SPECTRUM-BASED PREDICTION RULE FOR
PEAK STRUCTURAL RESPONSES DUE TO SEISMIC POUNDING

Part 1  SDOF systems pounding against rigid structures

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1 INTRODUCTION
1.1 Structural Pounding Problem

Pounding between adjacent buildings due to earthquake has been reported as one of the causes of severe structural damages. This seismic hazard occurs mainly in highly constructed area where the separation distance is insufficient to accommodate the peak relative displacement between buildings.

Structures used in studies on structural pounding are often modeled as either single- or multi-degree-of-freedom (SDOF or MDOF) systems. A large number of parameters involve in the MDOF modeling of pounding problem makes it too complicated that only a few studies have been reported and most of them in the form of case studies. On the other side, many other authors opt for SDOF modeling to simplify the problem. Their purpose is to gain insight into the overall pounding behavior, to develop simplified prediction methods for peak responses, or to form bases for extension to MDOF modeling.

Collision between the buildings is typically simulated by the use of contact element, which becomes active only when contact is detected. Several types of contact element such as linear or nonlinear spring element, and linear or nonlinear viscoelastic element have been used for pounding simulation. These types of contact element cannot simulate the complex phenomenon at contact points, which involve plastic deformations, friction, etc. They also cannot show a complete figure of how energy transfers during impact and how stress waves travel away from the region of contact. However, most of the studies on structural pounding still use these simple types of contact element, such as for experiment verification or for simulating the collision in which the overall response of structures is of main interest.

Time-history analysis has been used mainly to investigate pounding. However, using numerical methods for this strongly nonlinear dynamic problem poses many difficulties, whereas maintaining the numerical accuracy requires time-consuming efforts. Moreover, numerical method may not be an efficient choice because many repetitive simulations with different ground motions are often required during the design process.

1.2 Spectrum-Based Response Evaluation

Other methods using response spectra are more suitable for practical use, since it can give peak responses fast. Unlike time-history analysis, the use of spectrum can clarify the relationship among structural properties, earthquake characteristics, and peak responses, thereby it helps to understand the pounding characteristics. Furthermore, since current seismic design practice is using response spectrum to obtain peak seismic responses, it is highly desirable to develop similar spectrum-based methods for the structural pounding problem as well.

The spectrum-based approach for pounding problem, however, is only at the initial stage of development: Refs. 9, 14-15 linearize a SDOF system pounding against rigid structure(s), equivalent stiffness of the system is derived from its hysteretic. Their approaches lack simplicity and sufficient accuracy check; moreover, they consider only a linear contact spring. Ref. 16 studied also a case of pounding against an adjacent flexible SDOF system, with a linear viscoelastic contact
element of relatively small stiffness and damping is considered. Hence, it addresses weak pounding, and approximates the response history by a smooth function that otherwise could vary abruptly. Equivalent stiffness and damping are obtained from the relation between phase angle at time of contact and external harmonic excitation frequency. However, the approach does not include random input like earthquakes.

1.3 Objectives and Scopes

The objective of our research, therefore, is to propose simplified methods for estimating peak responses of buildings subjected to seismic pounding. In this paper, we study the pounding problem, where a SDOF system pounds against adjacent rigid structure(s) located on its side(s).

A viscoelastic contact element is used, because of its capability in simulating the dissipation of energy during impact, and a wide range of its stiffness and damping considered. Formulas for equivalent period and damping of the SDOF system under pounding are derived, and they are combined with elastic response spectra to obtain peak responses. The method proposed is applicable to both the harmonic and earthquake ground motions. Moreover, the method can be easily modified when contact elements other than viscoelastic contact element are used in the simulation. The method is validated over numerous combinations of the structural properties, contact element properties, and earthquakes.

2 MODELING OF POUNDING PROBLEM

2.1 Issues Related to Contact Modeling

The impact problem may be accompanied by the complex phenomena at contact points, which involve plastic deformations, friction, damages, etc. However, as reported from past incidents of actual structural pounding,\(^{16,17}\) plastic deformation and damages at contact region are often insignificant compared with other parts of the structures, whose deformation largely increase due to effect of pounding. Therefore, it is strongly felt that contact region can be simulated by relatively simple analytical element, which becomes active when contact is detected.

So far, only limited amount of experiments are conducted for verifying analytical models. Linear spring contact element is verified with the results of experiment on pounding of steel girders of elevated bridges.\(^{20}\) In another study, viscoelastic contact element is shown to be quite good in simulating experimental impact force, after being verified with some experiment results and compared with several other types of more sophisticated contact elements.\(^{18}\) Linear spring contact element is also used to model pounding between two elastic rods traveling in the same and opposite direction, and is found to reasonably represent the propagation of stress wave.\(^{20}\) From these limited results, some agreements have been obtained regarding the reasonableness of using linear spring or viscoelastic contact element to simulate collision, especially when damage at contact region is not an issue.

In analytical parametric studies, therefore, linear viscoelastic contact element is typically used to approximate behavior of contact region. Some studies use this type of element to examine the basic characteristics of the pounding phenomenon,\(^{18}\) or to investigate pounding of buildings in series during earthquakes.\(^{7}\) The use of linear viscoelastic contact element has become more convenient after a formula to calculate its dashpot property from the well known coefficient of restitution \(e\) was presented.\(^{19}\)

In this paper, based on these experiments and analytical studies, we use linear viscoelastic contact element, hereafter we call viscoelastic contact element. The contact element's stiffness and damping are varied so that they can cover wide possible range of contact region's characteristics as well as coefficient of restitution \(e\). However, it should be noted that the simplified method proposed later in this paper is not necessarily restricted to this type of element. Other types of contact elements can be conveniently applied as well, provided that we can obtain their equivalent stiffness and damping (see Sec. 3.2, 4.1).

2.2 Definitions in Modeling of Pounding Problem

A structure is modeled as a SDOF system with mass \(m\), stiffness \(k\), and viscous damping coefficient \(c\). It pounds against adjacent rigid wall(s) when magnitude of its displacement \(|u(t)|\) exceeds either the separation distance \(s^+ (>0)\) on the right side, or \(s^- (>0)\) on the left side.

