DYNAMIC INTERACTIVE BEHAVIOR BETWEEN A POTENTIAL FLUID AND ELASTIC CONTAINER IN LARGE DEFORMATIONS

– Nonlinear vibration responses of a two-dimensional tank (Part 2) –

ポテンシャル流体と弾性容器の大変形動的連成場における相互作用問題
二次元タンクの非線形振動応答分岐

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Liquid-filled tanks on a shaking table showed vibratory responses that contradicted the predictions of elementary vibration theory. The author has been studying the responses that might be caused by the dynamic geometric nonlinear behavior of the system and showed the Lagrangian function that governs the interactive behavior between a potential fluid and elastic container for large deformations. Arbitrary Lagrangian-Eulerian Elements (ALE) were applied to the functional of the two-dimensional model, proposed a new procedure to analyze the problem, and obtained various nonlinear vibration responses corresponding to the vibratory responses in experiments using a shaking table.

Keywords : *nonlinear response of tank, coupled vibration of fluid and container, 1/2 subharmonic oscillation, summed combination resonance*

タンクの振動, 円筒タンクのオバリング, タンクの分数調波振動応答, 定常応答の不安定領域

1. Introduction

Until the 1950s, for the problem related to interaction of a fluid and its elastic container, the formulation that appends the mass effect of the fluid to the container was used. Subsequently, a functional was developed for the interaction problem of a potential fluid and its elastic container on the basis of the infinitesimal deformation field. Based on this, a linear theory^{2,3,4)} was formulated that yielded useful knowledge for engineering. Recently, seismic response of floating roof^{5,6,7)} were examined to secure its safety.

J. C. Luke¹⁾ proved that the function that integrates the pressure formula for a potential fluid over the fluid field is a strict Lagrangian function governing the fluid movement. Considering the fluid field deformation accompanying the elastic displacement of the container, the author has shown the strict functional that governs the dynamic interaction problem of a potential fluid and its elastic container^{8, 10)} in large deformations.

The nonlinear response obtained from experiments using cylindrical tanks could be theoretically analyzed on the basis of the derived functional. In this study, an analysis by discretizing the system on the basis of this functional was attempted. Because the pressure integral for the potential fluid is given by Euler's method and that for the elastic container is generally given by the Lagrange method, precisely processing compatibility conditions for deformation and movement of the

interaction surface between the fluid and elastic container involved substantial difficulty.

When a cylindrical tank containing water is subjected to periodic ground motions at a certain frequency caused by an external force, subharmonic vibrations and a nonlinear vibration response occur, which significantly increase the response. The author has experimentally studied the subharmonic vibration that occurs in cylindrical tanks¹¹⁻²²⁾.

In the early 1990s, ALE elements and reference coordinates were developed. These elements introduced moving reference coordinates and enabled the measurement of fluid motion using a coordinate system that is independent of the motion of the fluid particles.

Limiting the investigation to incompressible, inviscid, and irrotational fluids by using the functional that governs the interaction behavior of a fluid and an elastic body, the equation for the discretized field can be derived. First, it is shown that the appropriate fundamental equations and boundary conditions of the system are acquired from the first variation of the functional. Subsequently, a direct method is formulated for discretizing the strict functional in the dynamic interaction problem of the potential fluid and the elastic container in large deformations. Both are reported in the previous paper²³⁾.

In this paper, the numerical analysis of the subharmonic vibrations generated by the container's response in the two-dimensional interaction

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problem of the fluid and elastic container is discussed, and the relationship between the frequency of the external force and the subharmonic vibration as well as the modes of oscillation are analyzed, and finally, the usefulness of the proposed analyzing method is demonstrated.

2. Functional of the elastic container in large deformations

An elastic container that exists on a moving base at velocity $\mathbf{v}_0$, which holds a potential fluid and its free surface, is considered. By introducing the velocity potential φ and the velocity $\hat{\mathbf{v}}$ of moving coordinates to measure fluid motion by an observer positioned on this base, the functional $I_m(\varphi, \boldsymbol{\eta}, \mathbf{u})$ ^{8, 10)} for the interaction field of the potential fluid and the elastic container in large deformations, which is seen by the observer on the base, is expressed by the following formula²³⁾.

$$I_m = \int_{t_0}^{t_1} \left[\iint_{A_L(\boldsymbol{\eta}, \mathbf{u})} \rho_L \{ \dot{\varphi} + \nabla \varphi \cdot (\nabla \varphi / 2 - \hat{\mathbf{v}}) + \dot{\mathbf{v}}_0 \cdot \mathbf{r} + g(y - y_f) \} dx dy - \int_0^\ell \{ (\rho_E A_E \dot{\mathbf{u}}(\dot{\mathbf{u}}/2 + \mathbf{v}_0) - \Pi(\mathbf{u})) \} ds \right] dt \quad (1)$$

