CROSS-INTERACTION OF SURFACE AND EMBEDDED STRUCTURES SUBJECT TO SPATIAL VARIATION OF GROUND MOTION

空間変動のある地震動を受けた地表基礎および埋込み基礎構造物のクロスインタラクション

Farhad BEHNAMFAR* and Yoshihiro SUGIMURA**

ファルハッド・ベナマファール、杉村義広

This study focuses on the effects arising from interaction between adjacent structures when spatial variation of ground motion is taken into account. In this respect, the ground excitation is modeled using two different approaches. The first approach is taken only by accounting for deterministic effects calculated by solving the equation of motion of an elastic homogeneous medium. In the second approach, the theory of random vibration in elastic media is applied in the shape of an additional term in the solution of the wave equation representing the coherence between motions at every two points of the ground surface. Effects of passage of body as well as surface waves on the adjacent structures are studied in detail. Effects of the ground motions identified as above on the response of the system are examined assuming a harmonic excitation and considering type of the wave and angle of incidence of the body waves. The structures are placed on the surface or embedded foundations to make it possible to study the effects of embedment of buildings in the latter case. It is shown that resonance frequency increases with closeness of structures. For some cases, depending on the distance between the adjacent structures, the structural response increases. Applying the random approach leads to more realistic results especially for the case of surface waves.

Keywords: cross-interaction, dynamic response, spatial variation, surface structures, embedded structures.

1. Introduction

It is common practice in earthquake engineering analysis to analyze a structure for seismic motion separately from its surrounding buildings, not taking into account the radiation of energy from them. The foundation of buildings is almost always embedded to some extent in the ground, but in terms of analysis effort it is more convenient to assume them as surface footings if the soil-structure interaction effects are to be calculated. Moreover, the time histories and the corresponding design spectra utilized in such an analysis depend on the assumption that all points of the ground surface are excited synchronously by vertically incident plane shear waves and experience the same free-field motion.

When following the above procedure, it can be questioned that how important the cross-interaction of the adjacent buildings, spatial variation of the ground motion, and embedment of the foundations are?

The theory and strong motion array data both show that the ground motion may vary from one point to the next. This can severely alter the pattern of dynamic response of certain structures including buildings with large foundations or with widely-spaced multiple supports.

Spatial variability of ground motion includes deterministic and random components. Known as the wave passage effect, the deterministic component is actually the solution of the wave equation in a medium consisting of homogeneous layers. In this case, the wave front is plane and sweeps different points of the foundation at different times since in general it propagates nonvertically. This effect was relatively early known. Kobori and Suzuki (1970) studied the response of a surface foundation to the wave passage effect. Their analysis method is called "the Dynamic Ground Compliance Method" and has been used widely for the design of nuclear power plants in Japan. Also, cross-interaction between embedded structures was examined by Kobori and Kusakabe (1978). Dominguez (1978) for the first time used the boundary element method to determine the response of embedded footings. The random component arises from spatial incoherence of seismic ground motion. The term spatial incoherence refers to a phenomenon in which motions at two different points of the ground surface tend not to vary together. The reason for such phenomena to occur is multifold. The individual wave trains may impinge the foundation at different instants and with different angles of incidence, or they may propagate through paths of different physical properties and may be affected differently in both amplitude and phase by the characteristics of the travel paths. Since 1980's this effect has been the subject of a number of studies including the response of buried pipelines [Hindy and Novak (1980)]; massless footings on elastic halfspace [Lucuco and Wong (1986)]; gravity dams [Ramadan and Novak (1992)]; and, idealized superstructures [Veletsos and Prasad (1989)]. Yoshida and Mita (1988) studied the case of an embedded foundation under spatially random ground motion.

It is well-known that soil-structure interaction may affect the dynamic response of a structure supported on flexible soil considerably decreasing the maximum response amplitude in many cases and reducing the resonance frequency of the system [Chopra & Gutierrez (1974), Bielak (1976), Iiguchi (1978)]. Analytical methods to calculate these effects are well established, e. g., in a textbook by Wolf (1985). When there is more than one structure in the medium, because of interference of the structural responses through the soil, the soil-structure problem of a single structure evolves to a cross-interaction problem between multiple structures [Kawase & Nakai (1984), Lin et. al. (1987)]. The embedment of foundations in the ground makes the real pattern of response of such adjacent structures a more complicated one, for which only a few studies can be found in the literature [Kobori & Kusakabe (1978), Ichikawa et. al. (1986), Imamura et. al. (1992)].
In the present work, it is aimed to assess the effects of two different approaches of modeling the ground motion on the dynamic response of adjacent structures with foundations resting on the surface or embedded in the body of a half-space.

