SYSTEM RELIABILITY REGULATION THROUGH
SENSITIVITY FACTOR

感度係数による系信頼度の調節

Sadaichi TERADA* and Takafumi MOROI**

寺田 貞一, 諸井 孝文

This paper develops a method to design a system that has prescribed reliability adjusting the capacity of suitably chosen element(s) directed by the sensitivity factor. The sensitivity factors for the reliability index generally work better than those for the failure probability. Approximate sensitivity factors in the examples afford the prescribed system reliabilities through simple calculation. Some additional calculations over ordinary reliability analysis are more than enough to attain the purpose.

Keywords: Performance function, Reliability adjustment, Sensitivity factor, System, Target reliability

性能関数, 信頼度調整, 感度係数, 系, 目標信頼度

1. INTRODUCTION

To ensure the prescribed reliability for steel structures, the load and resistance factor design known as LRFD has been developed^{1) 2)}. It ensures the prescribed reliability for the structural members and their joints to specified loads and their combinations. However, even if some members in a structure may fail, the system often survives as a whole. Therefore there needs a method to ensure the prescribed system reliability. This paper develops a simple method to attain the object adjusting the capacity of suitably chosen member(s) directed by the sensitivity factor.

2. SENSITIVITY FACTOR

In order to calculate the importance factor, the sensitivity factor of a parameter θ for the failure probability $\Phi(-\beta_s)$ of a system is defined by the differential that

$$\gamma_{1,\theta} = \frac{d\Phi(-\beta_s)}{d\theta} \quad (1)$$

where $\gamma_{1,\theta}$ = the sensitivity factor of θ for $\Phi(-\beta_s)$, β_s = the system reliability index, and $\Phi(\cdot)$ = the standard normal cumulative distribution function^{3) 4)}.

Another sensitivity factor of θ for β_s is defined by

$$\gamma_{2,\theta} = \frac{d\beta_s}{d\theta} = \frac{-1}{\phi(-\beta_s)} \frac{d\Phi(-\beta_s)}{d\theta} = \frac{-1}{\phi(-\beta_s)} \gamma_{1,\theta} \quad (2)$$

where $\phi(\cdot)$ = the standard normal probability density function.

The parameter θ_T that attains the target $\Phi(-\beta_T)$ or β_T can be obtained by the following equations.

* Prof Emeritus, Tokyo Metropolitan Univ, Dr Eng
** Kobori Research Complex, Kajima Corporation

東京都立大学 名誉教授・工博
鹿島小堀研究室

$$\Phi(-\beta_T) - \Phi(-\beta_S^{(0)}) = (\theta_T - \theta^{(0)})\gamma_{1,\theta} \quad (3)$$

or

$$\beta_T - \beta_S^{(0)} = (\theta_T - \theta^{(0)})\gamma_{2,\theta} \quad (4)$$

where $\beta_S^{(0)}$ and $\theta^{(0)}$ are β_S and θ of the preliminary design, respectively.

The sensitivity factor $\gamma_{1,\Phi(-\beta_1)}$ works well because it has the same unit as $\Phi(-\beta_S)$, in which β_1 is the reliability index of basic failure event F_1 . Though both $\Phi(-\beta_S)$ and β_S are nonlinear function of the average μ and standard deviation σ of a basic random variable, variation of $\gamma_{1,\theta}$ accompanied by the increase of μ and σ is larger than that of $\gamma_{2,\theta}$. Therefore, the sensitivity factor $\gamma_{2,\theta}$ is preferable for the parameters μ and σ .

3. PARALLEL SYSTEMS

The failure event F_S of a parallel system is expressed by the intersection of basic failure events F_i ($i = 1, 2, \dots, n$); that is, $F_S = F_1 \cap F_2 \cap \dots \cap F_n$. Its failure probability is expressed by

$$\Phi(-\beta_S) = \Phi(-\beta_{12\dots n}) = \Phi(-\beta_1)\Phi(-\beta_{23\dots n1}) = \Phi(-\beta_1)\psi = \Phi(-\beta_1)\Phi(-\beta_{211})\Phi(-\beta_{3112}) \dots \Phi(-\beta_{n112\dots(n-1)}) \quad (5)$$

