ESTIMATION OF GROUND SNOW LOAD USING SNOW LAYER MODEL
積雪層モデルによる地上積雪荷重の推定

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For useful estimation of ground snow load, the authors have built the snow layer model using data of daily precipitation, daily mean and daily minimum air temperature. Although the catch ratio of solid precipitation is important for estimating the snow load on the ground, the calculation results using the catch ratio proposed by Ohno et al. was not sufficient for some observation points. Therefore the authors proposed the new equation for the catch ratio using daily mean temperature. The results with newly proposed catch ratio was sufficient for every observation points.

Keywords: Snow load, Snow layer, Precipitation, Daily mean air temperature, Catch ratio of precipitation gauge
積雪荷重, 積雪層, 降水, 日平均気温, 捕獲率の推定

1 INTRODUCTION
In Japan, there are few stations at which snow weight on the ground is measured directly. To obtain snow weight, we normally use snow depth data and multiply them by an equivalent snow density. However, there are many automatic observation stations that observe precipitation. Therefore, it is necessary to estimate snow weight from precipitation data. Kamimura et al. estimated daily snow mass on the ground using precipitation data for road snow removal. Sakurai et al. also estimated ground snow load. However, for estimating annual maximum snow load, it is important to define the melting process and it's the melting quantity. The authors have developed a snow layer model for estimating ground snow depth and snow weight. Both estimated data are compared with real observed data to verify the validity of the model.

2 DEVELOPMENT OF SNOW LAYER MODEL
2.1 Snow layer
It is well known that snow on the ground is made up of layers comprising one layer for each snowfall. In this study, it was assumed that each day's snowfall formed one layer. After a snowfall, the snow consolidates and metamorphoses in proportion to the number of days and the temperature. Finally, it melts and it is assumed to percolate into the ground. These processes are constituted by snow density and a snowmelt coefficient, as shown in Figures 1 and 2. They are defined as follows.

2.2 Snow Density
During a snowfall at low temperature, snow with low density falls. When the temperature is high, the density varies. The authors defined the density of snow during a snowfall as shown in Figure 1a, based on engineering judgement. This is described as follows:

\[ \rho(t) = \begin{cases} \rho_{\text{min}} & (T \leq 0^\circ \text{C}) \\ \rho_{\text{max}} - \rho_{\text{min}} \times \frac{T - 0}{3} & (0^\circ \text{C} < T \leq 3^\circ \text{C}) \\ \rho_{\text{max}} & (3^\circ \text{C} < T) \end{cases} \]

where, \( m \) = number of days from the start of continuous snow cover, \( \rho_{0} \) = density of a snowfall on the \( m \)th day from the start of continuous snow cover, \( \rho_{\text{min}} \) = minimum density of snowfall as calculated in the following section and it is fixed in a winter, \( \rho_{\text{max}} \) = maximum density of snowfall fixed as 500 kg/m³ considering metamorphosis of snow, and \( T \) = daily mean air temperature of the corresponding day. The snow density of the snowfall is fixed as 500 kg/m³ when \( T \) is warmer than 3°C, but the snowfall might melt as described in section 2.4.
After the snowfall, snow is consolidated day after day. An empirical value for the density and thickness of a layer has been expressed as follows:

\[ \rho_n = \rho_{\text{min}} \sqrt{n} \]  
\[ \Delta h_n = \frac{m}{\rho_{\text{min}} \sqrt{n}} \]  

where, \( n \) = number of days from the snowfall of the layer, and \( m \) = precipitation of the corresponding snowfall. The authors considered the initial condition of snowfall and improved this process as follows:

\[ \Delta h' = \frac{m}{\rho_{\text{min}} \sqrt{n+k}} \]
\[ \Delta h'' = \frac{m}{\rho_{\text{min}} \sqrt{n+k}} \]

where, \( k \) = the correction number for the initial condition. \( k \) is obtained when \( n \) is put in Equation 4 with 0, as follows:

\[ k = \left( \frac{m \rho_n}{\rho_{\text{min}}} \right)^2 \]

This means that snowfall and consolidation starts from \( k \) days in Figure 1b. The density of the layer does not change when the consolidation progresses to 500 kg/m\(^3\). An example of the transition process of snow layers in snow depth and elevation of snow density of this model are shown in Figure 3 and Figure 4, respectively.