2. Analytical Models and Parameters

An idealized system as shown in Fig. 1 is taken under exploration. The two structural systems consist of rigid roofs that are connected to the rigid foundations resting on the surface or embedded in the body of a half-space to a depth \( h_i (i = 1, 2) \). The supporting columns are taken as massless and presumed to respond within their elastic range of behavior. The clearance between the structures, edge to edge, is represented by the parameter \( 2a \). The mass of each roof and its corresponding rotational mass moment of inertia with respect to the center of mass of the corresponding roof is denoted by \( m_i \) and \( I_i \), respectively. The dimension, \( 2l_i \), of a structure is assumed to be the same as that of its underlying foundation. Effects of cross-interaction between the two structures are taken into account exactly using the dynamic stiffness and kinematic response matrices of the coupled system of foundations.

![Fig. 1. System Considered. (a) Adjacent surface buildings. (b) Degrees of freedom. (c) Dimensions of the foundations for the case of the embedded structures.](image)

Propagation of two different types of waves in the medium is taken into account; the shear waves and the Rayleigh waves. The body shear waves that are polarized in-plane are called \( SV \)-waves and propagate vertically, i.e., in \( Z \)-direction. The surface Rayleigh waves propagate in \( X \)-direction and are abbreviated as \( R \)-waves. The flexible halfspace is defined with its material damping ratio \( \xi_s \), poisson ratio \( \nu \), and shear-wave velocity \( c_s \).

Taking into account only the deterministic component of the spatially variable ground motion is called deterministic approach in this study, while when the ground motion is considered to be consisted of both deterministic and random components, it is called random approach.

The parameters introduced as follows are required to identify the structural model and the underlying soil:

1) Dimensionless height \( \tilde{h}_i = h_i / l_i \), which is taken 0.5, 1, 2, 4 for 2, 5, 10, 20- story buildings, respectively [Sikaroudi & Chandler (1992)] and 1 for reactor buildings. \( h_i \) is effective height from the ground surface to the center of mass of the building \( i \).

2) The structure/surface soil mass ratio \( \tilde{m}_i = m_i / (\rho h_i l_i) \) with \( \rho = \) soil mass density. This is the mass ratio of the investigated structure to a same volume filled with underlying soil. Experience with practical buildings show that \( \tilde{m}_i \) equal 0.3 and 3 are appropriate values for multi-story buildings and massive containment buildings of nuclear reactors (reactor buildings), respectively [Chandler and Hutchinson, (1987)].

3) The lateral stiffness ratio \( \tilde{w}_{li} = w_{li} h_i / c_s \), in which \( w_{li} \) is fixed-base frequency of the structure \( i \) in lateral motion. This is regarded as a measure of soil-structure interaction. If the fundamental period of a n-story building is assumed to be 0.1 sec and height of each story to be 3 m so that \( h_i = 3n \), and also \( c_s = 100, 200, 400 \) m/sec for soft (clay), medium (sand), and stiff (hard clay) soils, then \( \tilde{w}_{li} = 0.5, 1, 2 \) for the same soils.

4) The dimensionless clearance between structures \( \tilde{a} = a / l \) in which \( a \) is half the real clearance between the two buildings and \( l \) is a reference length. For instance when \( \tilde{a} = 1 \), this means that the two buildings are apart by a distance equal to twice that of the reference length.

5) The non-dimensional length of each foundation \( \tilde{l}_i = l_i / l \).

6) The dimensionless depth of embedment of the foundation \( \tilde{H}_i = H_i / l \).

7) Hysteretic damping ratio of each building \( \xi_v \).

8) The foundation/structure mass ratio \( \alpha_i = m_{li} / m_i \), in which \( m_{li} \) is mass of the basement \( i \).

Throughout the numerical analysis of this study, the following values are set to be constant: \( \alpha_1 = 0.33 \), \( \xi_1 = 0.05 \), \( \xi_4 = 0.05 \) and \( \nu = 0.33 \). The value assumed for \( \alpha_i \) is a common value for multi-story buildings, that of \( \xi_4 \) is the value recommended in most building design codes, and finally those of \( \xi_1 \) and \( \nu \) are corresponding to medium soils. These parameters are set to be constant because as Chandler and Hutchinson (1987) stated, their variation has not been found to produce significant changes to the response trends. Also, \( l_1 = l_2 = l \) (\( \tilde{l}_1 = \tilde{l}_2 = 1 \)) is taken in this study.

