MECHANICAL MODEL FOR DIRECT SHEAR FAILURE OF CONCRETE

—Prediction for entire load-displacement characteristics—

コンクリートの直接せん断破壊の力学モデル

—全荷重-変形特性の予測—

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The objective of this paper is to develop a mechanical model for direct shear failure of concrete, focusing on the entire load-displacement characteristics. The method makes use of rotated smeared crack model, truss model theory and interlocking shear transfer model, combined in a simple model. The analysis employs the development of multiple diagonal cracks under far field uniform shear stress and macroscopic shear crack propagation. The model is verified through the comparison of the model's prediction with both experimental measurements and nonlinear Finite Element Method (FEM) analysis results. The model is found in good agreement with both experiments and FEM analysis, indicating that it can be used as a reliable tool for reasonable first estimate for the response. Specifically, the mechanical model can capture the complicated transition behavior although FEM analysis can not account for it. Finally, the present analytical study qualitatively shows the interaction between macroscopic shear strain softening phenomena and interlocking phenomena at the post-peak regime.

Keywords: Mechanical model, Direct shear failure of concrete, Rotated smeared crack, Truss model, Interlocking, Nonlinear FEM analysis

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1. INTRODUCTION

Mechanism of shear behavior has been subjected to heated debates in past decades. However, it has not been clarified sufficiently yet. Mechanism of nonlinear behavior in concrete structures mainly depends on cracking phenomena associated with strain softening. Therefore, it is necessary to examine strain softening behavior of shear fracture process in order to make clear the nonlinear mechanism.

The amount of experimental data on the mechanical properties of shear fracture process zone is too limited. For making clear the shear transfer mechanism and strain softening behavior of the fracture process zone, shear tests were performed by Ishihara and Mihashi¹⁾. In the test, it was found that approximately four stages of shear fracture mechanism were observed (see Fig. 1). The first was the stage before crack initiation(I), the second was the stage of crack propagation(II), the third was a transition stage after crack propagation ended(III), and the final stage was a stable aggregate interlocking stage(IV).

Based on the ground of the aforementioned study, the present work aims at developing a simple mechanical model for the analysis of entire load-displacement characteristics of shear failure of concrete under direct shear loading. The intent of this modeling methodology is based on the fact that it is often possible to find general analytical solutions, which are much

easier to handle than complex numerical solutions such as nonlinear FEM analysis. The proposed model is formulated by modifying the original model, which was developed to predict the entire shear stress-displacement characteristics for shear keys, deep beams and membrane elements^{2), 3), 4)}. Specifically, the present model employs the interlocking model and can account for the transition stage(III) and the interlocking stage(IV) which were not considered in the original model.

2. NUMERICAL FORMULATION OF MECHANICAL MODEL

2.1 Fracture Sequence of Shear Failure

Based on the experiment by Ishihara and Mihashi¹⁾ shown in Fig.2, the following simple fracture sequence of shear failure is macroscopically constructed and is schematically shown in Fig.3

Phase I At the first stage of loading, flaws near the joint are easily activated by tensile stress induced due to shear stress. Then, intensive multiple cracking occurs on the local principal stress trajectories, and forms a shear fractured zone roughly aligned (mode I initiation). The multiple cracks are assumed evenly distributed along the shear fractured zone with a certain angle of inclination. Although in reality such distribution is not uniform, this simplification could be supported by the 'critical zone' concept along the direct shear plane⁵⁾; within a cracked region, the extensive

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FIG. 1. Schematic Description of Shear Stress vs. Shear Displacement¹⁾

FIG. 2. Specimens and Loading System of Test series B¹⁾

FIG. 3. (a) Shear Failure Sequence at Shear Fractured Zone; (b) Representative Rotated Crack; (c) Fictitious Surface Roughness

cracking of the concrete has the effect of redistributing the shear stress and the transverse normal stress more evenly along the shear plane

Phase 2 With further shear loading, the multiple cracks are assumed to rotate following the principal stress axis under mode I condition. The shear stress along multiple cracks is thus zero. Although multiple tensile cracks propagate in reality as cracks winged from each multiple crack tip, this simplification could show the analogy between the winged crack and the representative rotated crack along the principal axis as shown in Fig 3b. In addition, it was reported by Rots⁶⁾ that the rotated crack concept could describe the distributed defects of gradually rotating orientation, supported by considerable experimental evidence by Mattock and Hawkins⁷⁾, Vecchio and Collins⁸⁾, Bhude and Collins⁹⁾, Kollegger and Mehlhorn¹⁰⁾. With further shear loading, both the multiple crack opening displacement and the compressive strain of the struts between cracks increase continuously.

Phase 3 Crushing failure of the compression struts occurs as a result of further increase of shear loading, leading to the macroscopic shear strain softening. Specifically, multiple cracks are coalesced into the macroscopic vertical shear crack with rough surface caused by the highly localized strain distribution.