### 2.3 Discrimination between rain and snow

When the daily minimum air temperature was warmer than 2°C, the precipitation of the day was judged as rain\(^a\), and all of the water was considered to have moved into lower layer, until the corresponding layer's density reaches to 500 kg/m\(^3\). When the calculated density of the layer exceed \( \rho_{\text{min}} \), the superfluous part moves into the next layer. Finally, the superfluous part that can not stay any layers percolates into the ground. This means that the snow layer model can consider rain load in the middle of winter season.

### 2.4 Snowmelt coefficient

A number of studies have been carried out to consider the mechanism of snow melt (for example, Yoshida\(^b\), Otoishi\(^b\)). In the present paper, the authors have assumed that the snowmelt quantity is proportional to the daily mean air temperature, as shown in Figure 2. Daily melting snow weight \( W_e \) is expressed as follows:

\[ W_e = C_e (T - T_0) \]  
\[ (T_0 < T) \]  

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\(^a\) where, \( m \) = precipitation of the day.
\(^b\) where, \( \rho_{\text{min}} \) = minimum density of the layer.
where, $C_w$ = snowmelt coefficient calculated in the analysis and assumed to be constant for one winter season, $T$ = daily mean air temperature of the calculating day, and $T_r$ = reference temperature for snowmelt. Snowmelt occurs when $T$ is warmer than $T_r$. In the present paper, $T_r$ is fixed at -2°C. Snow fundamentally melts when the temperature is higher than 0°C. However, the reference temperature was set to be the -2°C. This value was chosen because the temperature may sometimes exceed 0°C during the daytime, even though the daily mean air temperature is negative. The melted water of volume $W_c$ moves as the same as described in section 2.3.

2.5 Catch ratio of gauges

Precipitation is measured with precipitation gauges. In Japan, three types of gauges are conventionally used by the Japan Meteorological Agency. Ohno et al.\(^\text{11}\) reported that in the Hokuriku region, the catch ratio for solid precipitation for each type of gauge depends on the wind speed and the type of gauge, as shown in Figure 5.

They gave the regression formula of the catch ratio $CR$ as follows:

$$CR = \frac{1}{1 + \alpha V}$$

where, $\alpha$ = correction factor for each type of gauge, and $V$ = average wind speed in corresponding snowfall. For the normal type gauge RT-1, they gave $\alpha = 0.17$. For the warm water type gauge RT-3, they gave $\alpha = 0.24$. For the overflow type gauge RT-4, they gave $\alpha = 0.14$. The smaller the value of $\alpha$, the better the catch ratio. The authors decided to use these factors to verify the estimated snow weight and the snow layer model.

3 ESTIMATION OF SNOW DEPTH AND SNOW WEIGHT

3.1 Data for verification

Few direct observations of snow weight have been made in Japan. In the present paper, snow weight calculations are performed on the basis of snow pit observations observed from the viewpoint of snow mechanics. The extraction method for the observation data and snow weight used in the present paper are as follows.

Yoshida et al.\(^\text{12}\) collected continuous pit observation data in seven towns in Japan for 5 winters from 1964 to 1968. Although water equivalent information was not collected in the observation report 12), the author calculated the corresponding snow weight using the observed snow depth and snow density. For densities outside the observed layers, the values of the uppermost and lowermost layers are extrapolated. For intermediate measurements when an observation has been duplicated, the average measurement was calculated and used in the analysis. The data used in
the present paper were collected from 1964 to 1968 for Sapporo, from 1964 to 1968 for Takada, from 1964 to 1968 for Toyama, and in 1967 & 1968 for Fukui. An example of snow pit observation is shown in Figure 4.