Degree of freedom of the system in these dynamic calculations is 8: one horizontal motion per building roof (masses \( m_i \)), and, two translational (the horizontal \( X \) and the vertical \( Z \)) and one rotational motion for each foundation (masses \( m_{li} \)) (Fig. 1b).

3. Equations of Motion

Displacements of one of the two buildings of Fig. 1 when resting on the surface of the ground and subject to the ground motion, is shown in Fig. 2. For the building on the right also a similar figure can be imagined.

Dynamic equations of motion of the system are written in an order as follows:

1) Translation of the mass \( m_i \) in \( X \)-direction:
2) Translation of the left-side building in $X$-direction:

$$m_i\ddot{u}_i + c_i\dot{u}_i + k_iu_i = 0$$

where $m_i$, $c_i$, and $k_i$ are the mass, damping, and stiffness coefficients of the building, respectively. $u_i$ is the displacement of the building.

$$m_i\ddot{u}_i + m_o\ddot{u}_o + P = 0$$

Another possible option could be replacing Eq. (2) by one written for the mass $m_o$ only. However, such an equation contains additional terms corresponding to the damping and stiffness of the structure which have already been accounted for in Eq. (1). Moreover, writing Eq. (2) as above, leads to a simpler system stiffness matrix be defined later in Eq. (10).

3) Translation of the left-side building in $Z$-direction:

$$(m_i + m_o)\ddot{u}_o + R = 0$$

4) Rocking of the left-side building about $y$-axis:

$$m_i\ddot{h}_i + (l_i + t_o)\ddot{h}_o + M = 0$$

Another four equations can be written this time for the right-side building, analogously to above. In the above equations, $P$, $R$, and $M$ are horizontal, vertical, and rotational components of the interaction force, respectively, that are computed as:

$$\{F_x\}_{tot} = [K]_{10}\{U_1\}_{10}$$

in which

$$\{F_x\} = \begin{bmatrix} \{F_{x_1}\}; \{F_{x_2}\} \end{bmatrix}_T \quad \text{and} \quad \{U_1\} = \begin{bmatrix} \{U_{11}\}; \{U_{12}\} \end{bmatrix}^T$$

and

$$[K] = \begin{bmatrix} \{K_{11}\} & \{K_{12}\} \\ \{K_{21}\} & \{K_{22}\} \end{bmatrix}$$

where $\{F_x\} = \{P, R, M\}$, is the vector of interaction forces applied on the foundation $i$. $\{U_{1i}\}$ is a vector having the two translational (the horizontal $X$ and vertical $Z$) and one rocking degrees of freedom of the rigid foundation $i$ as its elements, and $[K]$ is the dynamic stiffness matrix of the coupled foundation. The submatrices $\{K_{ij}\}$ with $i \neq j$ contain the coupling terms between the adjacent foundations. Arranging the equations written as above and transforming to the frequency domain, the following system of equations in matrix form is result:

$$[A(\omega)]\{u(\omega)\} = \{P(\omega)\}$$

where $\{u(\omega)\}$ is a $8 \times 1$ vector of degrees of freedom of the total system as

$$\{u(\omega)\} = \{u_1, u_{a1}, w_{a1}, h_{1}, \theta_{a1}, u_2, w_{a2}, h_{2}, \theta_{a2}\}$$

in which letters $u$ and $w$ show translation in $X$- and $Z$-directions, respectively, and letter $\theta$ represents rotation about $y$-axis, and $h$ is effective height of the mass $M$ in lateral motion in $X$-direction (Fig. 1). The load vector $\{P(\omega)\}$ is consisted of appropriate combinations of elements of input motion vector of the foundations. The system stiffness matrix $[A(\omega)]$ is symmetric when the adjacent foundations and structures are the same. When the equations of motion is written as was explained in the above, it has a simpler form as follows:

$$[A(\omega)] = \begin{bmatrix} a_{11} & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & a_{22} & a_{23} & a_{24} & 0 & a_{26} & a_{27} & a_{28} \\ 0 & a_{32} & a_{33} & a_{34} & 0 & a_{36} & a_{37} & a_{38} \\ 1 & a_{42} & a_{43} & a_{44} & 0 & a_{46} & a_{47} & a_{48} \\ 0 & 0 & 0 & a_{55} & 1 & 0 & 1 \\ 0 & a_{62} & a_{63} & a_{64} & 1 & a_{66} & a_{67} & a_{68} \\ 0 & a_{72} & a_{73} & a_{74} & 0 & a_{76} & a_{77} & a_{78} \\ 0 & a_{82} & a_{83} & a_{84} & 1 & a_{86} & a_{87} & a_{88} \end{bmatrix}$$