Phase 4 At the start of macroscopic shear strain softening, the interlocking occurs simultaneously, and failure mechanism transfers into the contact mechanism. The interlocking phenomenon could be associated with a fictitious surface roughness induced by both crushed compression struts and aggregates as shown in Fig 3c. Here, the macroscopic shear strain softening described in Phase 3 is continuously advanced in this interlocking process, too.

2.2 Equilibrium and Compatibility Conditions

In the present approach, the fracture process of multiple cracks is modeled by means of a combination of a rotated smeared crack model and a truss model. The mechanical model satisfies three basic requirements: equilibrium, compatibility and material constitutive laws. Equilibrium condition in a cracked element reinforced by steel bars are constructed

based on the works of Vecchio and Collins⁸⁾ and Hsu et al.⁹⁾ as shown in Fig 4. In this modeling, stress and strain are assumed uniformly distributed as averaged ones over the entire shear fractured zone. After diagonal cracking occurs, a series of diagonal compression struts is formed in the compressive direction (c-direction).

The element takes only compressive stress σ_c in the c-direction of compression struts and only tensile stress σ_t in the tensile direction (t-direction) transverse to compression struts. Shear stress τ_{ct} along the cracked element is assumed zero. Thus, σ_c and σ_t are always principal stresses of this system. The angle between the x-y and c-t coordinate systems is designated as θ . This angle is also the angle of inclination of compression struts with respect to the x-axis. Considering the reinforcement ratio of ρ_x in x-direction, average stresses of reinforced concrete element in the two coordinate systems, x-y and c-t, are related by the following equations:

$$\sigma_x = \sigma_c \cos^2 \theta + \sigma_t \sin^2 \theta + \rho_x \sigma_{sx} \quad (1a)$$

$$\sigma_y = \sigma_c \sin^2 \theta + \sigma_t \cos^2 \theta \quad (1b)$$

$$\tau_{xy} = (\sigma_c - \sigma_t) \sin \theta \cos \theta \quad (1c)$$

where σ_x and σ_y are the normal stresses in the x-y coordinate system, τ_{xy} is the shear stress in the x-y coordinate system, and σ_c and σ_t are the normal stresses of concrete in the c-t coordinate system (principal stresses). σ_{sx} is the normal stress of reinforcement in the x-y coordinate system. Similar expressions can be obtained in terms of the average strains as follows:

$$\epsilon_x = \epsilon_c \cos^2 \theta + \epsilon_t \sin^2 \theta \quad (2a)$$

$$\epsilon_y = \epsilon_c \sin^2 \theta + \epsilon_t \cos^2 \theta \quad (2b)$$

$$\gamma_{xy} = 2(\epsilon_c - \epsilon_t) \sin \theta \cos \theta \quad (2c)$$

where ϵ_x and ϵ_y are the normal strains in the x-y coordinate system, γ_{xy} is the shear strain in the x-y coordinate system, and ϵ_c and ϵ_t are the normal strains in the c-t coordinate system (principal strains).

FIG. 4. (a) Crack Formation; (b) Stress Transformation Systems; (c) Equilibrium in Reinforced Concrete

2.3 Constitutive Models

Considering the good predictions by the original model for the shear failure of deep beams and membrane elements⁴⁾, the same constitutive models are adopted in the present mechanical model. The tensile stress-strain relation of plain concrete in the direction perpendicular to the compression struts shown in Fig 5a is assumed to be expressed by the following equations. The bilinear tensile stress-displacement relation originally proposed by Hillerborg,¹¹⁾ as a tension-softening concept is adopted for the descending branch based on the crack band theory

$$\sigma_t = E_0 \epsilon_t \quad \text{if } \epsilon_t \leq \epsilon_{cr} \quad (3a)$$

$$\sigma_t = f_t \frac{\left(\epsilon_{tu1} - \frac{\epsilon_{cr}}{3} - \frac{2\epsilon_t}{3} \right)}{(\epsilon_{tu1} - \epsilon_{cr})} \quad \text{if } \epsilon_{cr} < \epsilon_t \leq \epsilon_{tu1} \quad (3b)$$

$$\sigma_t = \frac{f_t (\epsilon_{tu2} - \epsilon_t)}{3(\epsilon_{tu2} - \epsilon_{tu1})} \quad \text{if } \epsilon_{tu1} < \epsilon_t \leq \epsilon_{tu2} \quad (3c)$$

$$\epsilon_{cr} = \frac{f_t}{E_0}, \quad \epsilon_{tu1} = \epsilon_{cr} + \frac{4G_F}{5f_t h}, \quad \epsilon_{tu2} = \epsilon_{cr} + \frac{18G_F}{5f_t h} \quad (3d, 3e, 3f)$$

where E_0 is the initial Young's modulus and f_t is the tensile strength of concrete. ϵ_{cr} is the cracking strain, ϵ_{tu1} and ϵ_{tu2} are the post-cracking characteristic strains. G_F is the fracture energy and h is the crack spacing. Here, we roughly assume that h is identical to $W_S/5.0$ (W_S is the width of shear fractured zone, see Fig 3a). The quantity of $W_S=15\text{mm}$ is determined so as to fit the predicted normal compressive stress due to restraint bars to that obtained in the experiment, which detail will be discussed later.