where

$$\begin{aligned} \psi &= \Phi(-\beta_{23\dots n1}) = \Phi(-\beta_S)/\Phi(-\beta_1), \quad A_1 = \phi(-\beta_1)/\Phi(-\beta_1), \quad B_1 = A_1(\beta_1 - A_1), \\ \beta_{211} &= (\beta_2 - \rho_{12}A_1)/\sqrt{1 + \rho_{12}^2B_1}, \quad A_{211} = \phi(-\beta_{211})/\Phi(-\beta_{211}), \quad B_{211} = A_{211}(\beta_{211} - A_{211}), \\ \beta_{3112} &= \frac{\beta_{311} - \rho_{2311}A_{211}}{\sqrt{1 + \rho_{2311}^2B_{211}}} = \frac{\beta_3 - \rho_{23}A_2}{\sqrt{1 + \rho_{23}^2B_2}} \Big|_1, \quad \rho_{2311} = \frac{\rho_{12}\rho_{13}B_1 + \rho_{23}}{\sqrt{(1 + \rho_{12}^2B_1)(1 + \rho_{13}^2B_1)}} \end{aligned} \quad (6)$$

and the like⁵⁾. The symbol ρ_{ij} above denotes the correlation coefficient between F_i and F_j .

Probable errors in the calculation depend on ordering of the reliability indices. Practical experience suggests ordering that $\beta_1 \geq \beta_2 \geq \beta_3 \geq \dots \geq \beta_n$. The failure probability $\Phi(-\beta_S)$ in Eq.(5) is differentiated by θ as follows.

$$\gamma_{1,\theta} = \frac{d\Phi(-\beta_S)}{d\theta} = \frac{\partial\Phi(-\beta_S)}{\partial\Phi(-\beta_1)} \frac{d\Phi(-\beta_1)}{d\theta} + \frac{\partial\Phi(-\beta_S)}{\partial\Phi(-\beta_{211})} \frac{d\Phi(-\beta_{211})}{d\theta} + \frac{\partial\Phi(-\beta_S)}{\partial\Phi(-\beta_{3112})} \frac{d\Phi(-\beta_{3112})}{d\theta} + \dots \quad (7a)$$

Since $\partial\Phi(-\beta_S)/\partial\Phi(-\beta_1) = \Phi(-\beta_S)/\Phi(-\beta_1) = \psi$, and $d\Phi(-\beta_1)/d\theta = [\partial\Phi(-\beta_1)/\partial\beta_1](d\beta_1/d\theta) = -\phi(-\beta_1)(d\beta_1/d\theta)$, the sensitivity factor of θ for $\Phi(-\beta_S)$ is expressed by

$$\gamma_{1,\theta} = \frac{d\Phi(-\beta_S)}{d\theta} = -\psi\phi(-\beta_1) \frac{d\beta_1}{d\theta} \left[1 + \frac{A_{211}}{A_1} \frac{d\beta_{211}}{d\beta_1} + \frac{A_{3112}}{A_1} \frac{d\beta_{3112}}{d\beta_1} + \dots \right] = -\psi\phi(-\beta_1) \frac{d\beta_1}{d\theta} \zeta \quad (7b)$$

$$\text{where} \quad \zeta = 1 + \frac{A_{211}}{A_1} \frac{d\beta_{211}}{d\beta_1} + \frac{A_{3112}}{A_1} \frac{d\beta_{3112}}{d\beta_1} + \dots \quad (8)$$

If $\theta = \Phi(-\beta_1)$, then

$$\gamma_{1,\Phi(-\beta_1)} = \psi\zeta \quad (9)$$

Substituting Eq.(7) into Eq.(2), the following sensitivity factors for β_S are obtained.

$$\gamma_{2,\theta} = \frac{d\beta_S}{d\theta} = \frac{-1}{\phi(-\beta_S)} \frac{d\Phi(-\beta_S)}{d\theta} = \frac{A_1}{A_S} \frac{d\beta_1}{d\theta} \zeta \quad (10)$$

$$\text{and} \quad \gamma_{2,\beta_1} = \frac{d\beta_S}{d\beta_1} = \frac{A_1}{A_S} \zeta \quad (11)$$