Various elements of the matrix $[A(\omega)]$ contain mass, damping and stiffness properties of the structures and soils.
4. Wave-Type Equations

As is well known, propagation of elastic waves in an elastic medium occurs in the shape of body waves and surface waves. As of body waves, in this study only shear SV-waves which have in-plane motion perpendicular to the direction of propagation are considered. Since these waves are assumed here to propagate vertically, particle motion in the medium under these waves will be in the \(x-Z\) plane and in the horizontal direction, i.e., along \(x\)-axis. The surface \(R\)-waves also vibrate the particles in the same plane, but both horizontally and vertically.

For introduction of \(SV\)-waves, the incidence angle is selected and as is explained in the Appendix, the identifier parameters are calculated. For \(R\)-waves the apparent wave velocity \(v_c\) in the medium under investigation must be calculated. For a half-space with a poison ratio of 0.33 (as is assumed in this study), \(c_o = 0.933c_r\) for Rayleigh waves where \(c_r\) is shear-wave velocity in the half-space [Wolf (1985)]. When the identifier parameters \(l_s\) and \(m_j\) (see Appendix) have been calculated, the dynamic stiffness matrix of the site is computed and used to calculate the foundations' dynamic stiffness and effective input motion matrices.

For each set of dynamic analysis of this study, it is assumed that the ground motion is consisted of only one type of the above-mentioned elastic waves. Amplitude of the horizontal component of the ground motion on the surface at a point on a vertical axis passing through the center of the (left, when applicable) foundation is presumed to be unity. As of the vertical component, it is zero for vertically propagating \(SV\)-waves. For \(R\)-waves, it is equal to \(2\pi/(1-i'F)\) with \(S\) and \(t\) given in the Appendix.

5. The Random Approach to Spatial Variation of Ground Motion

The \(2 \times 2\) covariance matrix of the ground motion \([\bar{B}]\) which is a measure of statistical relation between various components of the motion at two different points of the ground is assumed to have the form:

\[
[B(x_1, z_1, \omega)] = [C(\omega)] f(x_1, z_1, \omega) g(x_1, z_1, \omega)
\]

in which

\[
[C(\omega)] = \tilde{B} \left[ \tilde{k} (x_1, \omega) \right] \tilde{k}^T (x_1, \omega)
\]

represents the covariance matrix of the free-field ground motion on the surface. \(E[L]\) denotes the expected value, \(x_1\) and \(z_1\) are location vectors of points 1 and 2 and the tilde denotes complex conjugate. The spatial coherence function \(f\) is given by [Lucu & Wong (1986)]:

\[
f(x_1, z_1, \omega) = \exp \left[ -\left( \frac{\omega(x_1 - x_2)\beta}{|\beta|^2} \right)^2 \right]
\]

in which \(\gamma\) represents the coherence parameter and \(\beta\) denotes velocity of the wave. A perfectly coherent or deterministic ground motion is associated with \(\gamma = 1\) or \(\gamma = 0\). Value of \(\gamma/\beta\) was reported to be equal to \((2-3) \times 10^{-4} \ m/s\) by Lucu & Wong (1986) for certain sites. In Eq. (13) it is observed that variation of \(f\) for two points at a fixed distance and for a certain frequency with regard to different values of the coherence parameter \(\gamma\) is so that augmenting \(\gamma\) results in smaller \(f\) that represents lesser coherence (larger incoherence) between motion of different points of the ground.

In the numerical analysis of this paper \(\gamma = 0.5\) is used.

The function \(g\) is the deterministic part of the spatial variation of ground motion. It is a function of locations of the pair of points considered as:

\[
g(x_i, z_i, \omega) = F(z_i, z_j, W) \exp \left[ -i\omega(x_i - x_j)/\beta \right]
\]

in which \(F(z_i, z_j, W)\) is a function of the depth of the points under consideration and type of the wave (WT) showing variation of the displacements with depth for a specific wave. \(\beta\) is the phase velocity equal to \(\beta/\cos \alpha\) for a body wave propagating with an angle \(\alpha\) from the horizontal. It equals 0.933 \(c_r\) for Rayleigh wave in a halfspace with \(V = 0.33\) [Wolf (1985)]. Using the boundary element method, the interface between the two adjacent foundations and the ground surface is discretized into \(N\) elements. The covariance matrix of the kinematic response of the two foundations \([\bar{B}]\) is defined similar to Eq. (12) as the expected values of this kinematic response. Introducing the matrix of the transfer function between the free-field ground motion (Eq. 12) and the kinematic response of the foundations as matrix \([\bar{R}]\), substituting Eq. (12) in Eq. (11) and expanding the resulting equations, finally leads to the following equation for the elements of the covariance matrix of the kinematic response of the foundations:

\[
B_{ij} = \sum_{\alpha_1=1}^{2} \sum_{\alpha_2=1}^{2} A_{\alpha_1,\alpha_2} C_{\alpha_1,\alpha_2} f(x_i, z_i, \omega)
\]

in which

\[
A_{\alpha_1,\alpha_2} = \sum_{p=1}^{N} \sum_{q=1}^{N} R_{ij} f_{\alpha_1 \alpha_2} s_{\alpha_1 \alpha_2}
\]

and \(s_{\alpha_1 \alpha_2}\) and \(C_{\alpha_1 \alpha_2}\) are power-spectra of the \(i\) and \(j\) components of the foundation response and \(m\)-component of the free-field ground motion. Then \((A_{\alpha_1,\alpha_2})^{1/2}\) will be the amplitude of a transfer function between these two components. \(f_{\alpha_1 \alpha_2}\) and \(s_{\alpha_1 \alpha_2}\) are functions \(f\) and \(g\) computed at the points \(x_i\) and \(x_j\).

6. Analytical Results

In the following numerical analysis, a harmonic ground excitation has been used as the input to the system. Calculating the harmonic response is considered essential to understand the system response to the more complex seismic excitations.

Various lateral responses of the superstructure, i.e., total translations of the mass \(m_i\) in the \(x\)-direction that are identical to \(u_x\) in Fig. 2, are compared as shown in the followings. When cross-interaction is studied, it is presumed that two structures are placed near each other with identical foundations and superstructures.

The abscissa \(\omega/\omega_0\) shows the ratio of frequency of vibration to the natural frequency of the fixed-base building in lateral motion.

6.1. Single structures on halfspace subject to \(SV\)- or \(R\)-waves:

1) Lateral responses of a 2-story building resting on a soft soil under \(SV\) and \(R\)-waves are compared by deterministic and random approaches in Figs. 3 and 4. It is seen that responses of the structure are much smaller than the case of fixed-base for both deterministic and random approaches.

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While the responses of the two different approaches are near to each other for $SV$-waves, the difference is larger in the case of surface Rayleigh waves. The reason is that amplitudes of the random vertical motions at different points are less than those of the deterministic case because of the additional exponential decay represented by the function $f$ in Eq. (11). This leads to a smaller rocking motion decreasing the total lateral response.

2) Figure 5 shows the case of a 5-story building on soft soil. It is observed that the responses under $R$-waves are larger than those of shear waves. While the response of random approach is somewhat larger than deterministic response under $SV$-waves, deterministic response is much larger than the random one under $R$-waves. Three of the above four cases of response show values fairly near together but the fourth one, deterministic response under $R$-waves, does not show consistency with others and is the largest response differing considerably in magnitude from other three cases. The increase in the response with $R$-waves seems to arise from the additional rocking motion associated with vertical component of the free-field ground motion of such waves. Between the two cases of $SV$-waves, with the random approach even for vertical incidence of shear waves a rocking motion is present resulting in a higher response for taller buildings.

3) The same tendency in response as the case of 5-story building is seen more clearly when the number of stories rises to 10 and 20 as shown in Figs. 6 and 7, respectively. The maximum response under $R$-waves for a 20-story building in the deterministic approach reach to a large value being nearly twice that of the random approach. Two important points should be emphasized in these figures; first, the deterministic approach with $R$-waves shows much larger response amplitudes than the other three, i.e., random approach with $R$-waves and both approaches with $SV$-waves; and second, as the number of stories of the building increases, response of the soil-structure system also increases to the extent that for 10- and 20-story cases responses to $R$-waves with both approaches surpass that under rigid-base assumption, as shown in Figs. 6 and 7. The latter point should have appropriate consequences in design codes of high-rise buildings. It is interesting also to note that the traditional deterministic approach of calculating for vertically propagating shear waves gives the least response for all the cases except those of 2-story building.

4) Results of the case of a reactor building on soft soil are shown in Fig. 8 for $SV$- and $R$-waves. Order of the largest maximum responses is similar.
to those of 5-, 10-, and 20-story buildings with the response to \( R \)-waves with deterministic approach being followed by those of random \( R \)-waves, random \( SV \)-waves, and deterministic \( SV \)-waves. Contrary to multistory buildings, the reactor building with soil-structure interaction shows a gradually decreased response from stiff to soft soils.

