ESTIMATION OF SHEAR STRENGTH OF RC INTERIOR BEAM-COLUMN CONNECTIONS BASED ON THE STRUT MODEL
ストラットモデルによるRC造十字形柱梁接合部の強度評価

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This paper presents a new theoretical macro model developed for estimation of joint shear strength of RC interior beam column connections. In the model, a compressive strut is considered as failing in bending mode. Hence, the stress block concept based on the beam bending theory may be used to estimate the strut strength. The depth of compressive strut is calculated from the neutral axes in the beams and columns, which are derived from the bending theory considering axial forces in the connected members and the effect of transverse reinforcement bars in the joint panel. Finite element analyses were carried out in order to observe the behavior of the compressive struts. Analysis results showed that the process of the formation and disappearance of the strut is consistent with the adoption of the bending theory in the strut model. The joint shear strengths of specimens in a database collected from past RC beam-column joint experimental studies were calculated by the developed model and compared to the published experimental values. The results show that the model provides a theoretical explanation of joint behavior as well as a much more improved estimation than the methods currently recommended in both the ACI or AIJ codes.

1 INTRODUCTION
It is important to estimate the strength of beam–column joints under lateral forces due to earthquake loads because the joint shear failure mode often shows poor ductility and energy dissipation capacity. However, a simple and accurate macro model for designing joints is lacking because of the complexity of the actual failure mode. In current codes such as the ACI[1] or the AIJ[2], strength estimation is mostly based on empirical rather than theoretical relations. While the truss model is considered for the estimation of joint shear force in the New Zealand code[3], Kurose[4] pointed out that the strut mechanism, rather than the truss mechanism, is more dominant in the joint panel.

There are several studies estimating the joint strength based on the strut model.[5,6] In those models, strength reduction factors are used to estimate the compressive strength of the strut damaged by the deformations in the opposite direction. Agreeable estimations are obtained when the strength reduction factor is well selected, although it is not easy to determine the factor theoretically.

Shiohara et al.[7,8] developed a joint shear failure model based on the bending theory for the joint panel. The beam–column joint is divided into four parts and the failure mode is described by a nine degree of freedom system. With this model, many parameters including the transverse reinforcement in the joint and axial force in the columns can be considered and the strength of the joint can be estimated reasonably well, although the calculation is rather complicated for design practice.

In a previous study, we translated the nine degree of freedom model to the classic strut model to estimate the joint shear strength.[9]. The estimation, however, was not accurate enough because strut depths were estimated in a simplified way. In this paper, several specimens in past studies are reproduced by the finite element (FE) analysis and the stress distributions in the joint panel are investigated. The width of the strut is then calculated in a more detailed way based on the results obtained in FE analysis, in which the effects of joint transverse reinforcement, axial forces in the columns and yielding of the main bars in the connected members are considered.

2 FINITE ELEMENT ANALYSIS OF JOINT SHEAR BEHAVIOR
A series of 4 interior joint specimens in a study by Au et al.[10] are analyzed using finite element (FE) analysis to observe the behavior of the concrete inside the joint panels. Two of the specimens, named herein as E-0.0 and E-0.3, do not have transverse reinforcement in the joint panel while 2 others, H-0.0 and H-0.3, have them. The notation “0.0” means that the column axial stress is zero and “0.3” means that the axial stress is 30 percent of concrete compressive strength. All the specimens were reported as failing in the joint shear failure mode. The specimens are modeled by two dimensional plane stress triangular elements with sides around 1.5 centimeters long.

In the FE analysis, a parabolic curve for compression and an exponential curve for tension are used for the concrete stress-strain relationship as shown in Fig. 1. The reinforcement bars are modeled by a bilinear curve and the bond-slip behavior was neglected. A constitutive model based on total strain, also called ‘Total Strain crack model’ is used for the modeling of concrete.