Since $\beta_{211} = f(\beta_1, \beta_2, \rho)$, $d\beta_{211}/d\theta$ is represented as

$$\frac{d\beta_{211}}{d\theta} = \frac{\partial\beta_{211}}{\partial\beta_1} \frac{d\beta_1}{d\theta} + \frac{\partial\beta_{211}}{\partial\beta_2} \frac{d\beta_2}{d\theta} + \frac{\partial\beta_{211}}{\partial\rho} \frac{d\rho}{d\theta} \quad (12a)$$

Entering $\partial A_1/\partial\beta_1 = -B_1$ and $\partial B_1/\partial\beta_1 = A_1 + B_1(2A_1 - \beta_1)$ into the equation, the differential is expressed by

$$\frac{d\beta_{211}}{d\theta} = \frac{\rho_{12}C_{12} \frac{d\beta_1}{d\theta} + \frac{d\beta_2}{d\theta} - \frac{A_1 + \rho_{12}B_1\beta_{211}}{\sqrt{1 + \rho_{12}^2B_1}} \frac{d\rho}{d\theta}}{\sqrt{1 + \rho_{12}^2B_1}} \quad (12b)$$

$$\text{where} \quad C_{12} = B_1 - \frac{\rho_{12}\beta_{211}[A_1 + B_1(2A_1 - \beta_1)]}{2\sqrt{1 + \rho_{12}^2B_1}} \quad (13)$$

Similarly, the differential $d\beta_{312}/d\theta$ approximates to

$$\frac{d\beta_{312}}{d\theta} = \frac{\rho_{231} C_{231} \frac{d\beta_{21}}{d\theta} + \frac{d\beta_{31}}{d\theta} - D_{231} \frac{d\rho_{231}}{d\theta}}{\sqrt{1 + \rho_{231}^2 B_{21}}} \quad (14)$$

where

$$C_{231} = B_{21} - \frac{\rho_{231} \beta_{312} [A_{21} + B_{21} (2A_{21} - \beta_{21})]}{2\sqrt{1 + \rho_{231}^2 B_{21}}} \quad (15)$$

$$D_{231} = A_{21} + \frac{\rho_{231} \beta_{312} B_{21}}{\sqrt{1 + \rho_{231}^2 B_{21}}} \quad (16)$$

and

$$\frac{d\rho_{231}}{d\theta} = \frac{A_1 + B_1 (2A_1 - \beta_1)}{\sqrt{(1 + \rho_{12}^2 B_1)(1 + \rho_{13}^2 B_1)}} \left[\rho_{12} \rho_{13} - \frac{\rho_{231} (\rho_{12}^2 + \rho_{13}^2 + 2\rho_{12}^2 \rho_{13}^2 B_1)}{2\sqrt{(1 + \rho_{12}^2 B_1)(1 + \rho_{13}^2 B_1)}} \right] \frac{d\beta_1}{d\theta} \quad (17)$$

The inequality that $1 \geq \frac{A_{21}}{A_1} \frac{d\beta_{21}}{d\beta_1} \geq \frac{A_{312}}{A_1} \frac{d\beta_{312}}{d\beta_1} \geq \dots$ in Eq.(8) suggests several approximate estimates that

$$\gamma_{1,\theta} = -\Phi(-\beta_s) \frac{A_1}{A_s} \frac{d\beta_1}{d\theta} = -\Phi(-\beta_s) \frac{d\beta_s}{d\theta} \quad (18)$$

$$\gamma_{2,\theta} = \frac{A_1}{A_s} \frac{d\beta_1}{d\theta} \quad (19)$$

$$\gamma_{1,\Phi(-\beta_i)} = \frac{\Phi(-\beta_s)}{\Phi(-\beta_i)} = \psi \quad (20)$$

and

$$\gamma_{2,\beta_i} = \frac{\Phi(-\beta_s) \Phi(-\beta_i)}{\Phi(-\beta_i) \Phi(-\beta_s)} = \frac{A_i}{A_s} \quad (21)$$

[Example 1] Assume that the failure event F_s of a system is expressed by $F_1 \cap F_2 \cap F_3$, $\rho(F_i, F_j)=0.5$ and $P(F_i)=\Phi(-\beta_i)=\Phi(-\beta)$. Design for a target that $\beta_T=3.0$.

If $\rho=0$, then $\Phi(-\beta_T)=\Phi(-3)=\Phi(-\beta)^3$ and the corresponding $\beta=1.21$. If $\rho=1$, then $\beta=3$. In this connection, a straight line joining the two points is used to obtain an initial reliability index $\beta^{(0)}$; that is, $\beta^{(0)}=1.21+1.79\rho=2.11$.