Snow pit observations have been continually performed by the Science and Technology Agency, The National Research Institute for Earth Science and Disaster Prevention (NIED). Data from 1976 to 1980 in the Shinjo Branch of Snow and Ice Studies were used in the present paper. The Shinjo Branch of Snow and Ice Studies used a snow sampler to perform the observation, so the water equivalent was calculated directly and used in the present paper.

The daily observed snow depth, daily mean and daily minimum air temperature of corresponding observatories belonging to the Japan Meteorological Agency are used for verification. These meteorological data are offered as SDP (Surface Daily Processing) data by the Japan Meteorological Agency. Each pit observation point is located in the same city as SDP data observed. The winter seasons used in the present paper include ordinary winter and warm winter as shown in Figure 6. The extraordinary heavy snow seasons are not included.

3.2 Calculation method

The calculation is done for one day step using observed daily precipitation, and daily mean and daily minimum air temperature. As mentioned in section 2, only \( \rho_m \) and \( C_w \) were unknown factors in the analysis. These factors which made the following estimation error \( \epsilon \), a minimum were searched using the random search method to lead to the optimum solution for the transition process of snow depths.

\[
\epsilon = \sqrt{\sum (d_e - d_o)^2}
\]  

where, \( d_e \) = estimated daily snow depth, and \( d_o \) = observed daily snow depth.

The proposed snow layer model is used to calculate the snow weight transition process, using estimated coefficients \( \rho_m \) and \( C_w \). The snow weight is fundamentally accumulation of cumulative precipitation. When the catch ratio is less than 1.0, the precipitation value is divided by the catch ratio for data correction. When the daily mean air temperature is higher than -2°C, the weight is deducted by \( W_c \) as expressed by Equation 7. Snowmelt is assumed to occur from the top layer down, and the overflowing water is absorbed into the next layer and percolates underground from the bottom layer when it can not be absorbed.

3.3 Results with Ohno's catch ratio

Figure 6 compares observed and estimated transition processes of snow depth. Regardless of Ohno's catch ratio, estimated snow depth shows a good approximation to actual values.

Figure 7 shows the snow weight transition process corresponding to winter seasons shown in Figure 6. Although the snow depth estimations show similar results, the snow weight processes show different results with regard to coefficients \( \rho_m \) and \( C_w \). Generally, estimated snow weights increase as correction factor \( \alpha \) increases. However, whether the correction factor gives the most appropriate estimated result of the snow weight varies from site by site.

In Sapporo, \( \alpha = 0.24 \) seemed suitable. However, in Takada, Toyama and Fukui, \( \alpha = 0 \) seemed better, i.e., there was no need to use a correction coefficient. This result was not compatible with Ohno's proposal. Therefore, the authors decided to find another factor that would give an efficient catch ratio.

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Figure 7. Example of measured and estimated snow weight using Ohno's catch ratio.
4 CONSIDERATION OF CATCH RATIO

4.1 Estimated catch ratio

In considering catch ratio, the authors defined the following value as estimated catch ratio.

\[ CR = \frac{S_p}{S_p + (S_p - S_e)} \]  \hspace{1cm} (10)

where, \( S_p \) = snow weight directly calculated from cumulative precipitation data during observation interval, \( S_p \) = observed difference in snow weight during the observation interval, and \( S_e \) = estimated difference in snow weight during the observation interval. Therefore, Equation 10 indicates a catch ratio considering snowmelt during the corresponding observation interval.

4.2 Effect of wind speed and temperature

Figure 8a shows the relationship between the estimated catch ratio defined in Equation 10 and the average daily mean wind speed during the corresponding observation interval. It is hard to see a clear relationship. Its correlation coefficient \( R = 0.05 \).

Figure 8b shows the relationship between the estimated catch ratio and the average daily mean air temperature during the corresponding observation interval. It seems to indicate that the lower the temperature, the smaller the catch ratio. Its correlation coefficient \( R = 0.29 \).