Specimen parameters, maximum loads and load estimations by the ACI and AIJ codes and the element method (FEM) are shown in Table 1. The joint
Table 1 Parameters and calculated story shear of interior joint specimens using FE analysis and building codes

<table>
<thead>
<tr>
<th>Model name</th>
<th>( f'_c ) (MPa)</th>
<th>( L_b ) (mm)</th>
<th>( L_c ) (mm)</th>
<th>( D_b ) (mm)</th>
<th>( b_b ) (mm)</th>
<th>( b_c ) (mm)</th>
<th>beam bars</th>
<th>( M_c ) ( M_b )</th>
<th>( P_w ) (%)</th>
<th>( N/N_0 )</th>
<th>Maximum load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-0.0</td>
<td>43.1</td>
<td>1500</td>
<td>1330</td>
<td>300</td>
<td>300</td>
<td>250</td>
<td>300</td>
<td>4T16</td>
<td>1.87</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>H-0.0</td>
<td>50.6</td>
<td>1500</td>
<td>1330</td>
<td>300</td>
<td>300</td>
<td>250</td>
<td>300</td>
<td>16T16</td>
<td>1.87</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E-0.3</td>
<td>46.1</td>
<td>1500</td>
<td>1330</td>
<td>300</td>
<td>300</td>
<td>250</td>
<td>300</td>
<td>4T16</td>
<td>3.06</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>H-0.3</td>
<td>45.1</td>
<td>1500</td>
<td>1330</td>
<td>300</td>
<td>300</td>
<td>250</td>
<td>300</td>
<td>16T16</td>
<td>3.13</td>
<td>1.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Note: \( f'_c \) = concrete compressive strength; \( L_b \) = Beam length; \( L_c \) = Column length; \( D_b \) = Depth of beam; \( D_c \) = Depth of column; \( b_b \) = Width of beam; \( b_c \) = Width of column; \( M_c/M_b \) = Ultimate moment ratio of the column to the beam; \( P_w \) = Transverse reinforcement ratio in the joint panel; \( N/N_0 \) = Axial force ratio in the column.

Fig. 1 Finite element modeling of stress-strain relation of concrete, (a) parabolic curve for compression concrete and (b) exponential curve for tension concrete.

Fig. 2 Envelope curve of load–deformation relation obtained in an experimental study and FEM reproduction.

2.1 Strut Depth in FE Analysis

Compressive principal stress distributions reproduced by FE analyses at three stages around the maximum load (pre-peak, peak, post-peak) are shown in Fig. 3(a) to (d). A compressive strut can be clearly seen in each figure. The strut depth in the specimens with no transverse reinforcement in the joints is almost constant along the diagonal in the early pre–peak stages. In a contrast, the depth is higher at the center of the joints containing transverse reinforcement. Maximum stress at the center of the joints is seen at the maximum input load \( V_{max} \) in all specimens. After the maximum load (in the early post-peak), the stress at the center of the joint panel decreases and the strut starts swelling. This swelling of the strut makes it difficult to estimate the post-peak strength of the joint by the strut model. However, in the beginning of the swelling, the total force in the strut does not actually change significantly. This is appar-
ently due to the crushing of the concrete when the strut starts swelling and the consequent decrease in stress at the center of the joint.

The principal stress distribution along the opposite diagonal of the joint panel (ex. diagonal line AB at the top of Fig. 3(a) to (d)) for each specimen is shown at the bottom graphs in Fig. 3. Horizontal axes in the figure show diagonal distance from A and vertical axes show the principal stresses. The dashed lines show stress distributions at maximum load and the dotted lines show the post peak state. After the maximum load, two peaks appear in the stress distribution which start to move apart from the center. This phenomenon corresponds to the model by Shiohara et al. where the bending compressive failure occurs continuously from the center of the joint to the corners.

Comparing the stress distributions between specimens, the strut depth in specimen E-0.0 (with no transverse reinforcement or axial force in columns) is smaller than those in specimen H-0.0 or specimen E-0.3, suggesting that the confinement by transverse reinforcements in the joint panel or axial forces in columns increases the strut depth. The effects of the transverse reinforcement in the joint panel and the axial forces in the columns will be taken into account during the estimation of strut depth.