Since all the reliability indices should be equally adjusted, $d\beta_j/d\beta_i=1$. In this case, above approximate factors may have large errors for a system that $\Phi(-\beta_i) \gg \Psi$. Though scarcely ever in actual structural system, this occurs if a system involves a large number of identical β_i and the performance functions are almost independent. Therefore, this modifies the equations considering $d\beta_j/d\beta_i=1$. The value of ζ in Eq.(8) varies from 3 to 1 when ρ increases from 0 to 1. The values of $(A_{21}/A_1)(d\beta_{21}/d\beta_1)$ are equal to 1 and 0 for $\rho=0$ and 1, respectively. This observation yields an approximation that $(A_{21}/A_1)(d\beta_{21}/d\beta_1) \approx (1-\rho)$. Also, an approximation that $(A_{312}/A_1)(d\beta_{312}/d\beta_1) \approx (1-\rho)^2$ is acceptable because $\Phi(-\beta_{312})$ is doubly conditional failure probability. Hence, this employs an approximation that $\zeta \approx 1 + (1-\rho) + (1-\rho)^2 = 1.75$. Application of approximate sensitivity factors $\gamma_{1,\Phi(-\beta_i)} (=0.0896)$ and $\gamma_{2,\beta_i} (=1.1587)$ into Eqs.(3) and (4) yields the same result that $\beta=2.0$. Corresponding β_s equals 2.995 which is very close to the target.

4. SERIES SYSTEMS

The failure event F_s of a series system is expressed by the union of basic failure events F_i ($i = 1, 2, \dots, n$); that is, $F_s = F_1 \cup F_2 \cup \dots \cup F_n$. Ordering that $\beta_1 \leq \beta_2 \leq \beta_3 \leq \dots \leq \beta_n$ is recommended to minimize probable errors in the calculation.

The failure probability of the system is expressed by

$$\Phi(-\beta_s) = \sum_{i=1}^n \Phi(-\beta_i) - \sum_{i=1}^{n-1} \sum_{j=2}^n \Phi(-\beta_{ij}) + \sum_{i=1}^{n-2} \sum_{j=2}^{n-1} \sum_{k=3}^n \Phi(-\beta_{ijk}) - \dots \quad (i < j < k < \dots) \quad (22)$$

or

$$\Phi(-\beta_s) = \Phi(-\beta_1) + \Phi(-\beta_{1'2}) + \dots + \Phi(-\beta_{1'2' \dots (n-1)'n}) = \Phi(-\beta_1) \psi \quad (23)$$

in which $\beta_{1'2}$ is the reliability index of $F_1^c F_2$, and F_1^c is the complement of F_1 .

Since $F_s^c = F_1^c F_2^c \dots F_n^c$, the system reliability is expressed by

$$\Phi(\beta_s) = \Phi(-\beta_1) \Phi(-\beta_{(2' \cap 3' \cap \dots \cap n')1'}) = \Phi(-\beta_1) \psi' \quad (24)$$

The values of sensitivity factors in Eqs.(22), (23) and (24) are calculated employing formulae for parallel systems above. Approximations based on Eqs.(18) through (21) are also effective for series systems because

$$\psi \geq 1, d\psi / d\Phi(-\beta_1) < 1, \text{ and } 1 \gg \Phi(-\beta_1) > \Phi(-\beta_{12}) > \cdots > \Phi(-\beta_{1'2'} \cdots (n-1)'n)$$

[Example 2] Examine the same problem as Example 1 but $F_S = F_1 \cup F_2 \cup F_3$.

