DETERMINATION OF LOAD AND RESISTANCE FACTORS 
BY METHOD OF MOMENTS

モーメント法による荷重・耐力係数の算定

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The load and resistance factors are generally determined using the first order reliability method, in which the “design point” must be determined and derivative-based iteration has to be used. In the present paper, the general formula of the load and resistance factor based on the method of moments is deduced based on the review of basic principle of the load and resistance factor format for structural design. A simple formula for determining the load and resistance factors using the fourth moment method in which neither the design point nor derivative-based iteration is necessary, is proposed. From the investigation of the present paper, it is found that the present method gives good improvement upon the method based on the third moment method and although the load and resistance factors obtained using the method of moments are different from those obtained through the first order reliability method, the required structural resistances under a specific design condition are almost the same.

Keywords: Load and resistance factors, Third moment method, Fourth moment method, FORM

1. INTRODUCTION

As the assurance of the performance of a structure must be accomplished under conditions of uncertainty, reliability-based structural designs have been developed. There are generally two ways of reliability-based structural design. The first is developed based on probability of failure in which the complete probabilistic analysis will generally be necessary and the practical designers have to master the basic theory of probability and reliability. Another way is developed without a complete probabilistic analysis. If the required safety factors are predetermined on the basis of specified probability-based requirement, reliability-based design may be accomplished through the adoption of appropriate deterministic design criteria, e.g., the use of traditional safety factors.

For obvious reasons, design criteria should be as simple as possible; moreover, they should be developed in a form that is familiar to the practical engineers. This can be accomplished through the use of load amplification factors and resistance reduction factors, known as the LRFD format. That is the nominal design loads are amplified by appropriate load factors and the nominal resistances are reduced by corresponding resistance factors, and safety is assured if the factored resistance is at least equal to the factored loads. The appropriate load and resistance factors must be developed in order to obtain designs that achieve a prescribed level of reliability.

The load and resistance factors are generally determined using the first order reliability method (FORM), in which the “design point” must be determined and derivative-based iteration has to be used. Some simplifications are proposed in order to avoid iterative computation. The third moment method has been applied to for the same object. In this paper, the general formula of the load and resistance factor based on the method of moments is deduced based on the review of basic principle of the load and resistance factor format for structural design. A formula of determination of load and resistance factors using fourth moment method is proposed. From the investigation of the present paper, it is found that: (1) the present method gives good improvement upon the method based on the third moment method; (2) although the load and resistance factors obtained using the method of moments are different from those obtained through the first order reliability method, the required structural resistances under a specific design condition are almost the same.

2. REVIEW ON DETERMINATION OF LOAD AND RESISTANCE FACTORS

The LRFD format may be expressed as the follows.

$$\phi S_n = \sum \gamma_i S_i$$

(1)

where $\phi$ = the resistance factor; $\gamma_i$ = the partial load factor to be applied to load $S_i$; $R_n$ = the nominal value of the resistance; $S_n$ = the nominal value of load $S$.

In reliability-based structural design, the resistance factor $\phi$ and the load factor $\gamma_i$ should be determined on the basis of achieving a specified reliability. That is, the design format, Eq. 1, should be equivalent to the
following equations in probability terms.

\[ G(X) = R - \Sigma S_i \]  

(2)

where \( R \) and \( S \) are the random variables representing the uncertainty included in resistance and load effects. \( p_f \) and \( \beta \) are the probability of failure and reliability index corresponding to the performance function Eq. 2. \( p_f \) and \( \beta \) are the target probability of failure and target reliability index, respectively.

If \( R \) and \( S \) are mutually independent normal random variables, the second moment method is correct and the design formula becomes

\[ \beta_{2M} = \beta_f \]  

(4)

where

\[ \beta_{2M} = \frac{\mu_G}{\sigma_G} \]  

(5a)

\[ \mu_G = \mu_R - \Sigma \mu_{S_i} \quad \sigma_G = \sqrt{\sigma_R^2 + \Sigma \sigma_{S_i}^2} \]  

(5b)

where \( \beta_f \) is the second moment (2M) reliability index; \( \mu_G \) and \( \sigma_G \) are the mean value and standard deviation of the performance function \( G \), respectively; \( \mu_R \) and \( \sigma_R \) are the mean value and standard deviation of \( R \), respectively; and \( \mu_{S_i} \) and \( \sigma_{S_i} \) are the mean value and standard deviation of \( S_i \), respectively.