The assumed compressive stress-strain relation in the direction of compression struts is constructed based on the works of Soroushian et al.¹²⁾

and Hognestad¹³⁾. In addition, compressive deterioration (softening) of concrete struts related to tensile strain in the direction perpendicular to struts is considered based on the work of Vecchio and Collins⁸⁾. Thus, assumed stress-strain relations shown in Fig. 5b are described by the following equations

$$\sigma_c = \frac{f_c}{\lambda} \left[\frac{2\epsilon_c}{\epsilon_{c0}} - \left(\frac{\epsilon_c}{\epsilon_{c0}} \right)^2 \right] \quad \text{if } \epsilon_c \leq \epsilon_{c0} \quad (4a)$$

$$\sigma_c = \frac{f_c}{\lambda} \left[1 - Z(\epsilon_c - \epsilon_{c0}) \right] \quad \text{if } \epsilon_{c0} < \epsilon_c \leq \epsilon_{cu1} \quad (4b)$$

$$\sigma_c = \frac{0.2f_c}{\lambda} \quad \text{if } \epsilon_{cu1} < \epsilon_c \quad (4c)$$

$$\lambda = 0.8 + 0.34 \frac{\epsilon_t}{\epsilon_{c0}} \geq 1.0 \quad (4d)$$

$$\epsilon_{c0} = \frac{2f_c}{E_0}, \quad \epsilon_{cu1} = \frac{0.8}{Z} + \epsilon_{c0}, \quad Z = \frac{0.5}{\frac{3+145\epsilon_{c0}f_c}{145f_c-1000} - \epsilon_{c0}} \quad (4e, 4f, 4g)$$

where λ is the coefficient to take account of the softening phenomena, f_c is the compressive strength and ϵ_{c0} is the associated strain. The unit of f_c in Eq. (4g) is MPa.

FIG. 5. Stress-Strain Relationship in Mechanical Model for (a) Tension; (b) Compression

The stress-strain relation in reinforcement is assumed as a perfectly elasto-plasticity. Here, a special treatment is conducted in order to assure the experimental condition. In the experiment, steel bars are not bonded in concrete (see Sec. 3.1), and therefore the steel strain is modified as follows. The deformation of restraint bars is assumed as localized deformation at the shear fractured zone width W_S and the total strain is calculated as the ratio of the localized deformation to the total length of the bar

$$\sigma_{sx} = E_{st} \left(\frac{\epsilon_x W_S}{H} \right) \quad \text{if } \left(\frac{\epsilon_x W_S}{H} \right) < \epsilon_{fy} \quad (5a)$$

$$\sigma_{sx} = f_y \quad \text{if } \left(\frac{\epsilon_x W_S}{H} \right) \geq \epsilon_{fy} \quad (5b)$$

$$\epsilon_{fy} = \frac{f_y}{E_{st}} \quad (5c)$$

where, E_{st} , ϵ_{fy} , f_y are the Young's modulus, yield strain and yield stress, respectively. Also, H is the distance between both ends of restraint bars.

The interlocking model for Phase 4 is constructed based on the experimental work by Li and Maekawa¹⁴⁾ which calculates that the shear stress transferred across a crack with constant width under monotonic loading paths is governed by the ratio of shear displacement to the crack width as follows

$$\tau_{int} = \frac{m\delta^2}{w_{scr}^2 + \delta^2} \quad \text{with} \quad m = m_c \sqrt[3]{\frac{f_c'}{\lambda}} \quad (6)$$

where δ is the shear displacement, w_{scr} is the constant shear crack width and m_c is a constant associated with the surface roughness in joints. The unit of f_c' in Eq (6) is MPa. The macroscopic vertical shear crack width w_{scr} is fictitiously assumed as the horizontal deformation at the shear fractured zone and calculated as $w_{scr} = \epsilon_x W_s$

2.4 Model of Apparent Poisson's Ratio

In the present modeling, it is assumed that the relation between the tensile and compressive strain of a compression strut could be constructed using apparent Poisson's ratio $\nu_a (= \epsilon_c / \epsilon_t)$ defined as.