Here, $\dot{\mathbf{v}}_0$ is the acceleration of the base, $\boldsymbol{\eta}$ is the wave height vector on the free surface, $\mathbf{u}$ is the displacement vector of the elastic container, $\mathbf{r}$ is the position vector after deformations, Π is the potential energy of the elastic container, A_L is the fluid domain, ρ_L is the fluid mass density, A_E is the cross-sectional area of the container, y_f is the coordinate value of free surface, ρ_E is the container mass density, and g is the acceleration due to gravity.

The Part 1 of the paper²³⁾ has been dealt a discretizing procedure to analyze the equation (1), in which geometrical nonlinearity of the container is taken into consideration, though natural.

3. Numerical analyses

3.1 Numerical analysis model

The two-dimensional fluid domain has a rectangular shape of width 0.9 m and height 0.6 m and is contained by the bottom and side walls of the container. The side walls on the left and right are composed of steel element of identical thickness (height 0.99 m and Young's modulus $E = 205.8$ GPa). The mass densities of the steel and fluid are $\rho_E = 7850 \text{ kg/m}^3$ and $\rho_L = 1000 \text{ kg/m}^3$, respectively. In the finite element method, eight elements were used in the height direction, 16 elements were used in the horizontal direction, and 12 elements were used for the container as a two-dimensional linear element (Fig. 2). The linear element is fixed at the rigid base.

3.2 Natural frequencies

The natural frequencies obtained when adopting three different kinds of container thicknesses (5 mm, 10 mm, 30 mm) and the natural frequencies for a rigid container are shown in Table 1. The minimum natural frequencies just for containers with no water are shown in the same table, where the left and right sides of the containers have identical natural frequency values. The natural frequencies for the antisymmetric vibration modes and the symmetric vibration modes have been annotated with symbols from the smallest up to the 9th or 10th modes.

The natural frequencies for the model with a container thickness of 30 mm and the rigid container were basically identical and the difference was around 0.15% at the most.

In the model of the 30 mm and 10 mm thick containers, the vibration modes that induced major container displacement corresponded to the natural frequencies ω_{a9} and ω_{s10} . In response to the mass effect of the fluid, the natural frequencies were smaller by approximately 3% and 6% than the values for only the container.

On the other hand, when the container thickness was 5 mm, the interaction between the fluid and container had a considerable effect on all the natural frequencies shown in the table. The influence is small for the natural frequencies of sloshing modes. But it is hard to detect the natural frequency of “bulging mode”. This has an important effect on the lowest natural frequency corresponding to symmetric and antisymmetric vibration modes of the 5mm thick container. Let examine the natural vibration modes corresponding to the system quantitatively.

The wave height and displacement values at the typical positions for the normalized antisymmetric and symmetric vibration modes for the model of the 5 mm container thickness in Table 1 are shown in Table 2. The symbols A1 and S1 in Table 2 correspond to the natural frequencies ω_{a1} and ω_{s1} , respectively. The wave height corresponding to the left edge, center, and right edge of the free surface were displayed as R.End, C.Sur, and L.End and the horizontal displacement for the right and left sides of the container at the free surface position were represented as R.Disp and L.Disp.

The vibration modes for all the natural frequencies in Table1 have R.Disp and L.Disp components shown in Table2, which are not negligible magnitudes comparing to the magnitudes of wave heights R.Ens and L.End. Then, when considering coupled behavior between the fluid and thin containers, natural frequency modes of container may be subject to the interactive effects.

3.3 Response of the container

The phenomenon of a container holding water and displaying an intense nonlinear response has already been reported in the Part 1²³⁾. The occurrence of the responses that corresponded to this type of responses were examined. For this model, a sine wave acts as the external force on ground acceleration of amplitude a_x . All response of the following numerical models are analyzed using the time interval $\Delta t = 0.005$ s. In addition, introducing damping terms into the system guarantees the stability of the numerical analysis. A critical damping ratio $h_c = h_\eta = 0.0075$ was adopted. The vibration frequency for the Rayleigh damping was set as $\omega_c = 0.6 \text{ rad/s}$, $\omega_\eta = 5.76 \text{ rad/s}$.