5) Figures 9 and 10 show the maximum amplitudes of 2 and 10-story buildings, respectively, resting on stiff to soft soils under different waves and different approaches. Uniform decrease in response is seen in the case of a low-rise 2-story building as the underlying soil becomes softer while the opposite tendency appears for 10-story building under \( R \)-waves. Again unrealistically large values are obtained for deterministic approach under \( R \)-waves.

It is important to note that the response of the soil-structure system increases with the height of the building, as was seen above, mainly because of a larger lateral motion in top stories due to the rotation of foundation. Rayleigh waves produce a large vertical ground motion that amplifies the rocking motion of the foundation. When there is spatial incoherency in the motion of the ground, the vertical and consequently rocking motions decrease resulting in a lower structural response. On the other hand, under \( SV \)-waves this incoherency results in an additional rocking motion at the foundation arising a larger response with the random approach.

6.2. Two adjacent structures on halfspace subject to \( SV \)-waves or \( R \)-waves:

1) Analytical results of cross-interaction of twin 5-story buildings supported on surface foundations on soft soil are shown in Figs. 11 to 13. In the case of \( SV \)-waves, an increase in the lateral response of the adjacent structures (\( a/l=1 \)) is observed in Figs. 11 and 12 with both deterministic and random approaches compared to the single structure (\( a/l=\infty \)). Also the resonance frequency slightly increases exhibiting a stiffer system for the adjacent case. A harmonic change in the lateral response of the building is observed when the structures become nearer to each other for \( SV \)-waves with both deterministic approach and for \( R \)-waves with random approach, Fig. 13. The Green's-functions-like behavior of the maximum response of the structure in Fig. 13 leads to the fact that this variation is the effect of interaction forces of the second structure on the response of the first structure. By looking into the response trends of Fig. 13, a dimensionless wavelength of \( \lambda = 3.5 \) can be seen. Considering the fact that the resonance frequency at which these maximum values happen is \( \omega = 0.9\omega_0 \), \( \lambda = 3.49 \) is calculated that clearly confirms the above reasoning on the variational effect of the interaction forces. The response of the twin system to \( R \)-waves with random approach shows the same tendency as of \( SV \)-waves but is somewhat larger, Fig. 13. Response of the twin system under \( R \)-waves in deterministic approach is again inconsistent with other cases.

2) Figures 14 and 15 show the results of the case of twin 20-story buildings. The response decreases when the structures exist at a distance \( a/l = 1 \) from each other. A sharp decrease in the response under deterministic \( R \)-waves is seen for the cases of nearer distances but the
other three analytical perspectives, i.e., random $R$- & $SV$-waves, and deterministic $SV$-waves show quite similar tendencies (in Fig. 15 the case of $R$-waves is not shown to avoid congestion). Between the latter three cases, again random $R$-waves show a larger response.

An important point to note is that how well assuming some degree of incoherency in the ground motion modifies the response under $R$-waves so that it becomes consistent with the other cases.

Applying the deterministic cross-interaction effects of multistory buildings under $R$-waves results in lower rocking response as the twin system is stiffer than the single one. This in turn leads to a smaller total lateral response for these structures.

6.3. Structures embedded in a halfspace subject to $SV$-waves or $R$-waves:

In order to investigate effects of embedment on the lateral response of single and twin structures, analytical examples are added for the case of 2-story, single and twin buildings, resting on the surface or embedded foundations in homogeneous halfspace. It is emphasized that the part of the embedded 2-story building above the ground level is identical to the 2-story surface building. Results are shown in Figs. 16 & 17 for the deterministic case and in Figs. 18 & 19 for the random case.

1) Figures 16 & 18 show the results of the case of vertically incident shear waves. The embedment ratio is unity. It is seen that the resonance frequency increases in both approaches. Compared to the surface structure, the responses are somewhat increased and decreased in the deterministic and random cases of embedded structures, respectively. The decrease of the amplitude in the random case is consistent with the experimental results [Inamura et al. (1992)]. Cross-interaction has only nominal effects in this case.

2) Figures 17 & 19 exhibit the results of the $R$-waves case. It is seen again that the resonance frequency slightly increases and the responses increase and decrease for the deterministic and random cases of the embedded structures, respectively. Therefore, with $R$-waves too analytical results of the embedded case are in accordance to the experimental results. Also, cross-interaction at this distance between the buildings tends to increase response of the embedded low-rise buildings like the case of the surface structures with similar height (Figs. 11-13). This effect is more visible with $R$-waves.