$$\nu_a = 0.2 \quad \text{if} \quad \epsilon_c < \epsilon_{c0} \quad \text{and} \quad \nu_a = 0.5 \quad \text{if} \quad \epsilon_{c0} \leq \epsilon_c \quad (7)$$

FIG. 6. Verification for Model of Apparent Poisson's Ratio

The apparent Poisson's ratio ν_a is a salient feature in the present mechanical model. To quantify the apparent Poisson's ratio in the tensile strain-dominated field, the comparison is made with experimental results referred to the works of Vecchio and Collins⁸⁾ and Mansur et al.¹⁵⁾ as shown in Fig. 6. This figure shows the relation between the apparent Poisson's Ratio and the normalized compressive strain ($= \epsilon_c / \epsilon_{c0}$). In the figure, the heavy solid line stands for the proposed model. The scattered data stand for both experimental data and analytical predictions introduced in the literature. The points marked by a + are the test results of reinforced concrete elements subjected to in-plane shear and normal stresses⁸⁾. The circular points are the results of reinforced fiber concrete deep beam tests¹⁵⁾ (BEAM B3 shear span ratio = 0.865), while the normalized compressive strain is obtained by assuming $\epsilon_{c0} = -0.002$. The rectangular points are obtained from the parameters calculated by Vecchio and Collins⁸⁾ for the comparison of their analytical prediction with their test data of PV 20 specimen which was a reinforced concrete element subjected to in-plane shear stress. From the figure, it is obvious that the apparent Poisson's ratio controlled by the tensile strain could be applicable for tensile strain-dominated field as rough estimation. The apparent Poisson's ratios will be applied to the modeling of present crack propagation according to the procedure presented in the next section.

2.5 Solution Procedure

Using the apparent Poisson's ratio ($\nu_a = \epsilon_c / \epsilon_t$) and the specified σ_x (in the present analysis, $\sigma_x = 0$), the preceding 12 unknowns (σ_x , σ_y , τ_{xy} , ϵ_x , ϵ_y , γ_{xy} , σ_t , σ_c , σ_{sx} , ϵ_t , ϵ_c and θ) are reduced to 10. By selecting one of them as a known value, the remaining 9 unknowns can be obtained from a set of 9 equilibrium, compatibility and constitutive equations. Hence, one can develop the relation between the average shear stress τ_{xy} and the average shear displacement $\delta (= \gamma_{xy} W_s)$ following the procedure shown in Fig. 7. Here, it should be noted that the interlocking model activates when the macroscopic shear strain softening for Phase 4 in Fig 3 initiates. It is assumed that this initiation is defined as a moment when the normal compressive stress $\rho_x \sigma_{sx}$ is just reduced, which could imply softening.

FIG. 7. Solution Procedure

3. VERIFICATION OF MECHANICAL MODEL

3.1 Review of Experiments

The experimental work was originally conducted to confirm the performance of construction joints in the structures by means of inversed placing method¹⁾. Two series (A and B) of tests were performed. The specimens and loading system of test series B are shown in Fig. 2. Several degrees of roughness in construction joints were installed such as plain, a little rough and rough. The specimens of each test series had the same fundamental configuration. The main differences between two series were the shear plane area and the loading speed. In test series A, the shear plane area was 100 mm × 100 mm and the shear displacement rate was 0.02 mm/min. In test series B, the shear plane area was 90 mm × 90 mm and the shear displacement rate was 0.005 mm/min.

Four internal steel restraint bars passing through inner sheaths were inset

to apply axial compressive force upon the crack plane by pushing. At the start of the shear test, no axial compressive force was applied to the internal restraint bars. Dowel action and undesirable shear resistance due to the bars were avoided by making the sheaths 5mm larger in diameter and placing the bars in the center thereof. The reinforcement ratio of restraint steel bars was 1.34 percent. Normal compressive stress was measured by attaching strain gauges to the steel internal restraint bars.

For concrete mixing, normal Portland cement with water-cement ratio of 60%, and the maximum aggregate size of 15 mm were used. Six days after placing concrete in the first portion, the joint surface was roughened. Seven days after roughening, the second portion was placed. Concrete specimens without joint were cast at the same time as the second portion. The average compressive strength, tensile strength and Young's Modulus are 24.2 MPa, 2.04 MPa and 23.2 GPa for A series, and 26.5 MPa, 2.20 MPa and 23.1 GPa for B series, respectively.