3.3.1 Small-amplitude response

When the wave height and the container displacement are small under a periodic external force, the system response is mainly a linear vibration. The time history response for a sine wave external force frequency of $\omega = 65 \text{ rad/s}$ and a horizontal acceleration of $a_x = 5.5 \text{ m/s}^2$

were resolved into each mode response, which is described in the Part1. The responses obtained for the antisymmetric mode in the periods $t = 0\text{--}200\text{ s}$ and $t=198\text{--}200\text{ s}$ are shown in Figs. 2.1 and 2.2. The symbol A_k represents the response of the generalized coordinates ζ_{ak} corresponding to the k -th antisymmetric vibration mode, which was shown in section 4.1.2 in the Part1. After the transient response fades, a linear vibration response for the damped system can be observed in which it converges to the periodic response of the fundamental vibration. Fig. 3 shows the Fourier spectra for the response resolved into the A1,A2,and A3 modes

Fig.1 Model Mesh and Reference Points

Table1 Natural Frequencies(8x16M)

記号	5mm	10mm	30mm	Rigid
ω_{a1}	5.40	5.74	5.78	5.78
ω_{s1}	7.05	7.18	7.19	7.20
ω_{s2}	7.96	8.32	8.35	8.36
ω_{a2}	9.92	10.30	10.34	10.35
ω_{s3}	11.66	12.09	12.13	12.14
ω_{a3}	13.28	13.79	13.83	13.85
ω_{s4}	14.86	15.48	15.53	15.55
ω_{a4}	16.39	17.20	17.25	17.28
ω_{s5}	17.97	18.99	19.05	19.07
ω_{a5}	19.42	20.85	20.93	20.95
ω_{s6}	21.07	22.79	22.88	22.92
ω_{a6}	22.55	24.78	24.90	24.93
ω_{s7}	24.39	26.75	26.89	26.93
ω_{a7}	26.08	28.57	28.75	28.79
ω_{s8}	28.04	30.10	30.31	30.35
ω_{a8}	29.69	31.14	31.35	31.39
ω_{s9}	31.14	31.63	31.72	31.77
ω_{a9}	33.19	49.70	154.3	—
ω_{s10}	33.52	50.92	155.8	—
only Container				
ω_{ocl}	26.48	52.96	158.9	—

Table2 Natural Frequencies and Key Value of Vibration Modes($h=5\text{mm}$)

notation	A1	S1	S2	A2	S3	A3	S7	A7	S8	A8	S9	A9	S10
Frequency	5.40	7.05	7.96	9.92	11.66	13.28	24.39	26.08	28.04	29.69	31.14	33.19	33.52
R.End	0.505	0.427	-0.462	-0.469	0.458	-0.443	0.038	-0.092	0.112	0.166	-0.114	-0.479	0.473
C.Sur.	0.000	0.174	0.302	0.000	0.287	0.000	0.341	0.000	0.363	0.000	0.391	0.000	0.042
L.End	-0.505	0.427	-0.462	0.469	0.458	0.443	0.038	0.092	0.112	-0.166	-0.114	0.473	0.479
R.Disp.	0.032	0.023	-0.027	-0.027	0.026	-0.027	0.026	0.020	-0.016	-0.012	0.008	0.019	-0.021
L.Disp	0.032	-0.023	0.027	-0.027	-0.026	-0.027	-0.026	0.020	0.016	-0.012	-0.008	0.019	0.022

(which used 1024 response datapoints in the vicinity of the time period $t = 190\text{--}200\text{ s}$ for a 0.01 s time interval). The response shown is simply the periodic response for the spectrum of the fundamental vibration. This spectrum for the A1 mode is 1.44 mm . The spectrum of the symmetric vibration mode is shown in Fig. 4. Only the constant term is present and the vibration response components are not included.

Fig. 5 shows the Fourier spectrum of the container displacement in the horizontal direction at the initial free surface level. Here, the symbols R.Disp and L.Disp refer to the right and left sides of the container, respectively. These responses mainly have the spectra of the fundamental vibration (1.8 mm) and the constant term (0.6 mm), whereas that for the second harmonic vibration can be barely observed.

Fig. 6 shows the response curve over the period $t = 198\text{--}200\text{ s}$ near the time of 200 s for the difference bending moment (kNm/m) at the right and left sides of the container base. Here, the symbols R.Mbt and L.Mbt correspond to the right and left sides of the container, respectively. The extreme differential bending moment for the response near 200 s is around 46 Nm/m .

The Fourier spectrum for this difference moment response is shown in Fig. 7. The response spectrum is mainly the fundamental vibration while the second harmonic vibration component also occurs.

