THE MINIMUM WEIGHT DESIGN OF A CLASS OF TALL MULTI-STORY FRAMES SUBJECTED TO LARGE LATERAL FORCES-II

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Introduction

The second part of the present paper presents several particular classes of the minimum weight designs subjected to various restrictions. A simplified and practically useful solution is obtained in Sec. 6. As an example, a 30-story frame of two bays with equal span length is designed for trapezoidally distributed lateral loads.

4. A class of the minimum weight designs of the frames such that $2h_1 \leq l_m$.

In view of the fact that the Foulkes' mechanism condition has been violated only in the first story columns and that ψ_1 is large enough to eliminate θ_1 , it seems appropriate to modify the previous mechanism as shown in Fig. 6, where θ_1 does not appear. ($\theta_j, j \geq 2$ are not shown in Fig. 6) The pattern of the mechanism shown in Fig. 5 (d) is now considered only for that part of the frame which excludes the first story columns. The equal angle of rotation in the plastic hinges formed at all the lower end sections of the second story columns will be denoted by ψ_2 . The equal angle of rotation of all the joints except the first story joints is equal to $\psi_1 + \psi_2$. Corresponding to these modifications, a collapse mechanism of beam type will be necessary for the first story beam with the minimum span length l_m . This angle of rotation is denoted by ϕ_m . The sums of hinge rotations for the members of the frame may be tabulated as shown in Table 2.

The condition [A] may be satisfied if the angles of rotations are determined as follows.

$$\begin{aligned}\psi_1 &= h_1 \theta, & \psi_2 &= (l_m/2 - h_1) \theta, \\ \phi_m &= (l_m/2 - h_1) \theta, & \phi_k &= (l_k/2 - h_1) \theta, & \phi_k &= (1/2)(l_k - l_m) \theta, \\ \theta_j &= (h_j/2) \theta, & \theta_2 &= (1/2)(h_1 + h_2 - l_m/2) \theta.\end{aligned}$$

Since all the angles of hinge rotation must be non-negative, the condition [A] can be satisfied by the frames with those span lengths and story heights which satisfy

Fig. 6

Fig. 7

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TABLE 2

Member	Plastic moment	Length	Subscripts $j; k$	Sum of hinge rotations
$(1, m)$ -beam	$B_{1, m}$	l_m		$2 \phi_m + 2 \psi_1$
$(1, k)$ -beam	$B_{1, k}$	l_k	$; k \neq m$	$2 \phi_k + 2 \psi_1$
(j, m) -beam	$B_{j, m}$	l_m	$2, \dots, f;$	$2 \psi_1 + 2 \psi_2$
(j, k) -beam	$B_{j, k}$	l_k	$2, \dots, f; k \neq m$	$2 \phi_k + 2(\psi_1 + \psi_2)$
$(1, k)$ -column	$C_{1, k}$	h_1	$; 1, 2, \dots, s+1$	ψ_1
$(2, k)$ -column	$C_{2, k}$	h_2	$; 1, 2, \dots, s+1$	$2 \theta_2 + \psi_2$
(j, k) -column	$C_{j, k}$	h_k	$3, \dots, f; 1, 2, \dots, s+1$	$2 \theta_j$

$$\left. \begin{array}{l} 2l_m \geq l_k \geq l_m, \quad 4h_1 \geq l_k, \\ 2(h_1+h_2), \quad 4h_1 \geq l_m \geq 2h_1. \end{array} \right\} \dots\dots\dots (13)$$

Corresponding to this modified overcomplete collapse mechanism, the design must be modified as follows. First, the general procedure of the previous section may be applied only to that part of the frame which excludes the first story beams and columns. Secondly, all the first story beams shall be so designed as to collapse under the vertical loads with plastic hinges formed at their midspans and both ends. Thirdly, the first story columns may be designed almost in any manner to carry the shear resultant so far as the assigned plastic moment of each column exceeds the moment at its top end in conformity with the direction of the hinge rotation. This is due to the fact that the first story columns have the same length and that the weight function is linear.

Fig. 8

For a frame of single span, the frame moments are uniquely determined only by (3) and the solution for this case will be included in Sec. 6.

The regions of l_m and l_k for which the minimum weight designs have been obtained, may be represented as Fig. 8. The region R_1 is defined by the restriction (12), whereas R_2 by (13). In view of the restrictions in terms of h_1 and h_2 , it may be well expected that a similar modification may be extended to the remaining region of $l_k \geq l_m > 0$.