3.2 Finite Element Analysis

In order to supplement the model verification, FEM analysis is carried out using SBETA¹⁶⁾ Finite Element Program developed by V. Cervenka et al. A rotated crack model in the smeared crack approach is employed. In addition, the following effects of concrete behavior are considered: nonlinear behavior in compression including hardening and softening, fracture of concrete in tension based on nonlinear fracture mechanics, biaxial stress failure criterion according to Kupfer et al.¹⁷⁾ by means of equivalent uniaxial stress-strain relationship¹⁸⁾, reduction of compressive strength after cracking¹⁹⁾. The details of numerical formulation are given in SBETA¹⁶⁾. In the rotated crack model⁸⁾,²⁰⁾, the direction of principal stress coincides with the direction of principal strain. No shear strain occurs on the crack plane and only two normal stress components must be defined. If the principal strain axes rotate during loading, the direction of cracks rotates, simultaneously.

Regarding the constitutive models, the available models are limited in SBETA and we adopt the following models which are relatively similar to those used in the mechanical model. Behavior of concrete in tension before cracking is assumed linear elastic. After cracking, the exponential crack opening law shown in Fig. 8a is used in the present analysis for the crack opening. The function of exponential crack opening law was empirically derived by Hordijk²¹⁾ as follows.

$$\frac{\sigma_t}{f_t^{ef}} = \left[1 + \left(c_1 \frac{w}{w_c} \right)^3 \right] e^{\left(-c_2 \frac{w}{w_c} \right)} - \frac{w}{w_c} (1 + c_1^3) e^{(-c_2)}, \quad w_c = 5.14 \frac{G_F}{f_t^{ef}} \quad (8)$$

where w is the crack opening, w_c is the crack opening at the complete release of stress, σ_t is the normal stress in the crack and f_t^{ef} is the effective tensile strength. Values of the constants are $c_1=3$ and $c_2=6.93$. The crack opening displacement w is derived from strains based on the crack band theory.

For the ascending branch of concrete stress-strain law in compression, the formula recommended by CEB-FIP Model Code 90²²⁾, shown in Fig. 8b is adopted.

$$\sigma_c^{ef} = f_c^{ef} \frac{kx - x^2}{1 + (k-2)x}, \quad x = \frac{\epsilon_c}{\epsilon_{co}}, \quad k = \frac{E_0}{E_c} \quad (9)$$

where σ_c^{ef} is the concrete effective compressive stress, f_c^{ef} is the concrete effective compressive strength, ϵ_{co} is the strain at the peak stress f_c^{ef} , k is a shape parameter and the value of 2.0 is used in this analysis for parabola, E_0 is the initial elastic modulus, E_c is the secant elastic modulus at the peak stress.

Softening law in compression is linearly descending. The end point of the softening curve is defined by means of the plastic displacement w_d (see Fig. 8b). From the experiments of van Mier²³⁾, the value of $w_d=0.5$ mm is used in the present analysis.

FIG. 8. Stress-Strain Relationship in FEM Model for (a) Tension; (b) Compression

Reduction of the compressive strength after cracking in the direction parallel to cracks is considered in a similar way as found from experiments of Vecchio and Collins⁸⁾. The function was derived from the experimental data published by Kollegger et al.¹⁹⁾

$$f_c^{ef} = r_c f_c', \quad r_c = c + (1-c) e^{-(128\epsilon_t)^2} \quad (10)$$

where ϵ_t is the crack opening strain and f_c' is the concrete compressive strength. The constant c represents the maximum strength reduction under a large transverse strain. Here, the value of c is determined in the present analysis so as to fit the experimental results. $c=0.35$.

The FEM model employed here was chosen to describe the test configuration discussed previously. After some trials in the preliminary numerical studies, the geometric/loading configuration and mesh profile shown in Fig. 9a were adopted in the FEM model. The specimen is assumed to be in a state of plane stress. In the area of V-groove at the interface of joints, the minimum thickness was adopted in whole area of V-groove. Here, in the present FEM model, the construction joint could not be considered. While the vertical displacement of the bottom portion of the specimen is restrained, a prescribed vertical displacement is applied at the upper. The boundary conditions are modeled as shown in Fig. 9a. Finite plane stress elements consist of quadrilaterals composed of two four-node sub-triangles. The restraint bars are modeled as external cables in which only both ends are bonded with concrete elements.

3.3 Comparison with Experiments and FEM analysis

The compressive strength f_c and the tensile strength of concrete f_t are identical to the experimental results¹⁾. The quantity of the initial tangent stiffness E_0 is calculated by modifying the Young's modulus E_c empirically obtained with the following relation²⁴⁾

$$E_0 = \frac{E_c}{0.82} + 1.8776 \quad (\text{MPa}) \quad (11)$$

The fracture energy G_F was estimated as 0.1 N/mm, which is often used by many researchers and also suggested as the most effective^{25),26)}

First, the crack patterns predicted by FEM at the shear displacements of 0.04 mm (pre-peak), 0.11 mm (peak) and 0.32 mm (post-peak) for B-3 specimen without joint are examined as shown in Figs 9b, 9c and 9d. The crack patterns reveal that multiple cracks first initiate at the minimum thickness area and continue to propagate. Then, the localization occurred along the interface of joints, and large shear displacement occurs due to the crushing of cracked elements. Here, it should be noted that this predicted crack behavior could be similar to that assumed in the shear failure sequence in Fig. 3