As mentioned in previous section, a viscoelastic contact element is used to simulate collision. It has linear spring and dashpot in parallel (Fig. 1a). The elements on the right and left sides have stiffnesses \(k^+\) and \(k^−\), and damping coefficients \(c^+\) and \(c^−\), respectively.

![Figure 1. Pounding of SDOF System Against Rigid Structures.](image)

The system is considered to be in state 1 when not in contact and state 2 when in contact with the wall. Elastic force \(F_{e}\) and damping force \(F_{d}\) are governed by displacement \(u\) (Fig. 1b) and velocity \(\dot{u}\) as follows:

\[
\begin{align*}
F_{e} &= ku, \quad F_{d} = cu & \text{(State 1: } -s^\leq u \leq s^+) \quad \text{(1a,b)} \\
F_{e} &= ku + k^+(u-s^+), \quad F_{d} = (c + c^+)\dot{u} & \text{(State 2: } u > s^+) \quad \text{(1c,d)} \\
F_{e} &= ku + k^-(u+s^-), \quad F_{d} = (c + c^-)\dot{u} & \text{(State 2: } u < -s^-) \quad \text{(1e,f)}
\end{align*}
\]

Clearly, stiffness and damping are added by the contact element when the state changes from 1 to 2, and we hereby define the stiffness increase ratio \(\kappa^+\) and \(\kappa^-\), as well as damping increase ratio \(\zeta^+\) and \(\zeta^-\).

\[
\kappa^+ = 1 + k^+/k, \quad \kappa^- = 1 + k^-/k \quad \text{(2a,b)} \\
\zeta^+ = 1 + c^+/c, \quad \zeta^- = 1 + c^-/c \quad \text{(3a,b)}
\]

Equation of motion of the system due to ground acceleration \(\ddot{u}_g(t)\) is:

\[
\ddot{u}_m + F_{e}(u(t)) + F_{d}(u(t)) = -m\ddot{u}_g(t) \quad \text{(4)}
\]

The vibration frequency at state 1 is called as no-pounding vibration frequency \(\omega_{np}\), and those at state 2 as pounding vibration frequencies \(\omega_{pd}\) and \(\omega_{pd}'\). Hereafter, subscript 'np' and 'pd' stand for properties of no pounding system and pounding system, respectively.

The vibration frequencies and damping ratios are related as follows:

\[
\begin{align*}
\omega_{np}^2 &= \frac{k + k^+}{m} = \sqrt{k^+ \omega_{np}}, \quad \xi_{np}^+ = \frac{c + c^+}{2m\omega_{np}} = \frac{\zeta^+}{\sqrt{k^+}} \quad \text{(5a,b)} \\
\omega_{pd}^2 &= \frac{k + k^-}{m} = \sqrt{k^- \omega_{pd}}, \quad \xi_{pd}^- = \frac{c + c^-}{2m\omega_{pd}} = \frac{\zeta^-}{\sqrt{k^-}} \quad \text{(6a,b)}
\end{align*}
\]

In above expressions

\[
\omega_{np} = \sqrt{k/m}, \quad \xi_{np} = c/(2m\omega_{np}) \quad \text{(7a,b)}
\]

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3 FREE VIBRATION WITH POUNDING

From here, "two-side pounding" refers to the case of pounding on both sides of the system. Similarly, "one-side pounding at right" or "one-side pounding at left" refers to pounding on right side or left side only, respectively. In later sections, two-side pounding is discussed first, then one-side pounding will be mentioned briefly since it is a simplified case of two-side pounding. This chapter addresses undamped free vibration of SDOF system subjected to pounding. 29

3.1 Time History of Free Vibration Response

(1) Two-Side Pounding: we consider displacement \( u = 0 \) and velocity \( \dot{u} = \dot{u}_{\text{max}} > 0 \) at time \( t = 0 \) (Fig. 2). Magnitude of \( \dot{u}_{\text{max}} \) is given, and it satisfies \( \dot{u}_{\text{max}} > \omega_{\text{op}} s' \) and \( \omega_{\text{op}} s' \) such that displacement will exceed the separation distances \( s' \) and \( s'' \).

a) \( 0 \leq t \leq t_1 \): solution of Eq. (1)

\[
\ddot{u}(t) + k\dot{u}(t) + ku(t) = 0
\]

Displacement \( u(t) \) at state 1, therefore, is expressed as

\[
u(t) = \frac{u_{\text{max}}}{\omega_{\text{op}}} \sin \omega_{\text{op}}t
\]

Substituting \( u(t) = s' \) into Eq. 9, time \( t_1 \) (Fig. 2) is obtained

\[
t_1 = \frac{\sin^{-1} \left( \frac{\omega_{\text{op}} s'}{u_{\text{max}}} \right)}{\omega_{\text{op}}}
\]

Velocity \( \dot{u}(t) \) can be obtained easily

\[
\dot{u}(t) = \frac{u_{\text{max}}}{\omega_{\text{op}}} \cos \omega_{\text{op}}t
\]

b) \( t_1 < t < 2t_1 - t_1 \): solution of Eq. (2), equation of motion becomes as follows

\[
\ddot{u}(t) + k\dot{u}(t) + ku(t) = 0
\]

Positive state 2 displacement \( u(t) \) is obtained by using initial conditions \( u(t) = s'' \) as well as \( \dot{u}(t) \) given by Eq. 11.

\[
u(t) = s'' + \frac{s'}{k} \left[ 1 + \cos \omega_{\text{op}}(t-t_1) + \frac{\dot{u}(t_1)}{\omega_{\text{op}}} \sin \omega_{\text{op}}(t-t_1) \right]
\]

Positive peak displacement occurs at time \( t_2 \) (Fig. 2) which is obtained by solving \( du(t)/dt = 0 \) of Eq. 13. Thus,

\[
t_2 = t_1 + \frac{1}{\omega_{\text{op}}} \tan^{-1} \sqrt{k^2 \left( \left( \frac{u_{\text{max}}}{\omega_{\text{op}} s''} \right)^2 - 1 \right)}
\]

c) \( 2t_1 - t_1 \leq t \leq t_1 \) (state 1): solution can be obtained by using the initial condition at time \( 2t_1 - t_1 \) (Fig. 2). For simplicity, however, an alternative expression is obtained by using zero displacement and peak velocity \( -u_{\text{max}} \) at time \( 2t_1 \). Thus, 1 displacement \( u(t) \) is:

\[
u(t) = -\frac{u_{\text{max}}}{\omega_{\text{op}}} \sin \omega_{\text{op}}(t-2t_2)
\]

