分布荷重を受ける直線曲げ材の近似閉解とその性質について, 日本建築学会論文報告集, 第157号, 昭和63年, pp.52-65
- 14) A.Rutenberg, A.Scarlet: ROOF BRACING AND EFFECTIVE LENGTH OF COLUMNS IN ONE STORY INDUSTRIAL BUILDINGS, J. Structural Engineering, Vol.116, No.10, 1990, pp.2551-2566
- 15) C.L.Dym, I.H.Shames: Solid Mechanics: A Variational Approach, McGraw-Hill (Japanese Translation)
- 16) 皆川洋一: 多スパン山形フレーム構造の折板効果の解析, 日本建築学会大会学術講演梗概集, 構造I, 1995年, pp.657-658

和文要約

1. はじめに

多スパンを有する鉄骨造の山形フレームは屋根面に梁やブレースが適切に配置されると、折板構造としての三次元効果を発揮し、各山形フレームの変位や応力は著しく小さくなる。立体ラーメン解析を適用してこの折板効果を検討する事ができるものの、多くのデータを取り扱わねばならない。多スパン山形フレームは2枚の板を用いた折板構造(Folded Plates)と考えることも可能である。各板が等方性である折板構造の解析手法はいくつか提案されている。鉄骨造の多スパン山形フレームは通常折板効果を見捨てた二次元の山形フレームとして解析されているので、大きな断面を有する山形フレームに比較的断面の小さな梁やブレースが屋根面に取り付けられる構造となっている。このような多スパン山形フレームの屋根板は直交異方性の板として挙動しているものと考えられる。故に、この種の山形フレームに折板構造の解析法を適用することは適当ではないものと考えられる。山形フレームの軸方向の伸縮を見捨て、山形フレームのみが曲げ剛性を有すると仮定すると、屋根板は剪断変形を考慮した梁に置換できる。ここでは、棟に関して対称形を為す山形フレームが棟に関して逆対称変形の外力を受けて逆対称変形をする場合におけるこの構造体の梁への置換方法を示す。屋根板が剪断変形を考慮した梁理論(Timoshenko梁)に置換され、両側に配置される妻壁、および両端部に配置された桁行き方向の壁や山形フレームの弱軸周りの剛性も考慮できる多スパン山形フレームの挙動を支配する汎関数を誘導し、Ritz法を適用してこれを解析する方法を示す。この手法から得られる山形フレームの変位や応力は節点を剛とした立体ラーメンの解析から得られたそれらと良く一致する。

2. 逆対称変形する多スパン山形フレームの直交異方性板モデル

山形フレームの棟に関して逆対称変形する多スパン山形フレームの屋根面に直交異方性を導入した汎関数は(1)式のように表される。Lagrange乗数 R_1, R_2 を導入して、棟が鉛直方向に変位しないこと(逆対称変形変形)、および柱の伸縮がないと仮定したことに対応するふたつの拘束を加えている。また、柱頭から屋根板に伝達される曲げモーメント M_{Σ} 、および軒位置での水平力(柱から伝達される剪断力と軒位置での外力) H_{Σ} が形成する外力仕事を導入し、柱が存在することを考慮に入れている。山形フレームが伸縮しない条件を導入するため、膜力 N_y が関与するひずみエネルギーはHellinger-Reissner型のそれを用いている。(1)式の変分式は(2)式のように表される。屋根板の満足すべき3本のつりあい式は(2-1,2,3)式、棟および軒での境界条件はそれぞれ(4)式、および(6)式となる。変分式の与える境界条件式から(1)式において導入したLagrange乗数 $R_1, R_2, M_{\Sigma}, H_{\Sigma}$ の物理的な意味が明らかにされる。