The decrease of amplitude with embedment is due to the fact that a portion of vibrational energy radiates back to the soil through the vertical walls of
the foundation also. It seems that cross-interaction tends to decrease this radiation with exchange of energy between the adjacent vertical walls of the two foundations.

7. Conclusions

In this study interaction of adjacent structures through the underlying soil subject to a ground motion with spatial variations of deterministic as well as random nature was investigated. From analytical examples of structures changing their height and mass parametrically, the conclusions are derived as follows:

1) As the adjacent structures exist closer, the resonance frequency of the system becomes higher.

2) For surface structures, cross-interaction can result in larger responses than those of single buildings depending on the distance between the adjacent structures.

3) Response of tall buildings (10- to 20-story structures) considering soil-structure interaction under Rayleigh waves by the random approach is somewhat larger than the fixed-base case.

4) Opposite to the deterministic approach, with the random approach response of a low-rise structure resting on surface foundations is larger than that of the same structure but having an embedded part, with cross-interaction tending to increase the response. This is in agreement with experiments.

5) Applying the random approach leads to more realistic results especially for the case of surface waves.

8. References


Appendix. Dynamic Stiffness Matrix of A Halfspace

As was described in sec. 5, referring to Fig. 1 for notations, elements of the dynamic stiffness matrix of a halfspace $[S_6]$ are defined as amplitudes of the dynamic external loads needed to make dynamic displacements with unit amplitudes in desired directions. $[S_6]$ is a 3 × 3 symmetric matrix having the following as (diagonal and upper-diagonal) elements [Wolf(1985)]:

$$s_{12} = s_{21} = 0, \quad s_{13} = ikG' \frac{1 + t^2}{1 + st}, \quad s_{31} = kG' \frac{2 - \frac{1 + t^2}{1 + st}}{1}, \quad s_{23} = ikG', \quad s_{32} = ikG' \frac{1 + t^2}{1 + st}$$

$$G' = G(1 + 2\xi^2), \quad i = \sqrt{-1}, \quad k = \frac{\omega_0}{c}, \quad s = -i \left[ -\frac{1}{t}, \quad t = -i \left[ \frac{1}{m^2}, \quad c = \frac{c_p}{l}, \quad c_l = \frac{c_s}{m}, \right] \right.$$

$c_p = \sqrt{c_p \left(1 + 2\xi^2\right)}, \quad c_s = c_s \left(1 + 2\xi^2\right)$; for body waves: $l_c = \cos \psi_\xi$, $m_c = \cos \psi_\xi$;

for surface waves: $l_c = \frac{c_s}{c}, m_c = \frac{c}{c_s}$.

$G$, $c_p$, $c_s$, $\xi$, and $\xi_p$ are shear modulus, velocity of $P$-wave, velocity of $S$-wave, material damping for $P$-wave, and material damping for $S$-wave of the halfspace, respectively. $k$ is wave number, $C$ is phase velocity, $\theta$ is frequency of vibration, $\psi_\xi$, and $\psi_\xi$ are angles of incidence of $P$- and $S$-waves, respectively, in the halfspace with the horizontal.
1. はじめに

地震応答解析では、地盤との動的相互作用を考慮する場合でも建物は単独で、基礎は根入れがなく地表上にあるものと仮定することが多い。また、地震動は基礎下のどの位置で同じ相位で入る自由地動の動きとして仮定される。本論では、弾性解析の範囲に限られるがこの点を改良すべく、地震動のランダム空間変動がある場合とない場合について、基礎根入れ効果を導入した隣接建物相互作用（cross interaction: CI）を比較検討する。

2. 解析システムとパラメータ

解析モデルは、Fig.1に示すように半無限弾性体と考えた地盤（減衰定数$\xi_g$、ポアソン比$v$、せん断波速度$C_s$）上で2aだけ離れた幅2$I_0$（$I = 1, 2$）、質量$m_l$、回転慣性$I_l$をもつ屋根が高さ$h_l$の柱で支えられている2つの建物である。基礎は根入れがない場合（基礎基礎）と$I_0$の根入れがある場合（根入れ基礎）を考える。両者の動的剛性と応答を求める際には厳密な理論解を用いることにS吾法と導入する。質量と高さ、隣接距離などを変えたパラメトリック解析を行い、基礎固定時の挙動を基準として相互比較する。

3. 運動方程式

Fig.2に解析モデルの変形イメージを示すが、左側の建物について運動方程式をたてると次の式で表される。右側の建物も同様にして考えることができる。いくつかの変換式を行えば最終的な剛性マトリックスとして（10）式が得られる。