3 NEW MODEL FOR ESTIMATION OF JOINT SHEAR

3.1 Modeling Concept

Shiohara developed the nine degree of freedom model to explain the joint shear failure in terms of the bending failure theory. Joint shear failure is assumed to occur when the concrete at the center crushes due to the flexural ultimate state of the four triangular ends of beams and columns. In this study, the compressive areas which appear in Shiohara’s model are taken as a couple of struts in the developed model, as shown in Fig. 4(a). The two struts fail in bending, not in axial compression, when the joint fails. Under this as-
initial assumption, the compressive force of the strut can be calculated based on the stress block concept, which is common for the estimation of ultimate bending strength of beams. The stress distribution at the center of the struts (Fig. 4(b)) is simplified to a constant distribution with reduced width by coefficient $\beta_1$ as depicted in Fig. 4(c).

The strut width $r$ is calculated considering the force equilibrium at the ends of beams and columns, as described in the following section. However, the forces at the member ends can be estimated only after the force in the strut is given; therefore, an iterative procedure is taken in the developed method.

At the member ends, the concrete does not bear tensile force and is assumed to be elastic in compression. This elastic assumption is not met when the curvatures at the member ends are large. However, moment at the member ends do not increase significantly any more in this case and the strength of the joint may not be very different from the strength determined by the bending failure of the member ends. Therefore, the elastic assumption is taken in this paper for simplicity, targeting only the evaluation of the joint strength. However, this method may be extended in the future to clarify the failure mode considering inelasticity at the connected member ends in the iterations to determine the strut width.

### 3.2 Initial assumption of the depth of neutral axes $x_0$ and $y_0$

The neutral axes at the end of the beams and columns can be calculated accurately only when the curvature of the members are known. For the first step of the estimation, the concrete and main bars are assumed to be linearly elastic while stress in the transverse reinforcement of the joint panel and axial forces in the columns are neglected. With these assumptions, the unknown curvature value is eliminated and the depth of the neutral axis can be determined simply. Thus, the initial assumption of the depth of the neutral axis of the column, $x_0$ is calculated by the following equation

$$0.5E_c b_c x_0^2 + (A_{sc}E_c)x_0 - A_{sc}Ed_c = 0 \tag{1}$$

where $b_c$ and $d_c$ are the depth and effective depth of the column, respectively, and $A_{sc}$ is the area of main tensile bars. $E_c$, $E$ are the Young’s modulus of concrete and reinforcement, respectively. The tensile stress in concrete is neglected in all the equations in this paper. The initial assumption of the neutral axis depth in the beam, $y_0$ is also calculated in the same form

$$0.5E_b b_h y_0^2 + (A_{sh}E_b)y_0 - A_{sh}Ed_h = 0 \tag{2}$$
where \( b_h \) and \( d_h \) are the depth and effective depth of the beam, respectively, and \( A_{sh} \) is the area of main tensile bars.

Next, the depths of the neutral axes in the beams and columns are calculated iteratively considering the curvature, the stress in the transverse reinforcement in the joint and the axial force in the columns. The joint shear strength is then calculated using the final \( i \)th iterated values of the depths of the neutral axes of the columns \( x_i \) and beams \( y_i \). The calculations involved are introduced in the following sections.

### 3.3 Effective depth of the strut \( D_j \)

The depth of the strut \( r \), as shown in Fig. 5, is calculated using the depth of the neutral axes in the columns \( x_j \) and beams \( y_j \). The effective depth of the strut \( D_j \), which is to be used for calculating the compressive force in the strut, is derived by reducing the depth based on the stress block concept of beam bending as follows:

\[
D_j = \beta_j \sqrt{x_j^2 + y_j^2} = \beta_j r
\]

where factor \( \beta_j \) is the recommended value by the ACI design code\[10\] for reducing the height of the stress block, given as follows.

\[
\beta_j = \begin{cases} 
0.85 & f'_c < 28 \text{MPa} \\
1.05 - 0.05 \frac{145 f'_c}{1000} & 28 \text{MPa} \leq f'_c < 55 \text{MPa} \\
0.65 & f'_c \geq 55 \text{MPa}
\end{cases}
\]

Using the factor \( \beta_j \), the concrete compressive stress is assumed to be distributed uniformly with a value of 0.85\( f'_c \) as in the ACI code. As a result, compressive strength of the strut in the joint panel \( C_k \) is estimated according to the effective area \( A_j = \beta_j r b_j \) from the following equation.