Substituting Eq. 5 in Eq. 4, produces,

\[ \mu_G(1 - \alpha_R \beta_f \beta_f) = \Sigma \mu_{S_i}(1 + \alpha_S \beta_f \beta_f) \]  

(6)

Comparing Eq. 6 with Eq. 1, the load and resistance factors may be expressed as,

\[ \phi = (1 - \alpha_R \beta_f \beta_f) \frac{\mu_R}{R} \]  

(7a)

\[ \gamma_i = (1 + \alpha_S \beta_f \beta_f) \frac{\mu_{S_i}}{S_i} \]  

(7b)

where \( V_S \) and \( V_R \) are the coefficients of variation, respectively, of \( R \) and \( S_i \) and \( \alpha_R \) and \( \alpha_S \) are the direction cosines (also known as separating factors), respectively, of \( R \) and \( S_i \).

\[ \alpha_R = \frac{\sigma_R}{\mu_R} \quad \sigma_R = \frac{\sigma_R}{\mu_R} \]  

(8)

When \( R \) and \( S \) are non-normal random variables, the reliability index expressed in Eq. 5 is not correct. The reliability index corresponding to the performance function Eq. 2 is generally obtained by first order reliability method (FORM). The design format can be expressed as

\[ R^* = \Sigma S_i^* \]  

(9)

And the load and resistance factors can be obtained as

\[ \phi = \frac{R^*}{R}, \quad \gamma_i = \frac{S_i^*}{S_{i\text{e}}} \]  

(10)

where \( R^* \) and \( S_i^* \) are the values, respectively, of variable \( R \) and \( S \) at the design point of FORM. Since \( R^* \) and \( S_i^* \) are obtained using derivative-based iteration, explicit expressions of \( R^* \) and \( S_i^* \) are not available. Some simplifications are proposed in order to avoid iterative computation.

In the present paper, the reliability index in Eq. 3 will be obtained using the method of moments. Since the central moments of the performance function, as described in Eq. 2, can be obtained quite easily, the probability of failure, which is defined as \( \text{Prob}(G(X) < 0) \) can be expressed as a function of the central moments. The proposed method, therefore, is based on the premise that by finding the relationship between the failure probability and the central moments of \( G(X) \). Since no derivative-based iteration is necessary in the proposed method, the determination of the required load and resistance factors should be more simply accomplished.

### 3. LOAD AND RESISTANCE FACTORS BY METHOD OF MOMENTS

#### 3.1 Determination of Load and Resistance Factors using the Third Moment Method

In order to avoid the use of design point which has to be obtained from derivative-based iteration, the third moment (3M) reliability index has been used as the reliability index for the performance function of Eq. 2 to obtain the load and resistance factors based on third moment method.

Substituting the 3M reliability index in the design format described in Eq. 3 produces,

\[ \beta_{3M} = \beta_f \]  

(11)

where the 3M reliability index \( \beta_{3M} \) is expressed as

\[ \beta_{3M} = \frac{-1}{\sigma_G^3} \left( 3 - \sqrt{9 + \sigma_R^2 \sigma_{S_i}^2 - 6 \alpha_R \alpha_S \sigma_{3M}} \right) \]  

(12)

where \( \alpha_R \) is the 3rd dimensionless central moment, i.e., the skewness of \( G(X) \). The \( \alpha_{3M} \) of Eq. 2 can be computed by

\[ \alpha_{3M} = \frac{1}{\sigma_G^3} \left( \alpha_R \sigma_R^2 - \sigma_S \sigma_{S_i} \right) \]  

(13)

where \( \alpha_R \) and \( \alpha_{3M} \) are the skewness of \( R \) and \( S_i \), respectively.

Submitting Eq. 12 into the design format described in Eq. 11, it yields,

\[ \frac{-1}{\sigma_G^3} \left( 3 - \sqrt{9 + \sigma_R^2 \sigma_{S_i}^2 - 6 \alpha_R \alpha_S \sigma_{3M}} \right) \geq \beta_f \]  

(14)

After some arrangements, we have

\[ \beta_{3M} = \beta_f - \frac{1}{3} \left( \beta_f - 1 \right) \]  

(15)

Denoting the right side of Eq. 15 as \( \beta_{2T} \), one obtains

\[ \beta_{2T} = \beta_{2T} + 1 \]  

(16a)

\[ \beta_{2T} = \beta_{2T} + 1 \]  

(16b)

Equation 16a is as same as Eq. 3. It means that if the second moment reliability index \( \beta_{2M} \) is at least equal to \( \beta_{2T} \), the reliability index \( \beta \) will be at least equal to the target reliability index \( \beta \), and the required reliability is satisfied. Therefore, \( \beta_{2T} \) can be considered to be a target value of \( \beta_{3M} \) and it is denoted as the target second moment reliability index hereafter.