Substituting \( u(t) = -s'' \) into Eq. 15, time \( t_3 \) is obtained

\[
t_3 = \frac{\sin^{-1} \left( \frac{\omega_{\text{op}} s'}{u_{\text{max}}} \right)}{\omega_{\text{op}}} + 2t_2
\]

and the velocity at time \( t_3 \) \( \dot{u}(t_3) \) is

\[
\dot{u}(t_3) = -\frac{s'}{k} - \left( 1 + \cos \omega_{\text{op}}(t-t_1) \right) \frac{\dot{u}(t_1)}{\omega_{\text{op}}} \sin \omega_{\text{op}}(t-t_1)
\]

d) \( t_3 < t < 2t_4 - t_1 \) (state 2): similarly to Eq. 13, negative state 2 displacement \( u(t) \) is obtained as

\[
u(t) = -s' - \frac{s''}{k} \left[ 1 + \cos \omega_{\text{op}}(t-t_1) \right] \frac{\dot{u}(t_1)}{\omega_{\text{op}}} \sin \omega_{\text{op}}(t-t_1)
\]

Negative peak displacement occurs at time \( t_4 \) (Fig. 2) that is obtained like Eq. 14. Then,

\[
t_4 = t_1 + \frac{1}{\omega_{\text{op}}} \tan^{-1} \sqrt{k^2 \left( \left( \frac{u_{\text{max}}}{\omega_{\text{op}} s'} \right)^2 - 1 \right)}
\]

3.2 Peak Displacements and Equivalent Vibration Period

(1) Two-Side Pounding: we defined \( u' \) and \( u'' \) as values of positive and absolute negative peak displacements (Fig. 2). They can be expressed in terms of \( u_{\text{max}}' \), such as for \( u' \) by substituting \( t_1 \) (Eq. 14) into Eq. 13, and for \( u'' \) by substituting \( t_4 \) (Eq. 19) into Eq. 18. Accordingly,

\[
u' = s' + \frac{1}{k} \left( 1 \right)
\]

\[
u'' = s' - \frac{1}{k} \left( 1 \right)
\]

Rewriting Eq. 20 to express \( u_{\text{max}}' \) by \( u' \) or \( u'' \) as follows

\[
u_{\text{max}}' = \omega_{\text{op}} u' \sqrt{1 + \left( 1 - s'/u' \right)^2 \left( k^2 - 1 \right)}
\]

From Eqs. 14 and 19, durations of responses per cycle showing positive, \( \Delta' = 2t_2 \), and negative, \( \Delta'' = 2(t_4 - t_2) \), are obtained. They are expressed by \( u_{\text{max}}' \) as follows

\[
\Delta' = \frac{T_{2\text{op}}}{\pi} \left\{ \sin^{-1} \left( \frac{\omega_{\text{op}} s'}{u_{\text{max}}'} \right) + \frac{1}{\sqrt{k^2 - 1}} \tan^{-1} \sqrt{k^2 \left( \left( \frac{u_{\text{max}}'}{\omega_{\text{op}} s'} \right)^2 - 1 \right)} \right\}
\]

\[
\Delta'' = \frac{T_{2\text{op}}}{\pi} \left\{ \sin^{-1} \left( \frac{\omega_{\text{op}} s'}{u_{\text{max}}'} \right) + \frac{1}{\sqrt{k^2 - 1}} \tan^{-1} \sqrt{k^2 \left( \left( \frac{u_{\text{max}}'}{\omega_{\text{op}} s'} \right)^2 - 1 \right)} \right\}
\]

Thus, equivalent vibration period \( T_{eq} \) can be given as follows

\[
T_{eq} = \Delta' + \Delta''
\] (two-side pounding) 23

where \( T_{eq} \) is no-pounding period, \( T_{op} = 2\pi/\omega_{\text{op}} \). Eqs. 22 and 23 indicate that \( T_{eq} \) depends on the initial velocity \( u_{\text{max}}' \) as well as \( s' \) and \( s'' \).
(2) One-Side Pounding: peak displacements $u'$ and $u''$ are obtained based on above equations. For pounding at right, $u'$ is given by Eq. 20a, but, $u'' = \dot{u}_{\text{max}} / \omega_{\text{np}}$ by setting $\kappa = 1$ in Eq. 20b since pounding does not occur at left. Similarly for pounding at left, $u' = \dot{u}_{\text{max}} / \omega_{\text{np}}$ by setting $\kappa = -1$ in Eq. 20a, and $u''$ is given by Eq. 20b.

Durations $\Delta t'$ and $\Delta t''$ are obtained in an analogous manner. For pounding at right, $\Delta t'$ is obtained from Eq. 22a, whereas $\Delta t'' = 0.5T_{\text{np}}$ is obtained by setting $s' = u'$ and $\dot{u}_{\text{max}} = \omega_{\text{np}}u'$ (Eq. 21b) in Eq. 22b. Similarly for pounding at left, $\Delta t'' = 0.5T_{\text{np}}$ instead of Eq. 22a, and $\Delta t'$ is still given by Eq. 22b. Therefore,

$$\begin{align*}
T_{\text{eq}} &= \Delta t' + 0.5T_{\text{np}} & \text{(one-side pounding at right)} \\
T_{\text{eq}} &= \Delta t'' + 0.5T_{\text{np}} & \text{(one-side pounding at left)}
\end{align*}$$

(24)

4 HARMONIC VIBRATION WITH POUNDING

4.1 Equivalent Damping Ratio

Vibration period and damping ratio of a nonlinear system can be estimated by using harmonic excitation, and they depend on the excitation’s magnitude and frequency. In some cases, those at the resonant state of the system are the most important factors for predicting earthquake responses, they are called as equivalent period and damping ratio.\(^{20,21}\) This paper applies such findings to pounding systems.