\[
C_k = 0.85 \beta_j [f'_c] r b_j
\]

where \( b_j \) is effective width based on the AII code.

\[
b_j = b_h + b_{a1} + b_{a2}
\]

where \( b_h \) is the width of beam, \( b_{a1} \) and \( b_{a2} \) are the smaller of one-quarter of column depth \( D_c/4 \) and one-half of distance between beam and column face \((x/2)\) on either side of beam, respectively. These reductions of the depth and compressive strength of the strut corresponds to the reduction factor strengths used in past methods based on the strut model\[8,11\].

### 3.4 Curvatures at the member ends \( \phi_c \), \( \phi_b \)

The story shear force will be calculated from the equilibrium of forces in the strut. A half section of the beam-column joint connection is divided into three segments, the upper column, the right beam and the upper half of the joint panel, as shown in Fig. 5. Based on the flexural theory and the equilibrium of the right beam and upper column, the following equations are given

\[
\begin{align*}
C_{j,\text{sh}} &= V_h b_h + T_j f_j & \text{and} & & T_h &= C_{j,\text{b}} - T_j \quad (7) \\
C_{j,\text{sc}} &= V_h k_c + P_j & \text{and} & & T_c &= C_{j,\text{c}} - P \quad (8)
\end{align*}
\]

where \( T_j \) is the tensile force in the transverse reinforcement and \( f_j \) is the distance of the tensile load in the transverse reinforcement (at the middle of beam, assuming that all the transverse reinforcement bars have yielded) from the tensile beam bars. Although \( T_j \) actually works inside the transverse reinforcement, the distance between the main bars in the columns and the beam ends are neglected in this case for simplicity, \( f_j \) is the distance between the column axial load (assumed as a point load at the center of the column) and the column tensile bars. \( C_{j,\text{b}} \) and \( C_{j,\text{c}} \) are the concrete compressive forces for the beams and columns, respectively, as shown in Fig. 5 (b) and (c). From the equilibrium of the upper half of the joint panel, the following equations are derived

\[
\begin{align*}
V_c + C_{j,\text{b}} D_j / L - 2 C_{j,\text{c}} + T_j &= 0 \quad (9) \\
V_b + C_{j,\text{c}} D_b / L - 2 C_{j,\text{b}} + P &= 0 \quad (10)
\end{align*}
\]

where \( D_j \) and \( D_b \) are the depths of the column and beam, respectively, \( T_j \) is the force in the transverse reinforcement (assumed to be yielding) and \( P \) is the axial force in the column.

Now it is possible to solve the four unknown variables, \( \phi_c \), \( \phi_b \), \( V_b \) and \( V_c \), by using the above four equations from (7) to (10). When \( C_c \) and \( C_b \) are given, the curvatures at the ends of the column and beam \( \phi_c \) and \( \phi_b \) can be calculated using the relations shown in Fig. 5 (b) and (c).

\[
\phi_c = \frac{2 C_b}{E_b b_h x_j^2}, \quad \phi_b = \frac{2 C_c}{E_c b_h y_j^2}
\]

### 3.5 Update of the neutral axis depth estimations \( x_{i+1}, y_{i+1} \)

After estimating the curvatures at the member ends, the depths of neutral axes can be calculated by the bending theory considering the transverse reinforcement and axial forces in the column. The \((i + 1)\)th iterated values of neutral axis depths are calculated by the following equations.

\[
\begin{align*}
0.5 E_b b_h x_{i+1}^2 + (A_{sc} E_c) x_{i+1} - A_{sc} E_c d_c - P/\phi_b &= 0 \quad (12) \\
0.5 E_c b_h y_{i+1}^2 + (A_{sc} E_c) y_{i+1} - A_{sc} E_b d_b - T_j/\phi_c &= 0 \quad (13)
\end{align*}
\]

If the strain of the main bar exceeds the yield strain, the above equation is replaced by one or both of the following equations

\[
\begin{align*}
0.5 E_b b_h e_x y_j^2 + (A_{sc} E_c) y_{i+1} - (A_{sc} f_{yc} + P) d_c &= 0 \quad (14) \\
0.5 E_c b_h e_y x_j^2 + (A_{sc} E_b) x_{i+1} - (A_{sc} f_{yb} + T_j) d_b &= 0 \quad (15)
\end{align*}
\]

where \( e_x = f_{yc}/E \) and \( e_y = f_{yb}/E \) are the yield strain of the bars in the beam and column, respectively.