Substituting Eq. 5 in Eq. 16a, it yields,

\[ \mu_G(1 - \alpha_R \beta_f \beta_f) = \Sigma \mu_{S_i}(1 + \alpha_S \beta_f \beta_f) \]  

(17)

Comparing Eq. 17 with Eq. 1, the load and resistance factors may be expressed as,

\[ \phi = (1 - \alpha_R \beta_f \beta_f) \frac{\mu_R}{R} \]  

(18a)

\[ \gamma_i = (1 + \alpha_S \beta_f \beta_f) \frac{\mu_{S_i}}{S_i} \]  

(18b)

where \( V_S \) and \( V_R \) are the coefficients of variation, respectively, of \( R \) and \( S_i \) and \( \alpha_R \) and \( \alpha_S \) are the direction cosines, respectively, for \( R \) and \( S_i \), which are the same as Eq. 8.

Comparing Eq. 18 with Eq. 7, one may understand that after replacing \( \beta_f \) in Eq. 7 by \( \beta_{2T} \) in Eq. 18, the determination of load and resistance factors using the third moment method is essentially the same as that by the second moment method.
The variations of the target second moment reliability index $\beta_T$ with respect to the target reliability index $\beta$ are shown in Fig. 1. From Fig. 1, one can see that $\beta_T$ is larger than $\beta$ for negative $\alpha_G$ and smaller than $\beta$ for positive $\alpha_G$. When $\alpha_G = 0$, $\beta_T = \beta$, then, Eq. 11 becomes exactly the same as Eq. 4, and the load and resistance factors can be determined using Eqs. 7 & 8.

![Fig. 1 Target 2M Reliability Index](image)

3.2 General Expressions of Load and Resistance Factors using Method of Moments

Consider the performance function in Eq. 2, without loss of generality, standardize the performance function $G(X)$ using the following standardized variable

$$z_u = \frac{G - \mu_G}{\sigma_G}$$

(19)

Then the probability of failure corresponding to the performance function $G$ can be expressed as the following equation according to its definition.

$$P_f = \text{Prob}(G \leq 0) = \text{Prob}(z \leq \mu + \mu_G)$$

(20)

$$= \text{Prob}(z \leq \frac{-\mu_G}{\sigma_G}) = \text{Prob}(z \leq -\beta_{2M})$$

where $\beta_{2M}$ is the second-moment reliability index expressed in Eq. 5a.

Suppose the relationship between the standardized variable $z_u$ and the standard normal variable $u$ can be expressed as the following functions using the first several moments of $Z = G(X)$,

$$z_u = S(u, M)$$

(21a)

$$u = S^{-1}(z_u, M)$$

(21b)

where $M$ is a vector denoting the first several moments of $Z = G(X)$ and $S^{-1}$ is the inverse function of $S$.

Let the CDF of $z_u$ be $F$, then we have

$$F(z_u) = \Phi(u) = \Phi[S^{-1}(z_u, M)]$$

(22)

Therefore, according to Eq. 20, the probability of failure corresponding to the performance function of Eq. 2 is expressed as

$$P_f = F(-\beta_{2M}) = \Phi[S^{-1}(-\beta_{2M}, M)]$$

(23)

Therefore, the reliability index based on method of moments is expressed as

$$\beta = -S^{-1}(P_f) = -S^{-1}(-\beta_{2M}, M)$$

(24)

Note, here, the CDF of $z_u$ is only used to deduce the Eq. 24, no $F$ is used in the calculation of $\beta$. Since the first several moments of $G(X)$ is used in the Eq. 24, Eq. 24 is a moment reliability index.

In this study, the moment reliability index described in Eq. 24 is used as the reliability index for the performance function of Eq. 2, in place of that obtained by FORM. Substituting Eq. 24 into the design format described in Eq. 3, it yields,

$$\beta = -S^{-1}(-\beta_{2M}, M) \Rightarrow \beta_T$$

(25)

From Eqs. 23 and 24, one can easily understand that $\beta$ increases monotonously with the increase of $\beta_{2M}$, therefore, the following design formula is equivalent to Eq. 25.

$$\beta_{2M} = -S(-\beta_T, M)$$

(26)

Denoting the right side of Eq. 26 as $\beta_{2T}$, one obtains

$$\beta_{2M} \geq \beta_{2T}$$

(27a)

$$\beta_{2T} = -S(-\beta_T, M)$$

(27b)

Equation 27a is as same as Eq. 3. It means that if the second moment reliability index $\beta_{2M}$ is at least equal to $\beta_{2T}$, the reliability index $\beta$ will be at least equal to the target reliability index $\beta_T$, and the required reliability is satisfied. Therefore, $\beta_T$ is a general expression of the target second moment reliability index.