4. 入力波、5. 地震動のランダム空間変動アプローチ

入力波は2方向に伝わるせん断（SV）波とx方向に伝わるレイリー（R）波を考え、Luco & Won（1986）が提案した空間コーナーレンス関数$I_s$を導入し、ランダム空間変動がある場合（RAP）の解析を行い、ない場合（DAP）の結果と比較した。

6. 解析結果

調和入力波は複雑な地震動の場合の理解の基本であるので、ここで調和入力波を用い、建物の水平応答、すなわちFig.2に示す質量$m_l$の絶対変位$\delta$に関する結果を以下に比較した。

6.1 地表基礎単体の場合

軟弱地盤における地表基礎単体で2000における2つのSV、R入力時のDAP、RAPの応答結果を比較してFig.3～Fig.8に示す。軸軸は地盤の応答に対する比、横軸は入力の基礎固定時建物に対する振動数比として規定しているので、図中で地表固定時の曲線は共通の基準として扱っており、横軸が無限大のときに応答比1である。

図から以下の点が認められる。

1）相互作用を考慮すると、階数が少なくなると、地震動の影響は地盤固定時に比べて共振振動数は低くなる（Fig.3、Fig.8）。

2）応答振幅、フィルターを考慮してレベルでどちらの場合も小さいが（Fig.3、Fig.4）、動振が増すと大きくなり高層ほどはほぼ同等か（SV入力）、それ以上（R入力）になる（Fig.6、Fig.7）。

3）SVとR入力を比較すると、4階以上になるほどSVの傾向は弱くなる。これはSV入力が水平成分のみであるのに対し、R入力は軸方向成分があることに加えて高層ほどロッキング応答の要因が大きくなることによる（Fig.6、Fig.7）。

4）DAPとRAPを比較すると、SV入力ではややDAP>RAP（2階建での場合のみ）の傾向があるものの、それほどはどちらの入力に対して影響はDAP>RAPとなり、高層になればなるほど顕著になる（Fig.6、Fig.7）。

建物と地盤条件の組み合わせの影響を見るために2階と10階建ての場合について解析した結果（Fig.9、Fig.10）で、前者はいずれも軽軽となるほど最大応答値は下がるのに対し、後者は逆の傾向を示す、とくにR入力のDAPが顕著である。

6.2 地表基礎2種の場合

SVとR入力に対する地表基礎の単体と2種の場合を比較して、5階建ての結果をFig.11～Fig.13に、20階建ての結果をFig.14、Fig.15に示す（Fig.12、Fig.15ではR入力のRAP結果は殆ど重なってしまいため図を省略）。これらの図で最も特徴的なのは、Fig.13(RS入力)におけるDAPとRAP、R入力におけるRP&APで隣接建物に応じて無次元元波長k=3.5の周期性を示し、いわゆるCI効果が顕著になることであるが、基礎固定時の建物の共振振動数の約0.9倍の振動数の波長$k_0=3.0$に対応している。ただし、R入力におけるDAPでは周期性を示されても、隣接間隔が短いと単独の場合に近づくので、ロッキングの影響などにより応答が大きくなくなるという他の因子とは異なる傾向を示す。したがって、建物が隣接している場合にはCI効果が無視できないこと、さらにR入力の場合は空間変動のランダム性が建物の応答に与える影響が大きいことが知られる。

6.3 埋込み基礎2種の場合

2階建の地表基礎、埋込み基礎の単体と2種の場合を比較する（Fig.16～Fig.19）、いずれの場合も埋込みによる拘束効果で共振振動数は高めに増している。しかし、樫付基盤入力波と解析アプローチによる差異が顕著である。すなわち、DAPの入力では埋込み基礎が地表基礎より目立って大きくなり、CI効果が大きくなる。また、SV入力では埋込み、CI効果による変化は少ないが地表基礎よりは大きくなっている。一方、RAPではどちらの入力に対して埋込みによる応答値の減少が個性的であり、実験等で確認されている傾向が現れている。

7. 結論

本論で行った解析から得られた結論は次の通りである。

1）建物同士が接続すると共振振動数は高くなる。

2）地表基礎の建物の場合、CIを考慮すると隣接間隔に応じて単独建物より大きな応答を示すことがある。

3）RAPによるR入力時の高層建物（10～20階建）の応答は、相互作用を考慮すると基礎固定の場合より低減する傾向がある。

4）DAPは逆に、RAP地表基礎構造物の応答は埋込み基礎と比較して、とくにR入力時の大掛かりCI効果をさらに増大する。

5）R入力の場合、RAPは現実に近い結果を与える。