As typically considered, resonant period is set equal to the free vibration period. Thus, the period derived in Chap. 3 is considered as the “equivalent period” $T_{\text{eq}}$ (Eqs. 23, 24). On the other hand, the “equivalent damping ratio” $\xi_{\text{eq}}$ will be derived by considering resonant vibration, where equation of motion is given as

$$m\ddot{u} + F_u(u(t)) + F_s(u(t)) = -m\ddot{u}_{\text{eq}} \sin(2\pi T/T_{\text{eq}})$$

Based on previous findings but for the entirely different nonlinear systems, we think that the resonant behavior can be characterized by the details of undamped free vibration. Thus, we performed extensive numerical tests, and found that the formulas involving $\dot{u}_{\text{max}}, u', u', \Delta t', \Delta t''$, and $T_{\text{eq}}$ (Chap. 3) can characterize the resonant responses with reasonable accuracy. Considering such undamped modal relation, equivalent damping $\xi_{\text{eq}}$ is obtained as follows

(1) Two-Side Pounding: $u(t)$ is approximated to simplify calculation of hysteretic energy. Two distinct half-sinusoidal curves with vibration frequencies ($\pi / \Delta t'$), ($\pi / \Delta t''$) are assumed for each side (Fig. 3), respectively. Thus, energy $E_u$ dissipated for $u(t) > 0$ and energy $E_s$ for $u(t) < 0$ (Fig. 3) are calculated respectively as (see App. B)

$$E_u = \frac{c}{2\Delta t''} \left[ 1 + \frac{2c}{\kappa m} \left( 1 - s' / u' \right) \left( 1 - \frac{s^2}{u^2} \right) \right]$$
$$E_s = \frac{c}{2\Delta t'} \left[ 1 + \frac{2c}{\kappa m} \left( 1 - s' / u' \right) \left( 1 - \frac{s^2}{u^2} \right) \right]$$

(26a, 26b)

The strain energy $E_s$ on each side can be calculated using the stiffnesses and peak displacements (Fig. 1b)

$$E_s = \frac{1}{2} k(1 + (s'' - 1)(1 - s' / u')^2)[u']^2$$
$$E_s = \frac{1}{2} k(1 + (s'' - 1)(1 - s' / u')^2)[u''']^2$$

(27a, b)

The above strain energies on both sides are assumed to be equal, which was verified by some numerical experiments.

Based on these, equivalent damping ratio at resonance is given by

$$\xi_{\text{eq}} = \Delta \xi' + \Delta \xi''$$

(28)

$$\Delta \xi' = \frac{E_u}{4\pi E_s}, \quad \Delta \xi'' = \frac{E_s}{4\pi E_s}$$

(29a, b)

(2) One-Side Pounding: for one-side pounding at right, set $s'' = u''$ in Eq. 26b and use $\Delta t'' = 0.5T_{\text{np}}$ (Eqs. 24). For one-side pounding at left, set $s' = u'$ in Eq. 26a and use $\Delta t' = 0.5T_{\text{np}}$. Thus, Eq. 28 can be written as

$$\xi_{\text{eq}} = \Delta \xi' + 0.5\xi_{\text{np}}$$

(30)

Fig. 3. Steady State Response of Two-Side Pounding: (a) Time History, (b) $F_D - u$ Relationship.

4.2 Numerical Experiments of Harmonic Vibration

Numerical experiments are conducted using harmonic ground acceleration $\ddot{u}_{\text{eq}} \sin(2\pi T/T)$, where $\ddot{u}_{\text{eq}} = 0.6g = 588$ cm/s. Period $T$ is varied over a wide range in contrast to Sec. 4.1 that considered resonant vibration period $T_{\text{eq}}$ (Eq. 25).

We consider a system with $m = 50$ ton, $k = 1973.92$ kN/m, $c = 12.57$ kN-s/m. Thus, $T_{\text{eq}} = 1$ s, $\xi_{\text{eq}} = 0.02$, and peak steady state displacement of the no-pounding system at resonant excitation ($T = T_{\text{eq}} = 1$ s) is $u_{\text{eq}} = \ddot{u}_{\text{eq}} / 2g\xi_{\text{eq}} = 3.72$ m.

We consider both two-side and one-side pounding. For two-side pounding, separation distances are set equal on both sides, $s' = s''$. Four different values of $s = 0, 0.5, 1$, and $4$ m are considered. For contact element: $k_x = 5921.76$ kN/m ($\kappa = 4$, and $T_{\text{eq}} = 0.5$ s), and $c$ is varied to consider 6 values of $\xi_{\text{eq}} = 0.01$–0.52 (Table 1). Respective values of coefficient of restitution $e$ (see Appendix A) are given. By varying excitation frequency $\omega$ ($= 2\pi / T$) and conducting time-history analysis, steady state displacement spectra are plotted in Figs. 4 to 6.

Table 1. Variations of Damping Capacity of Contact Element.

<table>
<thead>
<tr>
<th>$c_x$ (kN/m)</th>
<th>77.10</th>
<th>174.66</th>
<th>304.72</th>
<th>496.20</th>
<th>643.34</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>1.0</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Fig. 4 shows general shapes of spectra for no-pounding and pounding cases over relatively large $s$ (e.g. $0.3u_{\text{np}}$), moderate $s$ (e.g. $0.1u_{\text{np}}$), and small $s$ ($0.005u_{\text{np}}$), respectively. The shapes of the curves for both one-side and two-side pounding cases are alike. In general, pounding
occurs only at a certain range of $\omega$, otherwise the spectrum curve is identical with that of no-pounding case (Fig. 4). Larger $\xi_{pd}$, or larger $c$, is more effective in reducing the peak displacement, especially when $s$ is smaller. Smaller $s$ results in higher resonant frequency (Fig. 4).

Fig. 5 shows two-side pounding cases, whereas Fig. 6 shows $u'$ and $u''$ spectra of one-side pounding, respectively. For $s = 4$ m ($> \xi_{pd} = 3.72$ m), pounding is avoided and spectrum curves are identical irrespective of different $\xi_{pd}$-values, and one-side or two-side pounding.