If $\rho=0$, then $\Phi(\beta_r)=\Phi(3)=\Phi(\beta)^3$ and the corresponding $\beta=3.32$. If $\rho=1$, then $\beta=3$. Therefore, assume an initial reliability index that $\beta^{(0)}=3.32 - 0.32\rho=3.16$. The sensitivity factors for Eq. (22) are expressed by, employing ζ in Example 1,

$$\gamma_1 = 3 - 3 \frac{d\Phi(-\beta_{12})}{d\Phi(-\beta_1)} + \frac{d\Phi(-\beta_{123})}{d\Phi(-\beta_1)} = 3 - 3\Phi(-\beta_{211}) \left[1 + \frac{A_{211}}{A_1} \frac{d\beta_{211}}{d\beta_1} \right] + \Phi(-\beta_{2311}) \left[1 + \frac{A_{211}}{A_1} \frac{d\beta_{211}}{d\beta_1} + \frac{A_{3112}}{A_1} \frac{d\beta_{3112}}{d\beta_1} \right] \quad (25)$$

$$\gamma_{1a} = 3 - 3[1 + (1 - \rho)]\Phi(-\beta_{211}) + [1 + (1 - \rho) + (1 - \rho)^2]\Phi(-\beta_{2311}) \quad (26)$$

$$\gamma_2 = \frac{\Phi(-\beta_1)}{\Phi(-\beta_S)} \gamma_1 \quad (27)$$

and

$$\gamma_{2a} = \frac{\Phi(-\beta_1)}{\Phi(-\beta_S)} \gamma_{1a} \quad (28)$$

Approximate γ is expressed by the subscript of a . Both $\gamma_{1a}(=2.7774)$ and $\gamma_{2a}(=1.0637)$ yield the same result that $\beta=3.31$ and corresponding $\beta_S=3.005$. The approximate sensitivity factors in this example work well as before.

[Example 3] This is extracted from the example by Ma and Ang⁶⁾. The structure involves four potential failure modes in Table 1. The system failure event F_S is expressed by $F_1 \cup F_2 \cup F_3 \cup F_4$. The constituent basic random variables X are normally distributed and statistically independent. Assuming that the coefficient of variation δ of X is constant, the standard deviation σ varies in proportion to the average μ . The variables X_1 and X_2 express the strength capacities of the structure whereas X_3 and X_4 are the external forces on it. Since natural forces are generally uncontrollable, capacities remain as adjustable variables. The parameters of X are given in Table 2. Table 3 represents the matrix of the correlation coefficients. Ordinary reliability analysis yields the estimates that $\Phi(-\beta_S)=0.0388$ and $\beta_S=1.765$. Design for a target that $\beta_T=2.0$ adjusting the average of X_1 or X_2 . Also, design for another target that $\beta_T=1.5$.

Table 1 Performance functions and reliability indices

F_i	Performance function	μ_i	σ_i	β_i	$\Phi(-\beta_i)$
F_1	$Z_1=4X_1+2X_2-10X_3-15X_4$	650	357.8	1.82	0.0346
F_2	$Z_2=4X_1-15X_4$	690	311.9	2.21	0.0135
F_3	$Z_3=2X_1+4X_2-10X_3-15X_4$	890	394.0	2.26	0.0120
F_4	$Z_4=4X_2-10X_3$	920	305.0	3.02	0.0013

Table 2 μ_X and δ_X

X_i	μ_r	δ_r
X_1	360	0.15
X_2	480	0.15
X_3	100	0.10
X_4	50	0.30

Table 3 ρ_{ij}

	$j=1$	2	3	4
$i=1$	1.00	0.87	0.89	0.47
2	0.87	1.00	0.60	0.00
3	0.89	0.60	1.00	0.77
4	0.47	0.00	0.77	1.00

Approximate values of γ_{2,μ_i} are 0.00968 and 0.00505 for X_1 and X_2 , respectively. To save the material used, the average of X that has the largest sensitivity factor should be increased. Hence, the average of X_1 is increased from 360 to 385 directed by the sensitivity factor. Corresponding system reliability index for the system is 1.9954, which agrees well with the target reliability. When $\beta_T=1.5$, which is less than that of initially assumed system, the average of X that has the smallest sensitivity factor should be decreased in order to minimize the material weight used. In this case, the sensitivity factor indicates the average value 430 of X_2 , which perfectly attain the purpose.

5. GENERAL SYSTEMS

The failure event F of a general system is expressed by the union of mutually exclusive intersections. This implies that the system unreliability is expressed by a sum of the probabilities of the intersections of basic failure events. Therefore, its sensitivity factor equals a sum of the sensitivity factors of the intersections involved.

[Example 4] A constant load T is suspended by n identical bars that equally share the load. Assume that the breaking strength of bar X_i ($i = 1, 2, \dots, n$) is normally distributed with the average μ and the standard deviation $\sigma(=1.0)$. Assume also that $\rho(X_i, X_j)=0$ and $T=1$. Let the target system reliability index β_T be 2.5. Express the failure event of the system and then compare the probable errors due to application of approximate sensitivity factors for $n=3$.