If the relationship between the standardized variable $z_u$ and the standard normal variable $u$ is expressed as the following equation\(^{10}\).

$$z_u = \sum_{i=0}^{\infty} a_i u^i$$

(28)

Then $\beta_{2T}$ is expressed as

$$\beta_{2T} = \sum_{i=0}^{\infty} a_i (-\beta_T)^i$$

(29)

Since Eq. 27a is the same as Eq. 4 except that the right side is $\beta_{2T}$, the load and resistance factors corresponding to Eq. 27a can be easily obtained by substituting $\beta_{2T}$ in the right side of Eq. 4 with $\beta_{2T}$. The design formula then becomes

$$\mu_G(1-\alpha_R V_R \beta_{2T}) \geq S\phi(1+\alpha_S V_S \beta_{2T})$$

(30)

and the load and resistance factors are obtained as,

$$\psi = \frac{(1-\alpha_R V_R \beta_{2T}) \mu_R}{\mu_R}$$

(31a)

$$\gamma_S = \left(1+\alpha_S V_S \beta_{2T}ight) \frac{\mu_S}{S_m}$$

(31b)

where

$\alpha_R$ and $\alpha_S$ are determined by Eq. 8;

$V_R$ and $V_S$ is the coefficient of variation, respectively, of $R$ and $S$;

$\alpha_R$ and $\alpha_S$ is the direction cosine, respectively, for $R$ and $S$; and

$\beta_{2T}$ is the target second moment reliability index calculated from Eq. 29. Since the formula above is based on the first few moments of the load and resistances, the LRFs can be determined even when the distributions of the random variables are unknown.

3.3 Determination of Load and Resistance Factors using the Fourth Moment Method

The standardized variable $z_u$ can be expressed as a polynomial function of the standard normal variable $u$ as the following equation, which was suggested by Fleishman \(^{10}\),

$$z_u = a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4$$

(32)

where $a_1$, $a_2$, $a_3$, and $a_4$ are the polynomial coefficients that can be obtained by making the first four moments of the left side of Eq. 32 equal to those of the right side.
According to Eq. 29, the target second moment reliability index is expressed as

\[ \beta_{2T} = \alpha_{T} - \alpha_{L} \beta_{T} - \alpha_{R} \beta_{T}^2 + \alpha_{D} \beta_{T}^3 \]  

(33)

Substituting Eq. 33 into Eq. 31, the load and resistance factors can be given using the coefficients of \( \alpha_{L}, \alpha_{R}, \alpha_{D}, \) and \( \alpha_{T}. \)

Eq. 32 is simple if the coefficients \( \alpha_{L}, \alpha_{R}, \alpha_{D}, \) and \( \alpha_{T} \) are known. However, the determination of the four coefficients is not easy, since the solution of nonlinear equations has to be found when using Eq. 32 (10). An alternative way may be the first two polynomials of the Cornish-Fisher expansion (10).

\[ z_a = S(u) = u + \frac{1}{6} \alpha_{4G} (u^2 - 1) + \frac{1}{24} (\alpha_{4G} - 3)(u^3 - 3u) \]  

(34)

where \( \alpha_{4G} \) is the 4th dimensionless central moment, i.e., the kurtosis of \( G(X) \) in Eq. 2, which is given by

\[ \alpha_{4G} = \frac{1}{\alpha_{R}^2} (\alpha_{R}^2 \alpha_{S}^2 + 6 \alpha_{R}^2 \sum_{i=1}^{n} \alpha_{Ri}^2 \alpha_{Si}^2 + 6 \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{Ri}^2 \alpha_{Si}^2 \alpha_{Sj}^2) \]  

(35)

where \( \alpha_{R} \) and \( \alpha_{S} \) are the kurtosis of \( R \) and \( S \), respectively.

One can see that Eq. 34 is in close form and quite easily to be used, however, since the first four moments of the right side of Eq. 34 are not equal to those of the left side, the transformation generates relatively large errors (10).