If the updated neutral axis depths obtained are close enough to the values in the previous step, the values of the neutral axis depths are regarded as accurate and the iteration is terminated. Otherwise, the iteration is continued using the updated depths, \( x_{i+1} \) and \( y_{i+1} \) in Section 3.3. For this paper, the tolerance is set to 5% and the convergence conditions are as follows.

\[
\frac{x_{i+1} - x_i}{x_i} < 0.05, \quad \frac{y_{i+1} - y_i}{y_i} < 0.05
\]

### 3.6 Story shear

Finally, the story shear force \( V_c \) may be calculated from Equations (7) - (10) using the converged neutral axis depths

\[
V_{\text{cal}} = V_c = \frac{C_n (2 b_h D_j k_c + D_h b_h j_c) / L + K_p + K_i}{4 h_b f_{yc} - f_{yb}}
\]

where \( K_p = 2 b_h (j_c - 2 j_p) P \) and \( K_i = j_c (j_h - 2 j_p) T_j \).

Equation (17) estimates the shear strength of interior joints based on the new strut model.

### 4 ESTIMATION OF SHEAR STRENGTH OF SPECIMENS IN PAST EXPERIMENTAL STUDIES

A specimen database made up of 102 internal beam-column joint specimens showing joint shear failure is compiled from 21 different experimental stud-
ies in order to evaluate various joint shear strength estimation methods. The specimen list is shown in Table 2.

Joint shear forces \( V_{\text{sd}} \) are calculated for the constructed database using the method described in the previous Section. The results are shown in Table 2 as a ratio of the experimental shear force over the calculated shear force \( V_{\text{exp}}/V_{\text{sd}} \), along with the calculated estimates given by the ACI and AII codes.

All estimates are calculated by the equation for the allowable shear force in beam-column joints given in Article 15 of the code[21]. ACI estimates are from an equation in Section 21.5.3 of the code[21]. The experimental values are plotted against the calculated shear forces by the three methods in Fig. 6.

The best-fit line and the linear correlation coefficient (CCo) are also shown in the figure. These results show that the new model gives the best estimates among the three methods. The plots also show that the code estimations are not on the safe side. This may be because the database includes specimens where the beam main bars yield before the joint failure or because the concrete strength is not taken as the design strength but the test piece strength.

The histograms of the ratio of experimental over calculated values of the story shear force are shown in Fig. 7, along with the average (av.) and the standard deviation (SD) of the ratios. The story shear is estimated most accurately by the new model. The standard deviation in the new model, 0.1, is smallest, implying that the model uncertainty is quite small for this database. However, for design practice, the model uncertainty together with the material uncertainty may have to be considered to prevent overestimation.

5 CONCLUSIONS

Non-linear finite element analyses were performed for RC beam-column joint specimens compiled from past experimental studies, in order to observe the formation and disappearance of compressive struts in the joint panels. A
straight strut appears in the early loading stages. After the maximum load, the peak of the stress distribution breaks up into two parts, the strut swells and becomes ambiguous. These behaviors correspond to the nine degree of freedom model by Shiohara et al. The analyses also show that confinement by the transverse reinforcement and axial forces in the columns make the strut wider.

A compressive strut macro model was developed to estimate the joint shear strength by applying the beam bending theory, in which the depth of the strut and the stress distributions in the connected member ends are calculated iteratively. The influence of the joint transverse reinforcement and column axial load were also considered through the estimation of the strut depth.

Using a database of beam–column joint specimens constructed from past experimental studies, the new model was applied to estimate the strengths of the joint specimens for investigation. The results show that the model provides a theoretical explanation of joint behavior as well as a much more improved estimation than the methods currently recommended in both the ACI or AIJ codes.