3. 多スパン山形フレームをTimoshenko梁に置換する方法

次の4個の仮定を導入して、逆対称変形する多スパン山形フレームをTimoshenko梁へ置換した汎関数を得る。

- 1) 屋根板の振りモーメントを見捨てると共に曲げ剛性 D_1 がゼロである。
- 2) 柱は伸縮しない。この仮定は(1)式にLagrange乗数 R_2 を導入して既に考慮に入れている。
- 3) 面内剛性 C_2 は十分大きく、屋根板はy方向に伸縮しない。故に、屋根板の変位 v は(9-1)式のように表される。
- 4) 桁行き方向の変位 u は棟からの水平方向の距離と棟位置での回転を用いて(9-2)式のように表される。

これらの条件を導入すると、変分式は(11)式のように表される。また、これらの仮定のもとで(2-2,3)式は(12-1,2)式のように表される。これらを(13), (14)式の境界条件のもとで解析する。得られた変位 $w(y), N_y(y)$ を(11)式へ代入すると、(17)式の変分式が得られる。これを変分式とする汎関数は(19)式のように表される。妻壁の剛性、柱の弱軸方向の剛性、桁行き方向端部の壁の剛性を追加して、多スパン山形フレームの汎関数は(21)、あるいは(22)式のようにふたつの未知関数 d_H, θ_z を用いて表される。前者は柱の剛性が桁行き方向に分布したモデル(C-model)であり、後者は柱がそれぞれの位置に離散的に配置されたモデル(D-model)に対応する。

4. 多スパン山形フレームの解析

4-1 解析モデルと外力

風荷重の逆対称成分(23)式を外力とする。フレームの構成部材は全て鋼であるとし、ヤング率を $E=2,100\text{tonf/cm}^2$ とする。屋根勾配0.5、梁間方向スパン2,400cm、軒高600cm、中間の山形フレームの断面を $I_y=33,500\text{cm}^4, A_y=96.76\text{cm}^2$ 、両側の山形フレームの断面は中間フレームの1/2とし、これらはFig.1のようにそれぞれ3本の間柱(断面積 32.68cm^2)で支えられている。桁行き方向のスパン数 n_x をパラメータとし、2から24まで変化させる。屋根の隅点に桁行き方向、および梁間方向にFig.1のようにブレース(断面積 8.727cm^2)が配置されている。これらのブレース壁の水平剛性は

$K_{ew} = K_{Lw} = 10.15 \text{ tonf/cm}$ と算定される。一枚の屋根板に等間隔で3本配置された梁の断面は $A_x = 23.18 \text{ cm}^2$, $I_{x1} = 1,580 \text{ cm}^4$, $I_{x2} = 114 \text{ cm}^4$ とする。屋根のブレースはふたつのタイプを考える。屋根の梁と山形フレームが構成する全ての矩形領域にブレース（断面積 3.7635 cm^2 ）をクロスに配置した T_1 タイプ、および一枚の屋根板に Warren トラス型にブレース（断面積 15.04 cm^2 ）を配置した T_2 タイプである。桁行き方向の任意の断面での屋根ブレースの断面積は両方のタイプとも同一となる。スパン数 n_x を4、および24とした T_1 および T_2 の屋根板の平面図を Fig.4 および Fig.5 に示す。これらのフレームは $n_x = 4$ 、および24のときそれぞれ41、および181節点を有する。屋根板の構成則として、(II-3)式が利用しているので、屋根ブレースがクロスに配置された T_1 は本報で対象としている多スパン山形フレームにふさわしいモデルであり、 T_2 は屋根ブレースが最も少ないモデルのひとつの例として採用された。立体ラーメン解析では柱脚はピンとし、数値を示した断面積および断面二次モーメント以外は解に影響を与えない小さな断面性能値を入れて連立一次方程式のイル・コンデションを解消した。

4-2 Timoshenko梁に置換された多スパン山形フレームの解析

Ritz法を適用して、汎関数(21)、および(22)式は解析された。この時、用いられた未知関数の表現を(24)式に示す。

4-3 解の収束性の検討

スパン数 $n_x = 8$ とした D-model において、Fourier 級数の項数 N_k を1から21まで変化させて得られた中央の山形フレームの水平変位 $d_H(0)$ と屋根板の端部剪断力 $Q_x^*(L/2)$ の値の変化を Fig.6 に示す。解の収束は良好であることが判る。以下の解析は項数 $N_k = 11$ とした。