In order to improve the Cornish-Fisher expansion, Winterstein (10) developed an expansion to be expressed as

\[ z_a = \tilde{k} \beta + \tilde{k} (1 - \tilde{k} \beta^2) \beta + \tilde{k} \beta^2 \beta^2 \]  

(36a)

in which

\[ \tilde{k} = \frac{\alpha_{4G}}{4 + 2 \sqrt{1 + 1.5 (\alpha_{4G} - 3) - 1}} \]  

(36b)

\[ \tilde{k} = \frac{1}{\sqrt{1 + 2 \beta^2 + 6 \beta^3}} \]  

(36c)

It has been shown that Eq. 36 gives much improvement (10) on Eq. 34, and it generally give reasonable results with sufficient accuracy in the following ranges of \( \alpha_{4G} \) and \( \alpha_{T} \) (9).

For \( 0 < \alpha_{4G} < 1.2 / \beta_{T} + 2.7 + 1.5 / \alpha_{R} \leq \alpha_{4G} \leq 5.2 - 2 \alpha_{R} \) (37a)

For \( \alpha_{4G} \leq 0, 2.7 + \alpha_{R}^2 \leq \alpha_{4G} \leq 5.2 + \alpha_{R}^2 \) (37b)

In the present study, Eq. 36 will be used as the fourth moment transformation function.

According to Eq. 29, the target second moment reliability index is expressed as

\[ \beta_{2T} = \tilde{k} \beta + \tilde{k} (1 - \tilde{k} \beta^2) \beta + \tilde{k} \beta^2 \beta^2 \]  

(38)

Since Eq. 38 is an explicit function of \( \alpha_{4G} \) and \( \alpha_{T} \), the load and resistance factors can be easily given by substituting Eq. 38 into Eq. 31, the load and resistance factors can be easily given.

Especially when \( \alpha_{4G} = 3 \), one has \( \tilde{k} = 0 \), \( \tilde{k} = \alpha_{4G} / 6 \) and Eq. 38 becomes

\[ \beta_{2T} = \tilde{k} \beta + \frac{1}{6 \alpha_{4G}} (\tilde{k} \beta^2 - 1) \]  

(39)

when \( \alpha_{4G} \) is small enough, \( \tilde{k} = 1 \), then Eq. 39 becomes

\[ \beta_{2T} = \beta_{T} - \frac{1}{6 \alpha_{4G}} (\beta_{T}^2 - 1) \]  

which is essentially the same as the target second moment reliability index obtained by the 3M method. This is to say that the formula based on the 3M method is a special form of that based on the fourth moment method (4M) when the performance function approaches normal.

![Fig. 2 Variations of target 2M reliability index with respect to \( \alpha_{4G} \) and \( \alpha_{T} \)]

In more particular, when \( \alpha_{4G} = 3 \) and \( \alpha_{T} = 0, \) Eq. 38 becomes \( \beta_{2T} = \beta_{T} \), which is exactly the same as Eq. 4, and the load and resistance factors can be determined using Eqs. 7 & 8.

Needless to say again, since the formula above is based on the first four moments of the load and resistances, the load and resistance factors can be determined even when the distributions of the random variables are unknown.

The variations of the target second moment reliability index \( \beta_{2T} \) with respect to the target reliability index \( \beta_{T} \) are shown in Fig. 2a, 2b, and 2c in the case of \( \alpha_{4G} = 0.2, 0, \) and 0.2, in Figs. 2d, 2e, and 2f in the cases of \( \alpha_{4G} = 2.8, 3.0, \) and 3.2, respectively. From these figures, one can see that for \( \alpha_{4G} = 0, \beta_{2T} \) is generally larger than \( \beta_{T} \) for \( \alpha_{4G} = 3.0 \) and smaller than \( \beta_{T} \) for positive \( \alpha_{4G} < 3.0. \) One can also see that for \( \beta_{2T} \) is generally larger than \( \beta_{T} \) for negative \( \alpha_{4G} \) and smaller than \( \beta_{T} \) for positive \( \alpha_{4G} \) and smaller than \( \beta_{T} \) for positive \( \alpha_{4G}. \)

3.4 Determination of the Mean Value of Resistance

Since the load and resistance factors are determined when the reliability index is equal to the target reliability index, the mean value of the resistance should be determined under this condition (hereafter referred to as the target mean resistance). In this study, the target mean resistance is computed using
the following approximate equation.

$$\mu_{RF} = \mu_{RF} + \frac{1}{2} \sigma_{G}^{2} \beta_{RF}$$

(40)

where

$$\mu_{RF} = \sum_{i=1}^{n} \mu_{Si} + \beta_{RF} \sqrt{\sum_{i=1}^{n} \sigma_{Si}^2}$$

(41)

where, $\mu_{RF}$ is the target mean resistance; $\sigma_{G}$ and $\sigma_{G}^{2}$ are the standard deviation and skewness of $G$ obtained using $\mu_{RF}$; $\beta_{RF}$ is moment reliability index obtained using $\mu_{RF}$.