If the failure event W_1 of a branch has failure sequence of bar 1, bar 2, $\dots$ and bar n in that order, then the event W_1 is expressed by

$$\begin{aligned}
 W_1 &= (X_1 < \frac{T}{n}) \cap (X_2 < \frac{T}{n-1}) \cap \dots \cap (X_n < T) \cap (X_1 < X_j) \cap (X_2 < X_k) \cap \dots \cap (X_{n-1} < X_n); \quad (j=2,3 \dots n, k=3,4 \dots n) \\
 &= \bigcap_{i=1}^n \Phi \left[\frac{-1}{\delta} \left(1 - \frac{T}{(n-i+1)\mu} \right) \right] \cap (X_1 < X_j) \cap (X_2 < X_k) \cap \dots \cap (X_{n-1} < X_n) \\
 &= \bigcap_{i=1}^n \Phi \left[-\beta_{n12} \dots (i-1) \right] \cap (X_1 < X_j) \cap (X_2 < X_k) \cap \dots \cap (X_{n-1} < X_n) \quad (29)
 \end{aligned}$$

There are $n!$ possible paths. Since the intersection of the constraints on the variables in every two branches is empty, the branches are mutually exclusive. Therefore, the failure probability of the system is $n!P(W_1)$.

If the constraints on the variables are eliminated, then W_i is positively correlated to W_j for all the combinations of i and j . As an example, let us examine the contents of F_s for $n=3$. The failure event of the system is expressed by

$$F_s = \xi_1 \cup \xi_2 \cup \xi_3 \quad (30)$$

where

$$\begin{aligned}
 \xi_1 &= E_1(F_2G_3 \cup F_3G_2), \quad \xi_2 = E_2(F_1G_3 \cup F_3G_1), \quad \xi_3 = E_3(F_1G_2 \cup F_2G_1) \\
 E_i &= (X_i < T/3), \quad F_i = (X_i < T/2), \quad G_i = (X_i < T) \quad (31)
 \end{aligned}$$

Since $E_i \subset F_i \subset G_i$, $E_i F_i = E_i$, $E_i \cup F_i = F_i$ and the like. Thus,

$$\xi_1 \xi_2 = E_1 E_2 G_3 \quad \text{and} \quad \xi_1 \xi_2 \xi_3 = E_1 E_2 E_3 \quad (32)$$

Substituting these relationships into Eq.(30), the following expression is obtained.

$$P(F_s) = 6P(E)P(F)P(G) - 3P(E)P(F)^2 - 3P(E)^2P(G) + P(E)^3 \quad (33)$$

Initially assumed reliability index $\beta_1^{(0)}$ of 1.3 is derived from a rough estimate that $\Phi(-2.5) = 6\Phi(-\beta_1)^3$. In this case, $\Phi(-\beta_s^{(0)})=0.008468$ and $\beta_s^{(0)}=2.39$. The sensitivity factors are expressed by

$$\gamma_{1,\mu} = \frac{d\Phi(-\beta_s^{(0)})}{d\mu} = \sum_{i=1}^3 \frac{d\Phi(-\beta_s^{(0)})}{d\Phi(-\beta_i)} \frac{d\Phi(-\beta_i)}{d\beta_i} \frac{d\beta_i}{d\mu} = \sum_{i=1}^3 -\phi(-\beta_i) \frac{d\Phi(-\beta_s^{(0)})}{d\Phi(-\beta_i)} \frac{d\beta_i}{d\mu} \quad (34)$$

$$\gamma_{2,\mu} = \frac{d\beta_s^{(0)}}{d\mu} = \frac{-\gamma_{1,\mu}}{\phi(-\beta_s^{(0)})} \quad (35)$$

Approximate sensitivity factors are estimated using Eqs.(18) and (19). Table 4 compares four sensitivity factors and corresponding system reliability indices. Note that the sensitivity factors for β_s work well and approximate sensitivity factor for β_s is preferable.