4-4 本論から得られた解と立体剛節フレーム解析から得られた解の比較

スパン数 $n_x = 2, 4, 8, 12, 16, 20, 24$ とした桁行き方向の長さの異なるフレームへ解法 D-model、および C-model を適用して解析した。得られた中央スパンの水平変位を Table.1 に示した。この表に剛節立体ラーメン解析を T_1 、および T_2 へ適用して得られた対応する位置の水平変位をも併せて示した。カッコ内の値は D-model から得られた水平変位との比である。これらの結果からふたつのことが分かる。ひとつは屋根のブレースの配置が異なるふたつの解析モデル T_1 、および T_2 の水平変位に大きな相違が見られないこと。他方は D-model が多スパン山形フレームのスパン長 $L = 1200 \text{ cm}$ （形状比 $\gamma_s = 0.5$ ）から $L = 14,400 \text{ cm}$ ($\gamma_s = 6$) において、立体ラーメン解析から得られた T_1 の変位と比較して差違が3.5%以内の極めて精度の高い値を与えていること。これら各種解法から得られた水平変位とスパン数を Fig.7 に図示した。また、中央の山形フレームの柱頭における曲げモーメントとスパン数の関係を Fig.8 に示した。これらの図には平面フレームと仮定して得られる変位、および曲げモーメントも示した。

4-5 多スパン山形フレームの折板効果

4-1において設定された断面を有する T_1 タイプの多スパン山形フレームを標準ケースとする。屋根のブレースの断面のみを2倍としたケース、および梁の断面のみを2倍としたケースを想定し、それらへ D-model を適用して得られた解をそれぞれ Case DA_b、および Case DA_s と表す。これらはそれぞれ屋根の曲げ剛性、および剪断剛性が標準のそれらの2倍となる。これらふたつのケースにおける中央スパンの水平変位と標準ケースのそれとの比を Fig.9 に示す。曲げ剛性の増大はスパン数の少ないフレームにおいてこの変位の縮小に寄与しないものの、スパン数が増加するに従い効果を発揮する。剪断剛性の増大はスパン数の少ない山形フレームの変位を小さくする効果を持つものの、スパン数が8 ($\gamma_s = 2$) を越えると効果が小さくなる。ここで採用した山形フレームではスパン数が8 ($\gamma_s = 2$) より小さい領域で屋根の変形よりも妻壁の変位が中央スパンの水平変位に占める割合が大きくなるので、剪断剛性の増大が変位を縮小する効果が小さくなる。これらの事から形状比が5-6以下の多スパン山形フレームの折板効果を高めるためには、屋根の曲げ剛性を高めるよりも屋根ブレースの断面を増し、剪断剛性を高めた方が効果が大きいこと、および形状比が小さいフレームでは妻壁の剛性を確保する必要があることが分かる。

5. 結論

1. 多スパン山形フレームが棟に関して逆対称変形をするとき、屋根板へ直交異方性板の理論を導入して、連続体に置換されたこのフレームの基礎式を誘導した。この基礎式において柱が存在することも考慮されている。
2. 屋根板の変形に適切な規制を加え、屋根板を剪断変形を考慮に入れた梁（Timoshenko梁）に置換し、柱や弾性の妻壁が存在するこの多スパン山形フレーム全体を Timoshenko梁 に置換する汎関数を示した。
3. Ritz法を適用して例題を解析し、この理論が極めて精度の高い解をもたらすことを実証した。
4. 屋根板の梁、あるいはブレースの断面積を増大させた多スパン山形フレーム・モデルを解析し、形状比が5-6以下のフレームでは三次元の折板効果を確保する上でブレースの断面を増大させ、屋根の剪断剛性増加させる方が梁の断面を増大させ屋根板の曲げ剛性を増加させるより効果が大きいことを示した。

(1995年5月10日原稿受理, 1996年1月9日採用決定)