4. NUMERICAL EXAMPLES

Example 1

Consider the following performance function,

$$G(X) = R - (D + L)$$

(42)

where $R$=resistance, a lognormal variable with $\mu_{R}=1.1, \sigma_{R}=0.15$;

$D$=dead load, a normal variable with $\mu_{D}=1.0, \sigma_{D}=0.1$; and

$L$=live load, a Weibull variable with $\mu_{L}=0.45, \sigma_{L}=0.4$.

The skewness for $R$, $D$, and $L$ are 0.453, 0, and 0.2768, respectively, and the kurtosis for $R$, $D$, and $L$ are 3.368, 3, and 2.78, respectively.

The load and resistance factors obtained using the proposed method of moments are illustrated in Figs. 3a and 3b for $\beta_{RF}=2$ and $\beta_{RF}=3$, respectively, compared with the corresponding factors obtained using FORM and 2M method. The target mean resistances obtained using the proposed method and those obtained by FORM and 2M method are illustrated in Figs. 3c and 3d for $\beta_{RF}=2$ and $\beta_{RF}=3$, respectively. From Fig. 3, one can see that although the load and resistance factors obtained by the present method are different from those obtained by FORM, the target mean resistances obtained by the present methods are essentially the same as those obtained by FORM. That is, the same design results will be obtained by the FORM and moment method even though the load and resistance factors of the methods are different. One can also see that the 2M method provides relative large errors and gives conservative results.

Fig. 4 Load and resistance factors and the target mean resistances for Ex. 1

Fig. 5 Investigation on the applicable range of 3M method

From Figs. 4b and 4d, one can see that the resistance factors, the load factors for snow load, and the target mean resistances obtained by the third and fourth moment methods have visible differences. This is because the third moment method in this case produces relatively larger error since only the first three moments of the performance function are used. In order to
clarify this problem, the skewness in the cases above are depicted in Figs. 5a and 5b, for $\beta_r=2$ and $\beta_r=3$, respectively. In the same figures, the application ranges of third moment method in terms of skewness are also depicted using the following equation (44):

$$-120r/\beta_{SM} \leq a_{5SM} \leq 40r/\beta_{SM}$$

where $r$ is the allowable error of reliability index in percentage.

From Figs. 5a and 5b, one can see that the range of skewness for 3M is applicable for $\beta_r=3$ is narrower than that for $\beta_r=2$ are beyond the application with $r=1\%$. This can explain why there are visible differences between the target mean resistance obtained by the third and fourth moment methods.

**Example 3**

The third example considers the following performance function

$$G(X) = R - (D + L + E)$$

where $R$ is the resistance; $D$ denotes the dead load effect; $L$ denotes the live load effect, and $E$ is the maximum earthquake load effect over 50 years which is taken as the main load here.

The probabilistic information of $R$, $D$, $L$, and $E$ is listed in Table 1. Since the third and fourth moments for Frechet distribution, which is generally used as the probabilistic model of earthquake load, do not exist according to the definition, the maximum value of earthquake load over 50 years is assumed to obey Gumbel distribution here. The distribution is directly used as the probabilistic model of $E$ in Eq. (45) according to Tarkostas's empirical load combination rule (5).

For target reliability level of $\beta_r=2.5$, the LRFDF format and the target mean resistances obtained using the 2M, 3M, 4M and FORM are listed in Table 2, from which one can see that 4M provides design results in good agreement with the FORM results while 2M provides results in larger error.

**Table 1.** Basic random variables for Example 3

<table>
<thead>
<tr>
<th>$R$ or $S_r$</th>
<th>CDFs</th>
<th>$\mu_0/\sigma_0$</th>
<th>$\alpha_0$</th>
<th>$\alpha_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ Log-normal</td>
<td>1.10</td>
<td>0.1</td>
<td>0.927</td>
<td>4.566</td>
</tr>
<tr>
<td>$D$ Normal</td>
<td>1.0</td>
<td>$\mu_0$</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>$L$ Log-normal</td>
<td>0.45</td>
<td>0.5$\mu_0$</td>
<td>0.4</td>
<td>1.264</td>
</tr>
<tr>
<td>$E$ Gumbel</td>
<td>0.64</td>
<td>$5\mu_0$</td>
<td>0.85</td>
<td>1.14</td>
</tr>
</tbody>
</table>