FIG. 9. FEM Analysis (a) Configuration of FEM Model; Crack patterns at (b) pre-peak, (c) peak and (d) post-peak

Next, the load-displacement characteristics predicted by the mechanical model are examined. Fig. 10 shows the comparison of shear stress-displacement relationships for test results of A series and B series, FEM

analysis results and the predictions given by the mechanical model. In the figures, A-3 and B-3 are specimens without joint. The surface roughness of joint for A-1, A-2, B-1 and B-2 were 0.61mm(a little rough), 2.19mm(rough), 0.075mm(plain) and 0.45mm(a little rough), respectively. Definition of surface roughness is the average height of both the convex face and the concave face¹⁾. The coefficient m_c in the interlocking model (see Eq.(6)) is determined for different surface roughness so as to fit the experimental curves. The mechanical model agrees with entire shear stress-displacement curves, especially for specimens without joint and those with joint of higher surface roughness. Thus, it could be expected that the appropriate formulation on m_c gives good predictions regarding the post-peak regime without fitting the experimental curves beforehand. Here, it should be noted that the stiffer pre-peak predictions by the mechanical model for A series are due to the rough estimation of W_S which influences the quantity of shear displacement. This is because the normal compressive stress due to restraint bars was not measured in the experiment of A series. The quantity of W_S for A series is therefore assumed as the same as B series.

FIG. 10. Predicted and Experimental Shear Stress vs. Shear Displacement Curves for (a) A series; (b) B series

On the other hand, FEM analysis does not trace the post-peak behavior, especially at the interlocking stage (stage IV in Fig.1), although the FEM analysis results agrees with both the experimental measurements and the predictions by the mechanical model in the pre-peak region and the post-peak region near the peak (stage I to stage III in Fig.1). This may indicate that the continuum-based cracking model in FEM analysis could not properly account for the transition phenomena.

In addition to the cracking behavior and the entire load-displacement characteristics, the following aspects in predictions by the mechanical model are examined. Here, it should be noted that the predictions by the mechanical model in Fig. 11 and 12 are independent of the interlocking model. Fig. 11 shows the comparison of normal stress and shear crack width relations obtained in the experiment, FEM analysis results and the predictions by the mechanical model for B series. The mechanical model agrees with both test results and FEM analysis results up to the dropping point of normal stress which is associated with the start of the interlocking model. Specifically, the predicted normal compressive stress agrees with the test results of B-2 and B-3 as well as FEM analysis, and the predicted shear crack width is the intermediate value between the test results

FIG. 11. Predicted and Experimental (a) Normal Compressive Stress vs. Shear Displacement; (b) Crack Width vs. Shear Displacement

FIG. 12. (a) Prediction of Inclination of Rotated Crack; (b) Effect of Parameters in Mechanical Model for B series

Fig. 12a shows the comparison of the inclination of rotated crack calculated by both mechanical model and FEM analysis for B series. Here, one can see the agreement between both calculations. In addition, this figure indicates that the inclination of rotated crack is associated with the shear transfer capability as shown in Fig. 10. Fig. 12b shows the effect of parameters on shear stress (τ_{xy})-shear displacement (δ) relation for B series

calculated by the mechanical model. Here, the shear stress τ_{xy} is obtained without the step (k) in Fig. 7. It is clearly observed that the shear stress τ_{xy} is associated with the compressive stress σ_c in strut and the dropping point of the normal compressive stress σ_x is related to the post-peak compressive stress σ_c . To examine the discrepancy between the peak stress of σ_c and the dropping point of σ_x , the compressive stress σ_c without softening ($\lambda=1.0$ in Eq. (4)) is calculated and shown in the figure. Then, it is obvious that the discrepancy between the two peaks is due to the softening parameter λ .

Fig. 13 shows the comparison of shear stress-displacement relationships for predictions by the mechanical model with and without the interlocking model, the interlocking model by Eq. (6) and experimental measurement for B-3 specimen. In the mechanical model without the interlocking model, the step (k) in Fig. 7 is ignored. The interlocking model drawn by a heavy dot-dash line is identical to the difference between the predictions by the mechanical model with and without the interlocking model. This figure also explains the interaction between macroscopic shear strain softening and interlocking. The hatched zone is defined here as the macroscopic shear strain softening zone. In order to examine the consumed energy in the shear strain softening behavior, the area in the hatched zone is calculated and obtained as 0.933N/mm (up to 1.0mm of shear displacement). This quantity is close to the mode II fracture energy of 1.049N/mm for the concrete with $f'_c=38\text{MPa}$ reported by Bazant et al.²⁷⁾, and a range of 0.7 to 1.0N/mm for the concrete with $f'_c=41\text{MPa}$ which was calculated from the mode II latent fracture toughness reported by Davies²⁸⁾.