For the case of $s = 0$: the system with two-side pounding (Fig. 5) becomes a linear system having vibration period of $T_{pd}$ and damping ratio $\xi_{pd}$ (Eqs. 5 and 6). The system with one-side pounding, in addition to the peak of harmonic response, the peak of the so-called subharmonic response exists at twice the resonant frequency of the pounding system (Fig. 6). However, this peak subharmonic response is insignificant.

### 4.3 Prediction of Peak Harmonic Responses

This section presents a simplified method to estimate maximum response under harmonic excitation of arbitrary frequency. Displacement $u'$ and $u''$ (Sec. 4.2) can be estimated from Eq. 20 if $u_{\text{max}}$ is known. We assume that $u_{\text{max}}$ represents the peak velocity of an equivalent linear system having $T_{ef}$ and $\xi_{ef}$ (Eqs. 23, 24, 28, 30), however, $T_{ef}$ and $\xi_{ef}$ depend on $\nu_{\text{max}}$. Therefore, we must do iterations to obtain the value $\nu_{\text{max}}$. The method is summarized below:

Given parameters are the initial period $T_{mp}$, damping ratio $\xi_{mp}$, separation distances $s'$ and $s''$, properties of contact elements, stiffness increase ratios $k^+$ and $k^-$, and damping increase ratios $\zeta^+$ and $\zeta^-$. First, we determine which pounding occurs by comparing $u_{\text{mp}}$ with $s'$ and $s''$:

- $u_{mp} > s': u_{mp} < s''$: pounding does not occur.
- $u_{mp} < s': u_{mp} > s''$: one-side pounding at right.
- $u_{mp} > s': u_{mp} < s''$: one-side pounding at left.
- $u_{mp} > s': u_{mp} > s''$: two-side pounding.

If pounding occurs, setting initial values $T_{eq}^{(1)} = T_{mp}$ and $\xi_{eq}^{(1)} = \xi_{mp}$, then follow the $i$-th iteration process given below:

1) Estimate $\hat{u}_{\text{max}}^{(i)}$ (Eq. 31), then $u'$ and $u''$ (Eq. 20).
2) Estimate $T_{eq}^{(i+1)}$ and $\xi_{eq}^{(i+1)}$ (Eqs. 23, 24, 28 and 30).
3) Estimate $\hat{\nu}_{\text{max}}^{(i+1)}$ (Eq. 31), and if $\hat{\nu}_{\text{max}}^{(i+1)} = \hat{\nu}_{\text{max}}^{(i)}$, the process ends.
4) Use accelerated estimate $\nu_{\text{max}}^{(i+1)} = (\hat{\nu}_{\text{max}}^{(i)} + \hat{\nu}_{\text{max}}^{(i)})/2$ and go to 1).

Note that $\hat{u}_{\text{max}}^{(i)}$ is given by

$$\hat{u}_{\text{max}}^{(i)} = \bar{u}_{\text{b},c} \sqrt{\left(\omega_{eq}^{(i)} - \omega^2\right) + \left(\xi_{eq}^{(i)} \alpha \omega^2\right)^2}$$

(31)

In step 4) above, we use the midpoint of the limits after each iteration, and the bounds containing exact value of $\nu_{\text{max}}$ decrease by a factor of two. This process will converge very fast, and normally it should take about 3 to 4 iterations if the tolerance limit is set to 1%.

We now investigate the accuracy of the proposed method. The values $u'$ and $u''$ obtained from time history are used to get $\nu_{\text{max}}^{(1)}$ (Eq. 21), thus step 1) is slightly modified. Then steps 2), 3), and 1) are followed and the results without further iterations are compared with the time-history analysis results (Figs. 5, 6).

Estimated spectra are plotted using broken lines also in Figs. 5 and 6. For both two-side and one-side pounding cases, the estimated curves match very well with exact ones from numerical analysis, especially near the peak of each curve. The method cannot predict peak subharmonic response that occurs in one-side pounding with small gap size (Fig. 6). Since peak subharmonic response shows no relevant effect compared with peak harmonic response and it cannot be seen from MDOF system, its effect will not be considered again in this paper.
5 PREDICTION OF SEISMIC POUNDING RESPONSES

5.1 Spectrum-Based Prediction of Peak Seismic Responses

The method proposed in Sec. 4.3 is also used for estimating peak displacements caused by seismic pounding. However, instead of Eq. 31, \( u_{\text{max}} \) is estimated by using elastic design velocity spectrum. Using such design spectrum, \( u_{\text{max}} \) still depends on equivalent \( T_{eq} \) and \( \varepsilon_{ep} \), and is expressed as (Fig. 7)

\[
u_{\text{max}} = S_{p}(T_{eq} \cdot \varepsilon_{ep}) = D_{2} \cdot S_{p}(T_{eq} \cdot \varepsilon_{ep})
\]

(32)

in above expression \( D_{2} \) is scaling factor to take into account the effect of damping on spectra,\(^{23,29}\) and is given as

\[
D_{2} = \frac{1 + \alpha \varepsilon_{ep}}{1 + \alpha \varepsilon_{ep}^{a}}
\]

(33)

where \( \alpha \) depends on each particular earthquake.\(^{23,29}\) Some values of \( \alpha \) are listed in Table 6 for several artificial earthquakes used in this study. Fig. 7 illustrates the changes of design velocity spectrum using \( D_{2} \) with two values of \( \alpha = 75 \) and 25, respectively.

![Figure 7. Effect of Damping on Elastic Design Spectrum.](image)

Using the peak displacements obtained after iteration, peak collision force will be calculated approximately as the square root of the sum of the squared peak forces developed in contact element's spring and dashpot element (Fig. 1, App. B). Accordingly, the peak collision force when pounding on the right or left side is written respectively as

\[
F_{c}^{+} = \sqrt{k'_{s}(u' - s')^{2} + c'_{s}(\pi / \Delta t)u'^{2}[1 - (s'/u')^{2}]}
\]

(34a)

\[
F_{c}^{-} = \sqrt{k'_{s}(u' - s')^{2} + c'_{s}(\pi / \Delta t)u'^{2}[1 - (s'/u')^{2}]}
\]

(34b)

5.2 Example Application

Considering a two-side pounding system having \( T_{eq} = 0.8 \) s (\( m = 50 \) ton, \( k = 3,084 \) kN/m, \( \omega_{p} = 7.854 \) rad/s, \( \varepsilon_{ep} = 0.02 \) (\( c = 15.71 \) kN/s/m). Initial separation distance \( s' = 9.6 \) cm and \( s = 6.5 \) cm. We assume same contact elements on both sides, which have \( k_{e} = 74,022 \) kN/m (\( k \approx 25 \)), and \( c_{e} = 272.61 \) kN/s/m (\( \zeta = 18 \) and \( \omega_{p} = 0.073 \), which is equivalent to coefficient of restitution \( e = 0.8 \)). Artificial earthquake simulating the phase of the El Centro NS record (PGA = 0.535g) is used for time-history analysis (Table 6).