**Table 2.** Results of LRFDF format for Example 3

<table>
<thead>
<tr>
<th>$R$ or $S_r$</th>
<th>LRFDF Format</th>
<th>Target mean $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2M</td>
<td>$0.99R_0 \geq 1.021D_0 + 0.469L_0 + 1.889E_0$</td>
<td>$\mu_R \geq 18.067\mu_0$</td>
</tr>
<tr>
<td>3M</td>
<td>$0.942R_0 \geq 1.026D_0 + 0.474L_0 + 2.182E_0$</td>
<td>$\mu_R \geq 21.472\mu_0$</td>
</tr>
<tr>
<td>4M</td>
<td>$0.936R_0 \geq 1.027D_0 + 0.474L_0 + 2.204E_0$</td>
<td>$\mu_R \geq 22.072\mu_0$</td>
</tr>
<tr>
<td>FORM</td>
<td>$1.036R_0 \geq 1.011D_0 + 0.426L_0 + 2.475E_0$</td>
<td>$\mu_R \geq 22.104\mu_0$</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

1. General formula of determination of load and resistance factors using the method of moments is derived and a simple method of estimating load and resistance factors by fourth moment method is proposed. Derivative-based iteration, which is necessary in FORM, is not required in the proposed method. For this reason, the proposed method is simpler to apply.

2. The present fourth moment method gives good improvement upon the method based on the third moment method.

3. Although the load and resistance factors obtained by the present method are different from those obtained by FORM, the target mean resistances obtained by both methods are essentially the same.

4. Since the present formula is based on the first few moments of the load and resistances, the load and resistance factors can be determined even when the distributions of the random variables are unknown.

It should be noted that random variable with Frechet distribution, for which the third and fourth moments do not exist sometimes, cannot be used in the present method. It is necessary to develop a suitable probabilistic model for such random variables in future study.

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和文要約

1. 序

近年、外力や抵抗に含まれている不確定性を考慮した構造信頼性設計が注目を受ける傾向にある。日本建築学会でも、建築物の限界状態設計指針が発表されている。限界状態設計法では、想定した限界状態に対して適切な信頼性を確保することを目的とし、確率・統計理論の取り扱い方によって確率基準設計法と耐荷・耐力係数を用いた設計の二つのレベルに分けられる。確率に基づく設計法は複雑な手法であるが、想定した限界状態に対する確率確率を正確に求める力がないことについては設計者にとって煩雑な部分である。荷重・耐力係数を用いた設計法では、確率確率の計算が不要となり、設計者が確率・統計理論を意識することを必要としても、既往の許容応力度設計法と同様の手順で構造設計を行うことができるという点において、極めて実用性の高いものである。

荷重・耐力係数の設計定法については、一次信頼性解析法（FORM）によることが一般的であるが、FORM設計点が一定の値にまで繰り返し計算により求められ、設計点及び分布係数を個別に表現できないことから、今後、性能設計に移行することに伴い、よりフレキシブルに設計するためには、設計者が独自に荷重・耐力係数値を評価する必要性がある。

より簡単には荷重・耐力係数を設定するために、3次モーメント法による算定式が提案されている。本論文では、モーメント法に基づく荷重・耐力係数算定の一般式を導き、4次モーメント法による荷重・耐力係数の簡単算定式を提案し、算定例を通じて従来手法による結果との対応を検討することを目的とする。

2. 既往の荷重・耐力係数算定法の回顧

モーメント法に基づく荷重・耐力係数の算定式を構築するために、まず荷重・耐力係数の設定原則及び既往の設計法について検討する。

荷重・耐力係数を設定する背景の設計法における設計式は(1)式(a)で表される。式(1)式は、既往の許容応力度設計法の設計式と同様の形式であるが、既往の安全性に対する荷重・耐力係数が目標信頼性レベルに荷重・耐力に含まれている不確定性を考慮して定められたものである。即ち、式(1)の荷重・耐力係数は式(2)の限界状態関数に対して、式(3)の設計式に満足するように設定されている。

相互独立な正規分布変数の場合、式(2)の限界状態関数に対して、2次モーメント信頼性指標は正確で、設計式式(4)のように、荷重・耐力係数が式(5)のように定められる。

非正規分布変数の場合、2次モーメント信頼性指標は正確で、設計式式(2)の限界状態関数に対して、設計式式(3)に満足する時の設計点を元の空間に変換させ、式(10)のように荷重・耐力係数を設定する。設計点を得るために繰り返し計算が必要となり、それを回避するために様々な試案が提案されている。