References
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3) Standard Association of New Zealand: Code of Practice for the design of Concrete Structures, 1982.
和文要約

1. はじめに
RC造柱接合部における応力伝達機構は複雑であり、簡便かつ精度の高い強度算定のためのマクロモデルの提案は少ない。本研究では、RC造柱接合部モデルの有限要素法解析を行なって圧縮ストラットの擬似数を検討し、ストラットモデルに基づく強度算定手法を見極める。また、RC造柱接合部に関する過去の実験結果から試験体データを収集し、開発手法によって算定した強度と実験値を比較し、手法の精度を検証した。

2. 柱梁接合部の有限要素法解析
Auらによって、接合部内の補強筋の有無と柱の軸力をパラメータとした4体の実験を有限要素法で再現した。荷重-変形関係の包絡線をよく捉えることにより、全体として妥当な解析ができていると考えることができる。

主応力の方向と大きさを線分で表すと、最大耐力前後において明確な圧縮ストラットを見ることのできる。接合部せん断補強筋の無い試験体では、幅が一定の比較的細いストラットが現れることに対し、補強筋のある試験体ではストラット力は大きく、中央で膨らんでいる。柱に軸力が導入されている場合で、ストラットの幅は大きくなる。最大耐力付近において、ストラット中央の応力度がコンクリート強度付近到達し、接合部中央でコンクリートの圧壊が進行することができる。最大耐力に達した後変形をさらに増大させると、ストラット中央の応力度は低下し、ストラットの幅が増大する。ストラットの膨らみの大きさを評価するのは難しいが、中央部分で耐力が低下するために、ストラットの膨らみに伴うストラット力の変化は大きないと考えられる。

3. 接合部せん断強度算定のための新しいモデル
塩原らは、9自由度のモデルを用いて、接合部せん断破壊を曲げ破壊の理論に基づいて説明した。本研究では、塩原らのモデルにおける圧縮領域を二個のストラットと考え、これらが曲げによって破壊すると仮定する。この仮定に基づいて、ストラットの耐力を、梁の曲げ理論におけるストレスブロックを用いて算定する。ストラットの幅は、柱と梁の各部における力の釣合いから求めた中立軸位置を用いて算定する。

部材端部の曲率が決まらないと中立軸位置は定まらない。そこでまず、柱の軸力と接合部補強筋を無視し、材料は弾性と仮定して中立軸位置を求め、これを初期値とする。次に、中立軸位置から求めたストラット幅と、曲げ理論に基づくストラット強度を用いてストラットの力を求め、これに釣り合うように柱と梁の端部の曲率を決定する。このような柱の軸力を考慮する。梁については、部材の端部を柱の最外側の主筋の位置とすることにより、接合部補強筋の効果を梁の中立軸位置に反映させる。ここで求められた各部材の曲率によって中立軸位置を計算し直し、前ステップの値と比較して収束計算を行う。

4. 既往の実験試験体の強度評価
RC造柱梁接合部を対象とした既往の21の実験的研究から、接合部せん断破壊した102体の試験体データを収集し、データベースを用いて試験体の強度を評価した。各試験体の接合部せん断力を、RC基準、ACI基準の計算式及び提案手法によって算定し、実験値と比較した。基準式による計算と比べて、提案手法による計算の精度は高い。実験値/計算値の分布を見ると、提案手法による計算では平均0.99、標準偏差0.1となった。

5. まとめ
RC造柱接合部の有限要素法解析を行い、圧縮ストラットの生成と圧壊を観察した。最大耐力前後で明確なストラットが見られ、幅は最大耐力時まであまり変化しない。最大耐力を超えると、ストラット中央の応力度は低下し始める。ストラットの幅が増大する。
RC造柱接合部の強度算定のためのストラットモデルを開発した。有限要素法の結果を参考にして、2つの圧縮ストラットが曲げ破壊すると仮定し、ストラットの幅は接続する柱と梁の端部の中立軸位置から求めるものとした。
既往の実験的実験から接合部破壊した試験体データを収集し、開発したモデルによって精度を評価して実験値と比較した。RC基準及びACI基準の設計式を比較して、精度よく強度を評価することができた。

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