Table 4 γ , μ , β_s and $\Phi(-\beta_s)$

	γ_1	γ_{1a}	γ_2	γ_{2a}
γ	-0.03694	-0.0393	1.61	1.71
μ	1.69	1.687	1.7	1.694
β_s	2.484	2.48	2.5	2.49
$\Phi(-\beta_s)$	0.0065	0.006563	0.006214	0.006385

6. CONCLUDING REMARKS

This paper develops a method to design a system that has prescribed reliability adjusting the capacity of suitably chosen element(s) directed by the sensitivity factor. The sensitivity factor for the reliability index generally works better than that for the failure probability. Approximate sensitivity factors in the examples afford the prescribed system reliability through simple calculation.

REFERENCES

- 1) American Institute of Steel Construction, Inc. Load & Resistance Factor Design Specification for Structural Steel Buildings, 1983
- 2) Architectural Institute of Japan. Recommendation for Limit State Design of Steel Structures, 1998 10
- 3) Bjerager, P. and Krenk, S. Parametric Sensitivity in First Order Reliability Analysis, Journal of Engineering Mechanics, ASCE, 115(7), pp.1577-1582, 1989
- 4) Ronold, Knut O. Sensitivity Measures in Nested Reliability Analyses, Journal of Engineering Mechanics, ASCE, 117(9), pp.2101-2113, 1991.
- 5) Terada, S. and Takahashi, T. Failure Conditioned Reliability Index, Journal of Structural Engineering, ASCE, 114(4), pp 942-952, 1988
- 6) Ma, H -F, and Ang, A H -S. Reliability Analysis of Redundant Ductile Structural Systems, Structural Research Series No. 494, University of Illinois at Urbana-Champaign, 1981

和文要約

1. 序

「荷重・耐力係数設計法」は、鋼構造部材とその接合部に所定の信頼度を与える設計法である。しかし仮に幾つかの部材が壊れたとしても、系全体としては生き残る可能性が高い。従って、所定の系信頼度を保証する簡単な設計法の開発が必要である。本論では、感度係数により系信頼度を調節する一法を誘導する。

2. 感度係数

系破壊確率に対するパラメータ θ の感度係数 $\gamma_{1,\theta}$ を、(1)式で定義する。ここに、 β_s は系信頼性指標、 $\Phi(\cdot)$ は標準正規累積分布関数である。別に、 β_s に対する θ の感度係数 $\gamma_{2,\theta}$ を(2)式で定義する。ここに、 $\phi(\cdot)$ は標準正規確率密度関数である。

パラメータとしては、性能関数中の係数、基礎確率変数の平均値 μ と標準偏差 σ 、基本破壊モード F_i の信頼性指標 β_i とその破壊確率 $\Phi(-\beta_i)$ 等がある。(1)式は、 $\theta=\Phi(-\beta_i)$ に適用し、それ以外の θ には(2)式を用いる。系信頼度は(3)、(4)式で調節される。

3. 並列系

並列系の破壊事象は、 $F_s=\bigcap_{i=1}^n F_i$ で表わされる。系の破壊確率 $P(F_s)$ は、(6)式の条件付き信頼性指標を用いて(5)式で表わされる。式中の ρ_{ij} は、 F_i と F_j の間の相関係数である。計算上、 $\beta_1 \geq \beta_2 \geq \beta_3 \geq \dots \geq \beta_n$ となるように符号をふろう。

$\Phi(-\beta_s)$ に対する感度係数 $\gamma_{1,\theta}$ は、(7)式で表わされる。式中の ζ は、(8)式で表わされる。 $\theta=\Phi(-\beta_1)$ の場合の感度係数 $\gamma_{1,\Phi(-\beta_1)}$ は(9)式となる。 β_s に対する感度係数 $\gamma_{2,\theta}$ は(7)式を(2)式に代入すれば求まり、(10)、(11)式で表わされる。

条件付信頼性指標の微係数 $d\beta_{21}/d\theta$ は、(13)式の係数 C_{12} を用いて、(12)式で表わされる。これと同様に、 $d\beta_{312}/d\theta$ は(15)～(17)式の係数を用いて(14)式で計算できる。(18)～(21)式は、感度係数の近似式である。

【例題1】系の破壊は、 $F_s=F_1 \cap F_2 \cap F_3$ で表わされる。 $P(F_i)=\Phi(-\beta_i)=\Phi(-\beta)$ 、 $\rho_{ij}=\rho=0.5$ とする。目標信頼度は $\beta_r=3.0$ である。近似式の誤差を調べる。