The elastic design spectrum (Sec. 5.3) gives

\[
u_{\text{eq}} = (T_{eq} / 2\pi)S_{p}(T_{eq} \cdot \varepsilon_{eq}) = (0.8 / 2\pi) \times 40 = 17.83 \text{ cm}
\]

Since \( u_{eq} > s' \) and \( u_{eq} > s \), two-side pounding occurs. Using a tolerance \( tol = 0.01 \), we follow the steps shown in Sec. 4.3

After setting initial values \( T_{eq}^{(1)} = T_{eq} = 0.8 \) s and \( \varepsilon_{eq}^{(1)} = \varepsilon_{eq} = 0.02 \),

1) Estimate \( \bar{u}_{(1)}^{(1)} = S_{p}(T_{eq} \cdot \varepsilon_{eq}) = 140 \) cm/s, since \( D_{1} = 1 \).

\[
u_{u}' = 12.24 \text{ cm} \quad \text{and} \quad u' = 9.57 \text{ cm}
\]

2) Estimate \( T_{eq}^{(2)} = 0.389 \) s and \( \varepsilon_{eq}^{(2)} = 0.036 \).

3) Estimate \( \bar{u}_{(2)}^{(2)} = 71.41 \) cm/s, where \( D_{2} = 0.838 \).

Compare \( \bar{u}_{(2)}^{(2)} \) and \( u_{\text{max}}^{(2)} \), \( \bar{u}_{(2)}^{(2)} / u_{\text{max}}^{(2)} = 0.490 > tol \).

4) Use accelerated estimate \( \bar{u}_{(2)}^{(2)} = (\bar{u}_{(2)}^{(2)} + u_{\text{max}}^{(2)}) / 2 = 105.70 \) cm/s and go to 1). The process is continued as summarized in Table 2.

---

### Table 2. Prediction of Peak Displacement by Iteration.

<table>
<thead>
<tr>
<th>Step</th>
<th>( u_{\text{max}}^{(i)} ) (cm/s)</th>
<th>( u' ) (cm)</th>
<th>( u'' ) (cm)</th>
<th>( T_{eq}^{(i)} ) (s)</th>
<th>( \varepsilon_{eq}^{(i)} )</th>
<th>( u_{\text{max}}^{(i)} ) (cm/s)</th>
<th>( u_{\text{max}}^{(i)} ) (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>140.00</td>
<td>12.24</td>
<td>9.57</td>
<td>0.389</td>
<td>0.036</td>
<td>71.41</td>
<td>0.490</td>
</tr>
<tr>
<td>2</td>
<td>105.70</td>
<td>11.14</td>
<td>8.61</td>
<td>0.475</td>
<td>0.031</td>
<td>91.19</td>
<td>0.137</td>
</tr>
<tr>
<td>3</td>
<td>98.45</td>
<td>10.87</td>
<td>8.40</td>
<td>0.503</td>
<td>0.030</td>
<td>97.91</td>
<td>0.005</td>
</tr>
</tbody>
</table>

### Table 3. Comparison between Estimation and Time History.

<table>
<thead>
<tr>
<th></th>
<th>Displacement (cm)</th>
<th>Collision force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation</td>
<td>10.87</td>
<td>8.40</td>
</tr>
<tr>
<td>Time History</td>
<td>10.60</td>
<td>8.14</td>
</tr>
<tr>
<td>Est./TH Ratio</td>
<td>1.02</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Using the peak displacements obtained after iteration (Table 2), peak collision force will be calculated by Eq. 34. Table 3 shows comparison for peak responses obtained by using the proposed method and numerical analysis. The proposed method gives very good estimation for both peak displacement and collision force.

5.3 Validation of the Method

Extensive numerical tests are conducted to verify the accuracy of the proposed method. The validation study includes three different pounding incidents: one-side pounding, symmetric two-side pounding that considers same contact elements on both sides, and asymmetric two-side pounding that considers different contact element on each side.

For each pounding incident, we uses 8 systems with different initial vibration periods \( T_{eq} \) (Table 4, Fig. 8) but having same initial damping ratio \( \varepsilon_{eq} = 0.02 \). For each system, separation distances are varied through 10 values of separation ratios \( s/T_{eq} = 0.1 \) to 1.0 at an increment of 0.1.

For one-side pounding and symmetric two-side pounding incidents, two different cases of contact element stiffness are considered: in stiff contact case the contact element is much stiffer than the system's stiffness; whereas in soft contact case the contact element is assumed quite flexible such that \( k = 4 \) for all systems (Table 4, Fig. 8). For asymmetric two-side pounding incident (Table 5, Fig. 9), the stiffness of contact element on the right side is considered relatively stiff (those of stiff contact case above), while on the left side is relatively flexible (those of soft contact case above).

For each analysis case above, dashpot of contact element is chosen to simulate three values of coefficient of restitution, \( e = 0.9, 0.6 \) and 0.2.

### Table 4. Systems in One-Side, Symmetric Two-Side Pounding.

<table>
<thead>
<tr>
<th>System</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{eq} ) (s)</td>
<td>0.4</td>
<td>0.8</td>
<td>1.2</td>
<td>1.6</td>
<td>2.0</td>
<td>2.4</td>
<td>2.8</td>
<td>3.2</td>
</tr>
<tr>
<td>Stiff Contact Case ( \kappa )</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>64</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Soft Contact Case ( \kappa )</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

### Table 5. Systems in Asymmetric Two-Side Pounding.

<table>
<thead>
<tr>
<th>System</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{eq} ) (s)</td>
<td>0.4</td>
<td>0.8</td>
<td>1.2</td>
<td>1.6</td>
<td>2.0</td>
<td>2.4</td>
<td>2.8</td>
<td>3.2</td>
</tr>
<tr>
<td>Stiff Contact Case ( \kappa )</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>64</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Soft Contact Case ( \kappa )</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
elastie design spectrum that has pseudo velocity \( S_{ps} = 140 \text{ cm/s} \) in the constant velocity domain (from 0.64 s).