Fig.2.1 Time History Res. of AntiSym.Modes s

Fig.2.2 Time History Res. of AntiSym.Modes s

3.3.2 Subharmonic vibration response

The response for a model was examined assuming a sine wave external force frequency of $\omega = 65 \text{ rad/s}$ and a horizontal acceleration of $a_x = 8 \text{ m/s}^2$. The time history responses for the antisymmetric vibration modes A1 and A2 in the periods $t = 0-200 \text{ s}$ and $t = 198-200 \text{ s}$ are shown in Fig. 8.1 and Fig. 8.2, respectively. The response for the antisymmetric vibration mode A1 converges with time. On the other hand, although the response for the A2 mode of the oscillation persisted in a convergent manner up to about 10 s, it then increased with time. The time history responses of the antisymmetric vibration mode A9 and the symmetric vibration mode S10 are shown in Fig. 9. Although the response of these modes is similar to that of the above-mentioned A2, the growth rate is large after 10 s.

The Fourier spectra of the response corresponding to the antisymmetric vibration modes A1, A2, and A3 are shown in Fig. 10. The spectrum ω for the fundamental vibration of mode A1 is 4.05 mm. The spectra for the fundamental vibration of modes A2 and A3 are 4.5 mm and 3.6 mm, respectively. On the other hand, the 1/2 subharmonic vibration components for modes A1, A2, and A3 are present with amplitudes that are 2.8, 4.0, and 5.2, respectively, times higher than those for the fundamental vibration components.

The Fourier spectra corresponding to the symmetric vibration mode responses S1, S2 and S3 are shown in Fig. 11. There is also a branching response into the 1/2 subharmonic vibration component with these modes as well. The constant-term spectrum has also increased. However, the spectra for the fundamental vibration component corresponding to these modes cannot be seen. Fig. 12 shows the response spectra for the vibration modes A9 and S10. These spectra are mainly 1/2 subharmonic vibration component and the response increase with time (Fig. 9) is due to the subharmonic vibration component.

Fig. 13 shows the time history response near 200 s for the displacement of the container at the free surface level in the horizontal direction. Fig. 14 shows their Fourier spectra. Apart from the fundamental vibration, this response also has 1/2 subharmonic vibration component and a constant-term component. In addition, 3/2 subharmonic vibration component can be observed. The maximum displacement for the response of this time history response is 8.2 mm.

Fig. 15 shows the time history response near 200 s for the difference bending moment at the container base. The extreme value of the bending moment for this time interval is 270 kNm/m. This value is about 5.9 times that for the model given in the preceding section 3.3.1. The Fourier spectrum of this response is given in Fig. 16. The spectra occur

for the $1/2$ subharmonic vibration, fundamental vibration, $3/2$ subharmonic vibration, and second harmonic vibration.

The amplitude of the external acceleration of this model is about 1.5 times that the model in section 3.3.1. In this system, it branches out the $1/2$ harmonic vibration response and the wave height and container response grow rapidly. The spectrum of the fundamental vibration for the wave height indicates a 2.8-fold value of response, which is mainly mode A1. And maximum response values for the displacement and for the difference bending moment of the container base show around 3.4 times and around 5.9 times respectively.

The response of this system is in the process of increasing and may possibly increase further to transform into something else.

3.3.3 Almost periodic vibrational response

In the model with a sine wave external force frequency of $\omega = 59 \text{ rad/s}$ and a horizontal vibration acceleration of $a_x = 8 \text{ m/s}^2$, the vibration response is associated with the beat vibration of an essentially almost periodic vibration. The time history responses over 0–200 s for the antisymmetric vibration modes A1 and A2 are shown in Fig. 17. In addition, Fig. 18 shows the time history response for the similar antisymmetric mode A9 and the symmetric vibration mode S10. These responses, following a 30 s time history for the initial transient response, show a stable amplitude increase up to around 100 s, but thereafter, an unstable response up to around 180 s. Then, a renewed stable response can be seen, and the response was analyzed in the vicinity of 90–200 s.

First, the time response in the vicinity of 90 s is presented. The responses over the time history, $t = 88\text{--}90$ s, for the antisymmetric vibration modes A7, A8, A9, and A1 are given in Fig. 19. In addition, in Fig. 20, the same time history response is shown for the symmetric vibration modes S7, S8, and S10. A beat vibration is seen in both vibration mode responses. The Fourier spectra corresponding to these responses

are shown in Figs. 21 and 22, respectively.

The special features of the response spectra can then be illustrated by organizing these results.