Therefore, the following approximation can be expressed for the catch ratio using the daily mean air temperature. The equation is assumed to yield 1.0 when the mean air temperature is higher than \( 0^\circ \text{C} \).

\[ CR = \exp (0.077 T) \] \hspace{1cm} (11.a)

\[ CR = 1.0 \] \hspace{1cm} \( (T \leq 0^\circ \text{C}) \) \hspace{1cm} (11.b)

The square correlation coefficient \( \eta^2 \), that is defined by Equation 12\textsuperscript{4}.
for Equation 11a is 0.19, as shown in Figure 8b.

\[ \eta^2 = 1 - \frac{\sum_{i=1}^{N} (y_i - \bar{y}_i)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2} \]  

(12)

where, \( y_i \) = observed value, \( \bar{y}_i \) = estimated value, \( \bar{y} \) = the mean of observed values.

4.3 Results for proposed catch ratio

Figure 9 compares calculation results with proposed catch ratio expressed by Equation 11. The results appear to be accurate for all observation points comparing with Figure 7. Therefore, the proposed catch ratio, as expressed by Equation 11, can be used for correction of catch ratio on precipitation gauges.

5 CONCLUSIONS

For quick estimation of ground snow load, the authors built a snow layer model using data of daily precipitation, daily mean and daily minimum air temperature. By using this model, we can get the estimated transition process of both ground snow depth and ground snow load.

Although the catch ratio of solid precipitation is important for estimating the snow load on the ground, the calculation results using the catch ratio proposed by Ohno et al. were insufficient for some observation points. Therefore, the authors have proposed a new equation for the catch ratio using daily mean temperature. The results with this newly proposed catch ratio was accurate for all observation points.

For the next step of the estimation, regional characteristics of coefficients \( p_{\text{max}} \) and \( C_a \) should be considered, and this will be useful for observation points without snow depth measurement.

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和文要約
1. はじめに
従来、積雪量を直接観測した資料が限られていることから、設計用積雪荷重の基となる地上積雪量の推定には地上積雪深の値に等値単位積雪重量を掛ける手法が用用れてきた。一方、気象庁のAMeDAS観測網は全国に1,300箇所ほど存在し、1時間毎の降雪量を自動的に記録している。積雪重量は、融雪分を除けば根本的には降雪量であるから、このデータを地上積雪量の推定に用いることが出来れば、より合理的な地上積雪量の推定にとって大いに役立つことが期待できる。これまでは、上村ら⑩や桜井らによる降雪量と積雪深を用いて地上積雪量を推定しているが、年最大地上積雪量の評価に重要となる積雪量の評価法に問題点を残しており、研究の余地があると考えた。本報では、積雪層モデルに基づく地上積雪量推定法を構築し、併せて降水量の補正について検討を行った結果を報告する。