Accordingly, a total of 14,400 cases are analyzed, in which:
- One-side pounding: (8 systems) \( \times \) (10 gap ratios) \( \times \) (2 contact stiffesses) \( \times \) (3 contact dashpots) \( \times \) (6 earthquakes) \( \times \) (2 directions) = 5760 cases.
- Symmetric two-side pounding: (8 systems) \( \times \) (10 gap ratios) \( \times \) (2 contact stiffesses) \( \times \) (3 contact dashpots) \( \times \) (6 earthquakes) \( \times \) (2 directions) = 5760 cases.
- Asymmetric two-side pounding: (8 systems) \( \times \) (10 gap ratios) \( \times \) (3 contact dashpots) \( \times \) (6 earthquakes) \( \times \) (2 directions) = 2880 cases.

The accuracy of the proposed method in estimating peak displacement is demonstrated through ratios between estimated and respective time-history analysis results. The graphs in Fig. 11 show the mean and standard deviation of these ratios vs. separation ratios, for each value of coefficient of restitution \( \epsilon \). Thus, each value at a separation ratio is calculated from 96 ratios (8 systems) \( \times \) (6 earthquakes) \( \times \) (2 directions).

For one-side pounding, Fig. 11a shows very close results between time-history analysis and estimation for \( u' \), ratios for \( u' \) are slightly scattered at \( s/sh_{mp} \) small but most of them still distributes near 1.0. Accuracy of the method does not significantly change between soft and stiff contact case. Similarly to results of one-side pounding, the method predicts very well peak displacement of symmetric two-side pounding (Fig. 11b) as well as asymmetric two-side pounding (Fig. 11c).

For time-history analysis, 6 artificial earthquakes\(^{29} \) (Table 6) are considered in both positive and negative directions. These artificial earthquakes, which are scaled to Level 2, have similar spectrum characteristics and similar to the elastic design spectrum. Thus, results obtained from numerical analysis can be used for investigating the accuracy of the proposed method. Fig. 10 shows the pseudo velocity spectra of these earthquakes (2% damping ratio), it also shows the

<table>
<thead>
<tr>
<th>Phase</th>
<th>PGA (cm/s(^2))</th>
<th>Design</th>
<th>Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hachinohe EW</td>
<td>435.07 75</td>
<td>640</td>
<td>40</td>
</tr>
<tr>
<td>JMA Kobe NS</td>
<td>539.78 25</td>
<td>640</td>
<td>40</td>
</tr>
<tr>
<td>Tohoku NS</td>
<td>370.97 75</td>
<td>640</td>
<td>40</td>
</tr>
<tr>
<td>El Centro NS</td>
<td>524.67 55</td>
<td>640</td>
<td>40</td>
</tr>
<tr>
<td>Taft N111E</td>
<td>548.50 75</td>
<td>640</td>
<td>40</td>
</tr>
<tr>
<td>BCJ-L2</td>
<td>346.59 75</td>
<td>640</td>
<td>40</td>
</tr>
</tbody>
</table>
For peak collision force, before making comparison, it is normalized with the respective peak shear force that would develop in the system if pounding did not occur (i.e. \( k_{up} \)). Such normalized forces estimated by the proposed method are plotted against the accurate time-history analysis result in Fig. 12, where each data point corresponds to one analysis case. These graphs show good estimation, especially for pounding with small coefficient of restitution and soft contact element. This is because contact element with large damping can effectively dissipate energy, whereas soft contact element reduces collision force. There are a number of points lie on vertical axis, these points belong to cases of large separation (\( u_{up} = 0.8-1.0 \)), where, according design spectrum pounding occurs but in fact it does not under an actual earthquake input (Fig. 10). Obviously, whether there is pounding or not is not important in these cases because pounding, if occurs, is always weak with such large separation.

6 CONCLUSIONS

This paper proposed the new spectrum-based prediction rule to estimate peak structural responses of a SDOF system pounding against rigid structures. The conclusions are as follows

1) The collision due to pounding is modeled using linear viscoelastic elements. This idealization is consistent with SDOF system and appears adequate for study effects of overall pounding responses.

2) The spectrum-based prediction rule is based on analytical solution of free and harmonic vibration of a system involving pounding to obtain equivalent period and damping, and they are used together with elastic design spectrum for estimation of peak responses.

3) Using 6 artificial earthquakes that resemble the elastic design spectrum, the method is validated through extensive numerical experiments by varying system vibration period, separation distance, and contact element properties. The applicability of the method is shown and the correlation of estimated peak responses to time-history analysis results is very good.

The proposed method can serve as a basis to extend for pounding of MDOF systems. The writer's past studies proposed modal time-history analysis approach for MDOF systems \(^{10-11}\) by using vibration modes of no-pounding or pounding state, depending on the contact condition. Thus, as the natural extension of this method, it is strongly felt that peak pounding responses can be estimated using response spectrum approach with equivalent period and damping of the system. The MDOF system is firstly converted to a SDOF system by using its vibration period and damping ratio of first mode in no-pounding state and pounding state, respectively. The method proposed in this study is utilized next to find seismic responses of the SDOF systems. Then finally, the responses are converted back to MDOF responses considering the modes described above. Details procedure of this extension is being formulated. Accuracy of the method is assessed, and the results will be reported elsewhere.