$d\beta/d\beta_1=1$ に基づく β の初期値として、 $\beta^{(0)}=1.21+1.79\rho=2.11$ を仮定した。 $\gamma_{1,\Phi(-\beta_1)}=0.0896$ と $\gamma_{2,\beta_1}=1.1587$ より求めた β は等しく、2.0である。系信頼度は2.995であり、目標値にほぼ一致した。

4. 直列系

直列系の破壊は、 $F_s=\bigcup_{i=1}^n F_i$ で表わされる。計算誤差を減らすために、 $\beta_1 \leq \beta_2 \leq \beta_3 \leq \dots \leq \beta_n$ となるように符号をふる。その破壊確率は、(22)式または(23)式で表わされる。系信頼度は、余事象を用いた(24)式で表わされる。これらの式の感度係数は、並列系で得られた結果を式中の積事象に代入すれば求められる。また、(18)～(21)式に基づく近似式も有効である。

【例題2】 $F_s=F_1 \cup F_2 \cup F_3$ とする。他は【例題1】と同じ条件で $\beta_r=3.0$ の系を設計する。

β の初期値として、 $\beta^{(0)}=3.32-0.32\rho=3.16$ を仮定する。(22)式の破壊確率について計算すれば、(25)～(28)式の感度係数が得られる。添字

a は近似を表わす。

$\gamma_{1a}(=2.7774)$ と $\gamma_{2a}(=1.0637)$ より求めた β は等しく、3.31である。これに相当する系信頼性指標 β_s は3.005であり、目標値にほぼ一致した。

【例題3】この構造物の破壊は、 $F_s=F_1 \cup F_2 \cup F_3 \cup F_4$ で表わされる。その性能関数と信頼性指標をTable 1に示す。確率変数 X は、互いに独立な正規確率変数である。変動係数を一定と仮定すると、標準偏差は平均値に比例する。 X_1 と X_2 は容量を、 X_3 と X_4 は需要を示す確率変数である。一般に需要は管理できないので、容量を加減する。変数のパラメータをTable 2に示す。Table 3は性能関数間の相関係数である。 $\Phi(-\beta_s)=0.0388$ 、 $\beta_s=1.765$ と計算された。 $\beta_r=2.0$ および $\beta_r=1.5$ に対して設計する。

$\gamma_{2,\mu_1}=0.00968$ 、 $\gamma_{2,\mu_2}=0.00505$ である。資材節約のために、数値の大きい μ_1 を385に増す。その時、 $\beta_s=1.9954$ となり、 β_r に良く一致した。 $\beta_r=1.5$ の場合は $\mu_2=430$ とするのが経済的であり、 $\beta_s=1.5$ となった。

5. 一般系

一般系の破壊は、排反な積事象の和で表わされる。従って、系の感度係数は排反な積事象の感度係数の和に等しい。

【例題4】荷重 T を、 n 本の並行な棒で吊る。荷重は総ての棒に等しく配分される。棒の破断強度 X の平均値は μ 、標準偏差 σ は1であり、正規分布を仮定する。 $n=3$ 、 $T=1$ として、 $\beta_r=2.5$ の系を設計する。

棒1、2、3、 $\dots$ 、 n の順に破壊が起こる事象 W_1 は、(29)式で表わされる。 $P(F_s)=n!P(W_1)$ である。

(29)式中の確率変数 X の大小についての条件をとり除けば、経路の破壊事象は互いに排反ではなくなる。 $n=3$ の場合の破壊事象は(30)式で表わされる。系破壊確率は(31)、(32)式を(30)式に代入して、(33)式のごとく求められる。

略算に基づき、初期値 $\beta_1^{(0)}=1.3$ を仮定する。感度係数は(34)、(35)式で表わされる。近似感度係数は(18)、(19)式を用いて求められる。結果をTable 4に示す。信頼性指標 β_s の感度係数が良い結果をもたらしている。

6. 結語

感度係数により系信頼度を調節する一法を導いた。系信頼性指標に対する感度係数は、破壊確率に対するそれに比べて優れている。近似感度係数は計算が簡単であり、例題では十分に機能を発揮した。通常の解析に若干の計算を付加すれば、信頼度の調整ができる。

(2000年3月9日原稿受理、2000年5月9日採用決定)