- (1) The response for the antisymmetric vibration mode A9 possesses spectra for the vibration frequencies of 31.9, 27.0, 54.0, 4.9 rad/s, and so on. The vibration frequency of 31.9 rad/s is the maximum spectrum with a size of about 11 times the spectrum for the fundamental vibration of 58.9 rad/s. The spectrum for the vibration mode A9 has a large value compared with that for the other vibration modes.
- (2) Although the response for the symmetric vibration mode does not include the fundamental vibration component, it includes spectra for the vibration frequencies of 27.0, 31.9, 54.0, and 4.9 rad/s. The vibration frequency of 27.0 rad/s is the main spectrum. The vibration modes S8 and S10 have main spectra of approximately the same size.
- (3) The difference of 4.9 rad/s ($=31.9-27.0$) in the spectra of the predominant vibration frequencies for the antisymmetric and symmetric vibration modes is the vibration frequency of the beat vibration with an approximate period of 1.28 s ($=2\pi/4.9$) (Fig. 19 and Fig. 20). The vibration frequency of this beat vibration matches the principal spectrum for the antisymmetric mode A1.

The time history response over $t = 88\text{--}90$ s for the horizontal displacement of the container at the fluid free surface level is shown in Fig. 23. The maximum displacement in the range shown in this figure is about 10.7 mm. In addition, the Fourier spectrum for this response is shown in Fig. 24. The spectra for the vibration frequencies of 27.0 and 31.9 rad/s corresponding to the subharmonic vibrations show larger values than the spectrum for the fundamental vibration of 59 rad/s. The value of the constant spectrum is about 85% of that of the fundamental vibration.

Fig. 11 Spectra of Modes S1, S2, S3

Fig. 13 Time History Response of H. Disp.

Fig. 15 Time History Res. Δ Moment at Bottom

Fig. 12 Spectra of Modes A9, S10

Fig. 14 Spectra of H. Displacement

Fig. 16 Spectra of Δ Moment at Bottom

The response near 90 s for the difference bending moment response at the container base and its Fourier spectrum are shown in Figs. 25 and 26, respectively. The maximum value for the difference bending moment within the range shown in these figures is 267 kNm/m. This response includes the spectra of several harmonic vibrations apart from the subharmonic vibrations. The spectra have the fundamental vibration of 65 rad/s, the subharmonic vibrations of 31.9, 27.0, and 4.9 rad/s. Next, the response near 200 s was analyzed.

The time history responses for the antisymmetric vibration modes A7, A8, and A9 and the symmetric vibration modes S8, S9, and S10 are shown in Figs. 27 and 28, respectively. A complex beat vibration response occurs. The Fourier spectrum was calculated using 1024 response datapoints for the period 189–200 s. The Fourier spectra for the antisymmetric and symmetric modes are shown in Figs. 29 and 30, respectively.

Compared with the Fourier spectra obtained from the response near 90 s (Fig. 21 and Fig. 22), a newly generated spectrum for a vibration frequency of 29.45 rad/s is seen to have increased in value. This spectrum is the 1/2 subharmonic vibration having half the vibration frequency of the external force. This subharmonic vibration has a large spectrum in the case of the antisymmetric vibration mode A8 and the symmetric vibration mode S9.

In addition, this vibration frequency is also the average of vibration frequencies 27.0 and 31.9 rad/s, which showed the spectra for the large subharmonic vibrations in Figs. 21 and 22, respectively.

The response near the 200 s time history for the horizontal displacement of the container at the fluid free surface level is shown in Fig. 31. The maximum displacement in the range shown in this figure is 14.9 mm. Fig. 32 shows the Fourier spectrum for this response. The spectra for the subharmonics show larger values than the spectrum for the fundamental vibration. Thus, there is a consistent tendency for the spectra corresponding to the subharmonic vibrations to show larger values than the spectrum for the fundamental vibration of 59 rad/s.

The difference bending moment response at the container base near 200 s and the Fourier spectrum are shown in Figs. 33 and 34, respectively. The extreme bending moment value is 328 kNm/m. This Fourier spectrum includes the spectra for several harmonic vibrations apart from the subharmonic vibrations. The main spectra are the fundamental vibration of 59 rad/s and the subharmonic vibration frequencies of 31.9, 27.0, 29.5, and 4.9 rad/s.

The values for the horizontal displacement of the container and the difference moment at the container base are 1.39 and 1.21 times larger than those for the maximum response near 90 s.

The system response accompanying the branching out into new subharmonic vibrations increases in amplitude and there is an increased instability in the system response. Even at the time of 200 s, the system response has not attained a steady state.