ACKNOWLEDGEMENTS

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REFERENCES

For model using coefficient of restitution, kinetic energy of the mass $m$ after impact is

$$KE = \frac{1}{2}m(\nu - \nu_0)^2$$  \hspace{1cm} (A1)

where $\nu$ = velocity at the start of collision

For model using viscoelastic contact element, kinetic energy of the mass after impact

$$KE = \frac{1}{2}m\nu^2 - 2\nu^0\omega_k\zeta$$  \hspace{1cm} (A2)

where $\omega = \omega_0 \sqrt{1 - \zeta^2}$

Kinetic energy of the mass at the beginning of impact is the same for both types of model, thus the kinetic energy at the end of impact must also be same in order to have same energy loss. Equating Eq. (A1) and (A2), we have

$$\zeta_1 = \frac{\ln e}{\sqrt{\pi^2 + (\ln e)^2}}$$ \hspace{1cm} (A3)

The $\zeta-e$ relationship is drawn in Fig. A1: $e = 1$ corresponds to $\zeta = 0$, which means no energy dissipated during impact, while $e = 0$ corresponds to $\zeta = 1$, which means all energy will be dissipated.

### Appendix A. Contact Element Properties

When viscoelastic contact element simulates collision, its spring controls contact duration and its dashpot accounts for energy dissipated during collision. However, there is still much uncertainty in determination of these properties, especially the dashpot. On the other hand, typical values of the coefficient of restitution $e$ have been determined experimentally for simple colliding bodies. Therefore, it is reasonable to have a relation between dashpot and $e$. Such relation can be obtained by equating the kinetic energies at the end of impact given by each model as follows

$$E_{k}\left(\frac{m}{d}\right) = \frac{m}{d} + A_{m}$$

in which $A_{m}$ = energy dissipated by damping of the system, $A_{m}$ = energy dissipated by viscoelastic contact element. Using Eq. (B1), we can obtain

$$A_{m} = \frac{m}{d}\left(\frac{\pi}{2}\right)\left\{1 - \left\{\frac{\nu_0}{u_s}\right\}^2\right\}$$ \hspace{1cm} (B2)

$$A_{m} = \frac{m}{d}\left(\frac{\pi}{2}\right)\left\{\frac{\nu_0}{u_s}\right\}^2$$ \hspace{1cm} (B3)

### Appendix B. Energy Dissipated by Viscoelastic Elements

To simplify calculation of hysteretic energy dissipation, the displacement history $u(t)$ is approximated by two distinct half-sinusoidal waves with vibration frequencies $(\omega_0\Delta t^2)$, $(\omega_0\Delta t^2)$ on each side (Sec. 4.1). Thus, the energy dissipated by viscoelastic elements can be obtained by adding the energy dissipated by each viscoelastic element. As for energy dissipated on the right side, we have

$$E_{k}\left(\frac{m}{d}\right) = \frac{m}{d} + A_{m}$$

$$A_{m} = \frac{m}{d}\left(\frac{\pi}{2}\right)\left\{1 - \left\{\frac{\nu_0}{u_s}\right\}^2\right\}$$

$$A_{m} = \frac{m}{d}\left(\frac{\pi}{2}\right)\left\{\frac{\nu_0}{u_s}\right\}^2$$

$E_{k}$ and $A_{m}$ are the kinetic energy and energy dissipated by each viscoelastic element, respectively.

### Figure A.1. $\zeta-e$ Relationship

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和文要約

1 はじめに
1.1 構造物の衝突問題

地震時に構造物が異なる種類をとる場合の衝突被害が起こり得る。この対策のため、重要で可能に正確な衝突最大応答の評価が必要である。衝突を模倣して時刻歴解析法に応じた予測が可能であるが、構造物特性の変化により衝突応答を変化させる。この変化は特に構造物の弾性と影響を及ぼす。衝突条件を表す変数と推定される変数を用いて、時刻歴解析結果を表することがにより検証する。

2 衝突問題のモデル化
2.1 衝突部モデルの問題

被害調査から、構造物の被災は軽微であり、衝突により増した構造物の変形による損傷が著しいことが分かれる。これと既往の知見に基づき、構造物の変形を表す。ただし、本論文で提案する評価法は、衝突要因を特定するのではなく、その要因が如何に影響を及ぼすかを明らかにすることである。

2.2 衝突の定義

1 予想される両側に構造物を考え、両側に衝突する場合と片側に衝突する場合を含めて、構造物の特性と構造間隔が異なった場合、構造物の変形が衝突要因である。これにより、衝突要因は構造物の特性と変数を考慮して、衝突を予測することを表す。

3 衝突を伴う自由振動
3.1 時刻歴法の解析

無減衰で、衝突を伴う自由振動を数学的に解き、変数1.2に変わる時刻をそれぞれそれぞれ。両側衝突、右(正)側衝突、左(負)側衝突の場合それぞれの定義が分割され、その場合も構造物に与える変数を含む。

3.2 変数最大、最小値および等価周期と速度最大値の関係

以上の解より、自由振動での変数変化を数値的に解き、状態1.2に変わる変数をそれぞれそれぞれ。両側衝突、右(正)側衝突、左(負)側衝突の場合それぞれの定義が分割され、その場合も構造物に与える変数を含む。1.1に示した時刻を含む等価周期を定義し、これを等価周期にした。これらを衝突と考える場合、ランダム振動の応答予測にいうことにする。

4 衝突を伴う調和振動と最大応答予測
4.1 等価減衰定数の定義

無減衰で自由振動から得た時間変数(3章)と、減衰を伴った調和振動を考えて等価減衰定数を求める。100Hzの内で変位が正・負の場合の時間帯、変位最大値・最小値の比が、自由振動から得た場合に比べて大きくなる。

さらに、正側・負側の変位応答を、それぞれ異なる周期をもつ正弦波で近似し、それぞれと構造間隔から、状態1.2の等価周期を定義する。これに基づき、1予想する波形に応じた最大応答の予測は可能である。予測に応じた手法を提案した。多質点系の衝突応答を調和振動の応答を用いて、時刻歴解析結果を検証する。

5 衝突を伴う地震最大応答予測
5.1 地震応答スペクトルの基 slump最大応答予測

4.3節で用いた物質振動のスペクトルの相関において、地震応答スペクトルを用いて多質点系の応答を検証する。これにより、4.3節と同様に地震応答を検証し、多質点系を用いて、時刻歴解析結果を検証する。

6 おわりに
1 予想する波形に応じた応答を検証し、地震応答を伴う場合と片側に衝突する場合を含めて、その予測応答を検証した。この手法の適用は今後の課題である。

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