FIG. 13. Mechanism of Interaction between Macroscopic Shear Strain Softening and Interlocking for B-3 Specimen

4. CONCLUSIONS

In this paper, the mechanical model for direct shear failure of concrete was developed, focusing on the entire load-displacement characteristics. The proposed formulation identified the transition phenomena of two main fracture mechanisms: continuous propagation of multiple diagonal cracks and macroscopic vertical shear crack propagation along a fictitious surface roughness induced by both crushed compression struts and aggregates. The model employed both the combination of rotated smeared crack model and truss model theory for the development of multiple diagonal cracks under far field uniform shear stress, and the interlocking shear transfer model for the fictitious macroscopic shear crack propagation. Then, the two models

were combined to predict the entire shear stress-slip displacement behavior of direct shear failure. It should be here emphasized that the salient features in the proposed model are both the model of apparent Poisson's ratio and the combination of the continuum-based cracking model and the interlocking model.

The proposed mechanical model is shown to be sound. Good agreement between predicted and measured entire load-displacement history as well as shear strength was obtained. Specifically, the mechanical model could predict the post-peak behavior consisting of the transition and interlocking phenomena which could not be traced by FEM analysis. The direct shear failure of concrete is a very complicated process and the present mechanical model leaves many aspects of the problem unexplained, but the approach appears to have much promise and to be capable of further development. Other simulations with a modified constitutive law could have resulted in an even better agreement between test data and the predictions. However, the current results already show that the fracture behavior of concrete under direct shear was reliably captured by the mechanical model, and any additional parameter refinement would be just an exercise in curve fitting. Since the necessary parameters (e.g., constitutive laws) are kept in the model, analytical sensitivities can be generated, parametric studies can be carried out, and reasonable first estimates for the prediction of response can be made.

Finally, the macroscopic shear strain softening was defined and the interaction between the macroscopic shear strain softening and interlocking was hypothesized. Although the present study is conducted based on the numerical consideration, this hypothesis could be verified qualitatively. In order to verify this aspect quantitatively, further analytical and experimental works are necessary.

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和文要約

1. 序

コンクリートのせん断破壊メカニズムは、これまでも数多く議論されてきたが、まだ十分に説明されていない。本研究では、せん断破壊進行領域におけるせん断力伝達機構とせん断歪軟化挙動を評価するために行われた既往の実験結果¹⁾に基づいて、コンクリートの直接せん断載荷での全荷重-変位特性を与える力学モデルを提案し、解析的にこの複雑なメカニズムの物理的解釈を試みる。

2. 力学モデルの定式化

2.1 せん断破壊の進行過程

力学モデルを構築するにあたって、せん断破壊の進行過程をせん断変位の増加に伴い、図3に示すように以下の4つの段階に分類して設定する：(段階1) 斜めひび割れ群の発生、(段階2) 斜めひび割れ群の伝播と回転、(段階3) 斜めひび割れ群間の圧縮ストラットの圧壊とそれにより生じる巨視的せん断破面の形成、(段階4) 巨視的せん断破面上の圧壊コンクリートと骨材によるインターロッキング現象。

2.2 釣合条件と適合条件

斜めひび割れ群の破壊進行過程は、回転分布ひび割れモデルとトラスモデルを組み合わせ、図4に示す釣合条件に基づいて定式化する。ここでは平均化された応力と歪が一樣に分布すると仮定する。斜めひび割れの発生後、一連の斜めストラットがc方向に形成される。コンクリート要素はストラット方向に圧縮応力 σ_c のみ持ち、直交するt方向に引張応力 σ_t のみ持つ。そのためひび割れたコンクリート要素に添ったせん断応力 τ_c はゼロとなる。このように σ_c と σ_t は常にこのシステムの主応力となる。鉄筋コンクリート要素の平均応力はコンクリート要素の平均応力と鉄筋要素の応力に分解している。x方向の鉄筋比を ρ_x とすると鉄筋コンクリート要素のx-y座標とc-t座標における平均応力並びに平均歪の変換は式(1)、式(2)となる。