2. 積雪層モデルの構築
2.1 積雪層
周囲の雪、地上積雪は一降雪毎に層を形成しており、層毎に密度や変化などの物理特性が異なる。そこで、本研究では1日の降雪が1つの層を形成する仮定し、以下のよう密度と積雪推定法を仮定推定を行った。
2.2 積雪の密度
降雪時の密度は冰点下ではほぼ一定だが、気温が0℃を越えると非常にばらつくことが知られている。ここで、筆者らは物理的成因により密度を決める気温を1式、すなわちFigure 1aに示すように仮定した。また、層毎の密度の日変化を既に研究⑪を基に(4)式、すなわちFigure 1bのように表され仮定した。このとき降雪からn日目の積雪層の厚さは(5)式のように表される。
ここで用いられるのは降雪の日密度の初期値を規定するための変数であり、(4)のαの値を0と仮定(6)式のようになることがわかる。この値は、降雪流体の平均温度が0℃よりも高い場合に、見かけ上Figure 1bのρと日に密度変化が開始されることを意味している。
2.3 雨雪判別
日最低気温が2℃以上の日の降雪を雨と判断され、全積雪層が表面下の層にしまい込むものと仮定している。このようにすることにより、雪による積雪層の増加として積雪量の変化を評価可能である。計算した層の密度が500kg/m³を超える場合は、さらにその層がしまい込むものとした。
2.4 融雪係数
日平均気温がTₐよりも高い場合に、(7)式の値に従って、最も上の層から順に融雪層がWₐになるまで融雪が起こるものと仮定した。融雪した水は以下の層に流出し、最終的には土に吸収されると仮定している。本研究では、日平均気温が-2℃を越えると、日中の数時間は外気温が0℃を越えることもあることを考慮して、Tₐ = -2℃と仮定した。
2.5 降水量と補正率
現在気象庁では主に3種類の降水量計が用いられているが、大野ら⑫はこれらの種類ごとに、降雪深の風速と補正率の関係を観測し、(8)式のような関係式を提案している。補正係数αは降水計の種類によって異なる値が提案されており、本研究では提案された3種類の係数と補正なしの場合について検討を行ったこととした。
3. 積雪深と積雪重量の推定
3.1 検証に用いたデータ
本研究で推定値の検証に用いたのは、吉田らによる1964年から1968年までの積雪断面観測データ⑬と、科学技術庁新庄雪水防災研究支所における1976年から1980年までの積雪断面観測データ⑭である。
3.2 計算方法
計算は1日単位で行われ、(1)式に示される一年間の降雪の最小密度ρₚと、(7)式に示される密度係数C₀の2つをパラメータとし、(9)式に示される積雪深の変化過程の推定値と実測値の誤差が最小となるように、推定した上で、地上積雪重量変化過程を推定した。
3.3 大野らの観測に基づく計算
Figure 6にはこの補正率に基づいて補正された降水量を用い、積雪層モデルを用いて推定された積雪深の変化過程を示す。積雪層モデルに基づく積雪深の推定は補正率に関わらず推定値と良く一致している。Figure 7には、補正率に基づく積雪深の推定値と、実測値の比較を示す。これによると、札幌ではRT-3用の補正係数を用いた方がより良い推定結果を与え、逆に米沢、富山、福井では補正しない方が実測値の整合性が良い結果となっている。紙面の都合などで他の年度については載せられないので、傾向は同様であるが、地方については補正しない方が良い傾向が観察され、降水量の積雪の種類よりも地点ごとの傾向の傾向が顕著であった。
4. 本論文に関しての考察
4.1 推定補正率
降水量計の補正率は気象因子の相関を調べる目的で、(10)式によって補正率を逆算して検討した。

4.2 検証率補正率の提案
Figure 8bの結果に、日平均気温Tₐ = -2℃で補正率が1となるように指数関数を当てはめて図示を求めるとき(11)式のようになった。(12)式で定義されるこの回帰式の二乗相関係数は0.19であった。この補正率に関する回帰式を用いて補正した降水量を用いて積雪量を推定した結果をFigure 9に示す。
Figure 8bには積雪深の実測値と推定値の対比も併せて示した。Figure 9の結果をFigure 7と比較すると、Figure 9の推定値はいずれもFigure 7の推定値において最も良好な近似結果に近い結果となっている。

5. まとめ
日本国内の約1,300箇所あたりで観測されているAMeDASの降水量データを地上積雪重量の推定に用いることを目的として、積雪層モデルの構築を試み、過去の地上積雪重量観測データと比較することを通じて、その検証を試みた。その結果、地盤によって推定に過不足が生じるが、その傾向は既往の研究による降水量の積雪層毎の補正率では説明がつかないこともわかった。そこで、積雪層モデルによる推定結果と実測積雪重量を対比させて新たな推定補正率を計算して検討した結果、観測期間及び平均面積からの推定に有効な関係を見出すことはできず、平均気温と補正係数の関係を用いて補正することで良好な積雪重量の推定結果を得ることができた。

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