4. Domain for subharmonic vibrations and their frequency

The subharmonic vibration response is not included in the initial transient response. The response in which there is subharmonic vibration generated in this transient response is the typical nonlinear vibration response. The parameters involved in its generation are the vibration frequency of the external force, its amplitude, and the damping in the system. Varying the vibration frequency of the external force and its amplitude, the unstable domain was investigated for a transient response that includes a subharmonic vibration and also its vibration response.

4.1 Unstable domain for the initial response

In the comparatively high-vibration frequency domain of the sine wave external force for which the container response becomes large, the response up to 100 s was analyzed and established the unstable domain, in which subharmonic vibrations are generated (Fig. 35). The horizontal axis represents the vibration frequency of the external force

and the vertical axis shows the minimum external force acceleration amplitude to generate the subharmonic vibration. From this line, the domain with a large external force is unstable with responses from vibration components different to the transient response. In addition, a subharmonic vibration response is generated; however, in the domain with a small external force, a subharmonic vibrational response was not observed.

Next, with the frequency of the external force as a parameter for 52–70 rad/s, there was a subharmonic vibration response, and Fig. 36 shows the vibration frequencies of the antisymmetric and symmetric vibrational modes for the subharmonic vibrations obtained under external force amplitudes that gave a comparatively small response (0.2–0.3 m/s² was added to the minimum external force amplitude that generated a subharmonic vibration response). For each vibrational mode, the vibration modes that showed maximum amplitudes are also shown in Fig. 36 annotated with symbols (the principal antisymmetric and symmetric vibration modes in the unstable domain are given).

The horizontal axis is the frequency of the external force and the vertical axis is the vibration frequency of the subharmonic vibration response for each vibration mode.

4.2 Subharmonic vibration response

For the model with a sine wave external force frequency of 65 rad/s, the horizontal axis in Fig. 35 is 65 and for an acceleration of approximately 6.2 m/s², subharmonic vibration occurs. In section 3.3.2, the response was investigated with an external force acceleration of 8.0 m/s². In this model, for the generated subharmonic vibration response, both the antisymmetric and symmetric vibration modes had 1/2 subharmonic vibration component. In the vibration frequency domain for the external force frequency of 52–55 rad/s and 63–70 rad/s, the response branches out directly into the 1/2 subharmonic vibration component.

On the other hand, in the subharmonic vibration response for the model with an external force frequency of 59 rad/s, the antisymmetric vibration mode had a frequency of 31.9 rad/s and the symmetric vibrational mode had a frequency of 27.0 rad/s. For the external force frequency of 53–62 rad/s, the generated subharmonic vibration responses have different frequencies depending on the antisymmetric or symmetric modes.

From Fig. 36 of the vibration frequency domain for the external force, in which the unstable domain occurs, and the diagram of the unstable domain itself in Fig. 35, in the region in which this unstable domain occurs, the unstable domain is generated under small external force amplitudes.

The principal antisymmetric vibration mode in this vibration frequency domain is A9. In addition, the principal symmetric modes for external force frequencies of 56–57 rad/s and 58–61 rad/s are S8 and S10, respectively. For each external force frequency, the sum of the frequencies of the subharmonic vibration responses for the antisymmetric and symmetric modes matches the frequency of the external force. This response is one example of the summed combination resonance that forms a nonlinear vibrational unstable domain.

The principal spectra of the subharmonic vibrations in this domain have frequencies that increase or decrease by about 2.45 rad/s compared with the 1/2 subharmonic vibration component. Because the difference in these vibration frequencies is 4.9 rad/s, the response is seen as a beat vibration with an almost periodic response of about 1.28 s.

The response of this unstable domain grew in amplitude and 1/2 subharmonic vibrations occurred. This time it almost has a periodic response of about 2.56 s because the difference in the vibration frequencies is 2.45 rad/s. That is, comparing the response after branching out into the 1/2 subharmonic vibration component (Fig. 27 and Fig. 28) with the response before this vibration component occurred (Fig. 19 and Fig. 20), the beat period is twice the length.

From these figures, a response is seen that is a mixture of two beats with periods 1.28 and 2.56 s.

Conclusion

1. The natural frequency of sloshing vibration is generally little affected by rigidity of container. However, when a thickness of container is

thin, a fluid has considerable and important effects on the natural frequency and corresponding vibration mode of the container.