3. モーメント法による荷重・耐力係数の算定法

既往の荷重・耐力係数の算定法は FORM 信頼性指標に基づく性格上、FORM に関する設計点を繰り返し計算を必要とする。そこで式(3)の設計式中の式(2)の限界状態関数に対する信頼性指標であり、FORM にこだわらず、他の信頼性解析手法でも、式(2)に対して適切な信頼性指標を評価することが可能である。荷重・耐力係数は同様に計算する。本節では、モーメント法に基づく荷重・耐力係数算定の一式を著者、4次モーメント法による荷重・耐力係数の簡単算定式を構築する。

3.1 3次モーメント法による荷重・耐力係数の算定

式(3)の設計式に式(12)の3次モーメント信頼性指標を代入することにより、式(15)の設計式が得られる。式(15)の右辺をβrで表し、式(15)は式(16)のように表される。式(16)は形式式(3)の設計式と全く同じである。即ち、2次モーメント信頼性指標βrにβrを上回ることと3次モーメント信頼性指標βrにβrを上回ることと全く意味合いを持っている。ここでは、βrはβrの目標値となるので、βrを目標二次モーメント信頼性指標という。

目標二次モーメント信頼性指標βrをFig.1に図示する。Fig.1により、σα(0)がより大きい場合、βrはより小さい、σα(0)がより小さい場合、βrはより大きい、σα(0)=0の時、βr=βrとなることが分る。

3.2 モーメント法による荷重・耐力係数の設定の一式

限界状態関数を式(19)のように標準化し、定義された確率変数は式(20)で表すことができる。標準化限界状態関数は標準正規確率変数の間に式(21)の関係があると仮定すると、βの確率変数より、破壊確率と信頼性指標はそれぞれ式(22)と式(24)で表すことができ、式(24)のモーメント信頼性指標は式(3)の設計式に代入することにより、式(22)の設計式が得られる。式(26)の辺りをβrで図示し、式(26)は式(27a)のように表される。式(27a)は形式式(3)の設計式と全く同じである。前述と同じように、ここでは、βrを目標2次標準化限界状態関数を指す。

特に、標準化限界状態関数は標準正規確率変数の関数は式(28)のような多項式式で表す場合、目標2次モーメント信頼性指標は式(29)のように、式(27)と式(3)を比較することにより、式(31)のモーメントの算定式が得られる。式(31)では、破壊変数のモーメントのみ使用しており、確率変数の分布形状が分からなくても、荷重・耐力係数を設定することができる。

3.3 4次モーメント法による荷重・耐力係数の算定

式(32)の標準化限界状態関数は標準正規確率変数の関数と関係し、4次モーメント法に基づき目標2次モーメント信頼性指標は式(33)で表される。未定係数がα1, α2, α3は方程式式(32)の左右辺の4次までのモーメント等しいという条件から解かれると、未定係数がα1, α2, α3を解くには複雑な非線形方程式を解く必要があり、これを回避するため、本研究では、式(36)の遅延式を用いる。目標2次モーメント信頼性指標は式(32)で表され、それと目標信頼性指標の関係をFig.2に示す。

4. 構造・耐力係数の算定法

例1、式(42)の限界状態関数に対して得られた荷重・耐力係数と目標平均耐力をFig.3に示す。Fig.3によりモーメント法による荷重・耐力係数は FORM によって得られた荷重・耐力係数と全く同じである。それらによって得られた耐力平均値の目標値はほぼ一致することが分る。

例2、式(43)の限界状態関数に対して得られた荷重・耐力係数と目標平均耐力をFig.4に示す。Fig.4によりβ=0の場合、3次モーメント法の計算結果が得られることから、その原因は3次モーメント法の適用範囲である。

例3、式(45)の限界状態関数に対して得られた目標平均耐力をTable 2に示す。4次モーメント法を FORM によって得られた耐力平均値の目標値はほぼ一致することが分る。

5. 結論

(1) モーメント法に基づく荷重・耐力係数算定の一式を導出し、4次モーメント法による荷重・耐力係数の簡単算定式を提案した。算定法は設計点の概念を用いず、直線式で表している。

(2) 4次モーメント法による荷重・耐力係数の算定は3次モーメント法による算定法を改良している。

(3) 本手法および既往の FORM で得られた荷重・耐力係数にはどのようなもの、荷重・耐力係数を用いた設計効果は一致している。

(4) 提案した算定法では、確率変数の分布形が分からなくても、荷重・耐力係数を設定することができる。

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