2.3 構成則モデル

ここでは、モードI型で主応力軸に沿って回転し、一樣に分布するひびわれ間のコンクリート要素に対し、ひびわれ開口変位をコンクリート要素の引張歪として評価し、次に示す構成モデルを適用する。引張構成則モデルは、軟化部分をバイリニアとしたHillerborg¹¹⁾のモデルに基づき、式(3)で定式化する。圧縮構成則モデルは、Soroushian等¹²⁾とHognestad¹³⁾の研究に基づき、VecchioとCollins⁸⁾のコンクリートストラットの圧縮軟化パラメータを導入し、式(4)で定式化する。鉄筋の応力-歪関係は、完全弾塑性モデルとする。なお、実験¹⁾では、鉄筋の付着を完全に切っているため、せん断破壊ゾーンの局所的水平変位を鉄筋の変位と仮定し、式(5)のように定式化する。圧壊コンクリートと骨材のインターロッキングによるせん断応力とせん断変位の関係は、ひび割れ幅一定下でのせん断実験で得られた李と前川¹⁴⁾のひび割れ面の応力伝達構成式に基づいて、式(6)のように定式化する。

2.4 疑似ポアソン比モデル

コンクリートストラットの圧縮歪と引張歪は、式(7)に示す疑似ポアソン比 $\nu_a (= \varepsilon_c / \varepsilon_t)$ により関係付ける。疑似ポアソン比モデルは、引張歪に支配される構造実験結果に基づいて構築する。図6は疑似ポアソン比と規準化した圧縮歪 $\varepsilon_c / \varepsilon_{co}$ の関係を示したものであり、実線は疑似ポアソン比モデルを示し、分散点は既往の実験的研究^{8) 15)}から得られた結果を示している。図の比較において多少のバラツキはあるが、疑似ポアソン比モデルは実験値の分布のほぼ中間的な値を示している。

2.5 解法過程

疑似ポアソン比と与えられた軸方向力 σ_x を使って、12変数($\sigma_x, \sigma_y, \tau_{xy}, \varepsilon_x, \varepsilon_y, \gamma_{xy}, \sigma_t, \sigma_c, \sigma_{xt}, \varepsilon_t, \varepsilon_c, \theta$)は10変数となる。そこで一つの変数を選択すると、残る9変数は釣合条件、適合条件、構成則モデルからなる9関係式より解が得られる。こうして、平均せん断応力 τ_{xy} と平均せん断変位 $\delta (= \gamma_{xy} W_d)$ が図7に示すようにして求まる。なお、インターロッキング現象は、せん断歪軟化開始点から考慮される。この開始点は、鉄筋による拘束圧縮力の減少開始点と一致している。

3. 力学モデルの検証

3.1 実験の概説

図2に示す既往の実験¹⁾は、一体打ちも含めた種々の打継ぎ面荒さをせん断面に施して行われたものであり、せん断面積と載荷速度を変えた2種類の試験体を採用している。なお、拘束鉄筋は完全に付着を切った状態で使用されている。

3.2 有限要素法解析

有限要素法解析は、Cervenka等により開発されたSBETA¹⁶⁾有限要素法プログラムにより、式(8)から式(10)の構成則と回転ひび割れモデルを用いて行う。解析モデルは、図9aに示すように四角形平面応力要素を用い上部より強制変位を与える。

3.3 実験結果及び有限要素法解析結果との比較

有限要素法解析で得られたプレピーク荷重時、ピーク荷重時、ポストピーク荷重時のひび割れ状況(図9b,c,d)は、力学モデルで構築した破壊過程(図3)をほぼ再現している。有限要素法解析で評価できなかったポストピーク荷重時の挙動も含めて、力学モデルは実験で得られた全せん断力-せん断変位関係をうまく評価している(図10)。有限要素法解析結果と同様に力学モデルは、実験で得られた鉄筋による拘束圧縮力-せん断変位関係、及びせん断ひび割れ幅-せん断変位関係をうまく評価している(図11)。力学モデルは有限要素法解析結果とほぼ同等な回転ひび割れモデルの勾配-せん断変位関係を与えている(図12a)。力学モデルで評価されたせん断応力特性は、圧縮構成則モデルに強く依存している(図12b)。力学モデルで評価されたせん断歪軟化現象とインターロッキング現象の相互関係は、図13のようになる。図のハッチ部分で消費されたエネルギーは既往の研究^{27) 28)}において報告されたモードIIの破壊エネルギーとほぼ同等な値を示している。

4. 結論

本研究では、コンクリートの直接せん断破壊に対して全荷重-変位特性を与える力学モデルを提案した。ここでは、回転分布ひび割れモデル、トラスモデル、及びインターロッキングモデルを組み合わせた単純なモデルを構築し、遠方で一樣なせん断応力をうけて伝播する斜めひび割れ群とこれらの局所化により発生する巨視的せん断ひび割れの伝播に対して定式化している。複雑な破壊挙動を単純化した本モデルは、実験結果、及び有限要素法解析結果を比較的うまく評価した。特に、有限要素法解析でとらえられなかったポストピーク荷重時の破壊状態の遷移現象を評価することができた。さらに、本モデルを使ってせん断歪軟化現象とインターロッキング現象の相互関係を定性的に示した。

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