2. Resolving this system response into antisymmetric and symmetric modes with regard to the bisector of the container, responses of the antisymmetric vibration modes, apart from the spectrum for the fundamental vibration, also has spectra for the odd-order harmonic vibrations. On the other hand, the response of the symmetric vibration modes possesses even-order harmonic vibrations and a drift spectrum.
3. Under a given external force frequency region and external force amplitude, it was shown that subharmonic vibrations occurred and grew into a large response. Furthermore, it was revealed that there are two distinct types for this typical nonlinear vibration response.
4. One type is the nonlinear vibration response⁽⁵⁾ with principal components that are the 1/2 subharmonic vibration component of the external force frequency. With this response, for both symmetric and antisymmetric vibration modes, the principal spectra are the 1/2 subharmonic vibration components, in which the spectra for the $k/2$ subharmonic vibration components (for k odd) are also present.
5. For the other type, there is a disparity between the predominant frequency ω_a of the subharmonic vibration components for the antisymmetric vibration mode and the predominant frequency ω_s for the symmetric vibration components. The sum $\omega_a + \omega_s$ is simply the external vibration frequency, in other words, the fundamental vibration. In this case, the difference $\omega_a - \omega_s$ is the value near the minimum natural frequency ω_{a1} for the antisymmetric modes, which corresponds to the fundamental sloshing vibration mode. Moreover, for this type of subharmonic vibration response, it was shown that branching out into the 1/2 subharmonic vibration response of foregoing item 5 occurred, which is another beat response of these two types of subharmonics. To the best of the author's

Fig.35 Frequency of Base Acceleration rad/sec

Fig.36 Frequency of Base Acceleration rad/sec

knowledge, no report exists concerning this kind of nonlinear response occurring in fluid-containing tanks.

6. The solving method formulated here enables numerical analyses, and then, computation of the nonlinear vibration response in the dynamic interaction problem of a potential fluid and its container. It was demonstrated that this numerical analysis method can be applied to a model that branches out to subharmonic vibration responses. The method is also applicable to get response of tanks under seismic force.
7. The solving method shown in this paper is applicable for analyzing interactive behavior of the potential fluid and the container in a three-dimensional cylinder tank²⁸⁻³⁰⁾ under any external force. Then, this enables us to get seismic response of the cylindrical tank.

Acknowledgments

By setting up a shaking table and performing vibration experiments with a tank, an interaction response was observed that cannot be explained by any time theory. I constructed a functional that incorporates the deformation of the fluid field associated with the elastic displacements of the container. However, for more than 29 years ago, I could not have enough confidence about solving this functional with numerical analysis methods. I am indebted to Dr. Shimomai for his effort to discretize the equation during this time. Many students and graduates were involved in the experiments and participated in the search for the features of this nonlinear response. Their devoted and continued efforts have been a motivation to conduct research to elucidate this response from a theoretical aspect. I extend my sincerest thanks to them.

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和文要約

1. はじめに

Euler の圧力式を容器の弾性変形後の流体領域で積分して Hamilton 関数に取り込むと、この連成問題の厳密な汎関数となることを示した。この汎関数を用いると、容器が変形し、流体場も変形するこの連成問題の本質的な非線形応答を解析することが可能となる。二次元問題の定式化、および基本的な数値解析に引き続き、本論文では周期外力の下で容器に発生する動的不安定問題、分数調波振動応答を解析し、典型的な非線形振動の特徴を示し、提案する解法の有用性を検証する。

2. ポテンシャル流体と容器の動的相互作用の汎関数

ポテンシャル流体を入れた容器が載っている基盤が水平方向にある速度で移動する系をモデルとする。基盤に立つ観測者が観察する流体に速度ポテンシャルを導入する。流体運動を測定する遷移座標系の移動速度 $\dot{\mathbf{y}}$ を導入した二次元の汎関数を(1)式に示す。

3. 数値解析

流体の速度ポテンシャルは有意でない一様膨張の解を持つ。これを避けて、流体と容器の連成系の固有振動数を解析する方法を示す。

3.1 数値解析モデル

水高 0.6m、幅 0.9m、および容器高さ 0.99m を解析モデル (Fig.2) とする。この流体領域を 8×16 の流体要素メッシュにモデル化した。容器は板厚 5mm, 10mm, および 30mm の直線材を 6 要素に分割し、合計 12 要素を利用した。容器の最小固有振動数に対して臨界減衰比 0.0075 の減衰を導入し、応答を計算する。

3.2 固有振動数の数値解析

容器の剛性（板厚）をパラメータとした系の固有振動数を Table 1 に示す。板厚 30mm の容器と剛な容器のスロッシング振動の固有振動数はほぼ同一である。板厚 10mm のスロッシング振動の固有振動数は、ほぼ板厚 30mm のものと一致する。板厚 30mm, および 10mm モデルの容器の最小固有振動数は容器のみの振動数と比較してそれぞれ 3% および 9% 低下する。

板厚 5mm の容器の固有振動数はスロッシング振動とバルジング振動の区別は困難となる。すべてのスロッシング固有振動数に容器の剛性が影響し、固有振動数の値に大きな変化を与えないものの、すべての固有振動モードに容器の変形の成分が発生し、流体が板厚の薄い容器の振動に無視できない影響を与える。

3.3 容器の応答

応答スペクトルは原則として時刻歴 190-200 秒間の時間刻み 0.005 秒の 1024 個のデータを用いて算出した。

3.3.1 小振幅の応答

外力振動数 65rad/s, 外力加速度 $a_x=5.5\text{m/s}^2$ を作用させた系の時刻歴応答を Fig.2 に示す。対称モードと逆対称モードの応答スペクトルを Fig.3, Fig.4 に示す。応答は逆対称モードのみであり、応答は基本振動成分（外力と同一の振動数を持つ成分）のみである。これらの応答は線形振動理論で説明できる。

3.3.2 分数調波振動応答

外力振動数 65rad/s, 外力加速度 $a_x=8\text{m/s}^2$ を作用させた系の時刻歴応答を Fig.8, および Fig.9 に示す。このモデルは時刻 10 秒

近くまで、過渡応答は振幅が減少するのであるが、その後応答が増加し始める。時刻 200 秒も増加の過程にある。この系の A9, および S10 の時刻歴応答を Fig.9 に示す。逆対称モードの応答 Fig.10 に 1/2 分数調波振動応答が分岐する。この応答の振幅は基本振動成分のそれの 3-5 倍に達する。対称モードの応答 Fig.11 にも 1/2 分数調波振動応答が分岐するものの。基本振動成分の応答はほとんど観測されない。容器の水平変位、および容器脚部の差分モーメントも基本振動成分の振幅より大きな 1/2 分数調波振動応答が出現し、同時に 3/2 倍分数調波振動、および 2 倍高調波振動の応答も出現する。

3.3.3 概周期応答

外力振動数 59rad/s, 外力加速度 $a_x=8\text{m/s}^2$ を作用させた系の時刻歴応答を Fig.17, および Fig.19 に示す。これらの応答は、時刻 90sec 近傍、および時刻 200sec 近傍で応答を分析する。

逆対称モード、および対称モードの応答スペクトル (時刻歴応答 80-90 秒の時間刻み 0.01 秒の 1024 個のデータから算出した) をそれぞれ Fig.21, および Fig.22 に示す。逆対称モードは 31.9rad/s が卓越する応答を示す。一方、対称モードの応答スペクトルは 27.0rad/s が卓越し、この最大 1/2 程度となる 27.0rad/s のスペクトルが発生する。両方のモードの卓越スペクトルの振動数の和は外力振動数となる和型の分数調波振動が出現する。これらの差 ($31.9-27.0=4.9$) の振動数成分は最低次のスロッシング振動のスペクトルと符合する。近接する二つの振動数を有するので、時刻歴応答はうなり振動、概周期振動を示す。

次に、時刻歴応答 190-200 秒の特徴を示す。この時間帯の応答もうなり振動応答を示す (Fig. 27, Fig.28)。逆対称モード、および対称モードの応答スペクトルを Fig.29, および Fig.30 に示す。これらの応答スペクトルは、それぞれ Fig.21, および Fig.22 のスペクトルに振動数 29.5rad/s の振動成分、すなわち 1/2 分数調波振動成分を新たに追加したようなスペクトルとなる。これらの分数調波振動の分岐に伴い、容器の変位、および応力も増大する。

4. 分数調波振動が分岐する不安定領域

4.1 初期定常解の不安定領域

時刻歴 100 秒まで、時間刻み 0.005 秒の応答に基づいて、外力振動数と外力振幅をパラメータとする領域において、分数調波振動応答が発生する領域 (Fig.35) を定めた。

4.2 分数調波振動応答

分数調波振動が分岐する外力の振動数、および加速度振幅の不安定領域において、1/2 分数調波振動応答、および和型の分数調波振動が出現する領域を Fig.36 に示した。和型の分数調波振動が出現する領域では、逆対称振動モードと対称振動モードが異なる振動数を有する分数調波振動として、分岐する。これらの振動数の差はスロッシング振動の卓越振動数と一致する。

5. 結論

流体と容器の連成系の動的問題を数値解析し、容器に発生する代表的な非線形振動応答の特徴を明らかにした。この解析法は流体と容器の連成に起因する応答を解析するのに有用であることを示した。この解法は地震応答に利用できる。

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