5. Frames with equal span length.

Consider a special class of frames such that $l_k=l$ for all k . The linearized weight function (5) may then be written as

$$G = g \left[\sum_{j=1}^f 2 h_j \left(\sum_{k=1}^s M_{j,k} \right) + l \left(\sum_{k=1}^s M_{1,k} \right) + 2 l \sum_{i=2}^f \left(\sum_{k=1}^s M_{j,k} \right) \right] \dots \dots \dots (14 \text{ a})$$

Substitution of (3) into (14 a) furnishes

$$G = g \left[\sum_{j=1}^f (1/2) h_j^2 \sum_{i=1}^f P_i + (1/4) l h_1 \sum_{i=1}^f P_i + (1/2) l \sum_{i=2}^f h_j \sum_{i=1}^f P_i \right] \dots (14 \text{ b})$$

In other words, the weight is uniquely determined only by (3), without regard to the other constraints (4). This implies the following geometrical significance. The weight hy perplane is parallel to the intersection of f hyperplanes defined by (3). The constraints (4) truncate this in tersection. G achieves the minimum weight when it coincides with this intersection bounded by (4). Unless this bounded intersection is a point, any point therein represents a minimum weight design. Even though G is unique, there exist many minimum weight designs. The corresponding overcomplete collapse mode need not include the beam type mechanism such as shown in Fig. 5c. The Foulkes' mechanism condition may readily be examined for both of the case $2h_1 \geq l$ and the case $2h_1 \leq l$. Included in this class of designs is the special case where

$$M_{j,k}=M_j \text{ for all } k \text{ with the same } j.$$

satisfying all the constraints (4). This is a design such that all the beams and all the internal columns in a story have the same plastic moments, respectively and that the two external columns in a story

have half the plastic moment of the internal columns in the story.

6. Frames such that $M_{j,k}=M_j$ for all k with the same j .

The aforementioned simple example leads us to the special class of designs for the case of unequal span lengths such that

$$M_{j,k}=M_j \text{ for all } k \text{ with the same } j \dots\dots\dots(15)$$

This implies that all the beams in a story have to be assigned the same plastic moment and altogether form a large total span with the length defined by $L=\sum_{k=1}^s l_k$, and that, in view of eqs. (1), the plastic moment of an internal column in a story must be twice as large as that of the external columns in the story. In view of (15), the equations of sway equilibrium (3) may be written as

$$M_j=(h_j/4s)\sum_{i=j}^f P_i \dots\dots\dots(16)$$

Substitution of eq. (16) into the weight function (5) provides

$$G=g[(1/2)\sum_{j=1}^f h_j^2 \sum_{i=j}^f P_i + (L/2s)\sum_{i=1}^f P_i + 2\sum_{j=2}^f h_j \sum_{i=j}^f P_i]$$

Thus G is uniquely determined only by eq. (16) as far as the constraints (4) are all satisfied. The corresponding plastic moments are given by

$$\left. \begin{aligned} B_j &= (1/4s)(h_j \sum_{i=j}^f P_i + h_{j+1} \sum_{i=j+1}^f P_i) \\ C_j &= (1/2s)h_j \sum_{i=j}^f P_i, \quad \tilde{C}_j = (1/4s)h_j \sum_{i=j}^f P_i \end{aligned} \right\} \dots\dots\dots(17)$$

where B_j, C_j and $\tilde{C}_j$ denote the plastic moments of the j -th story beam, internal column and external column, respectively.

Since there are only three different plastic moments in a story, the $3f$ sums of hinge rotations for the $3f$ spans must satisfy the Foulkes' mechanism condition. These sums are tabulated in TABLE 3.

TABLE 3

Member	Plastic moment	Length	Sum of hinge rotations
Beam	B_j	L	$2s\psi_1$
Internal column	C_j	$(s-1)h_j$	$2(s-1)\theta_j$
	C_1	$(s-1)h_1$	$(s-1)(2\theta_1+\psi_1)$
External column	$\tilde{C}_j$	$2h_j$	$4\theta_j$
	$\tilde{C}_1$	$2h_1$	$4\theta_1+2\psi_1$

The Foulkes' condition can be satisfied if we determine ψ_1 and θ_j as follows.

$$\psi_1=(L/2s)\theta, \quad \theta_j=(1/2)h_j\theta, \quad \theta_1=(h_1/2-L/4s)\theta \dots\dots\dots(18)$$

In view of the limiting value of θ_1 , the present design gives the minimum weight for the frames satisfying

$$2h_1 \geq L/s \dots\dots\dots(19)$$

This condition corresponds to the previous limitation (12). A slight modification is necessary for those frames in which the restriction (19) is violated. The same technique as in Section 4 may be employed again to obtain the modified mechanism shown in Fig. 7. The symbols similar to those used in Fig. 6 are employed here in Fig. 7. Each first story beam carries a different vertical load and has the limiting smallest value to which its plastic moment may be reduced in order to achieve the minimum weight. Now that all the first story beams are to be assigned the same plastic moment here, the actual limitation is given by the largest value among all these limiting smallest values. As soon as the plastic moment of the first story beam is reduced to this largest value, there exists at least one beam at whose midspan a plastic hinge may form. Let $\bar{s}$ be the number of beams of the first story at whose midspans plastic hinges may form. Let this angle of hinge rotation be $2\phi_1$. The sums of hinge rotations of the members may then be tabulated as in Table 4.

TABLE 4

Member	Plastic moment	Length	Sum of hinge rotations
Beam	B_j	L	$2s(\psi_1 + \psi_2)$
	B_1	L	$2s\psi_1 + 2\bar{s}\phi_1$
Internal column	C_j	$(s-1)h_j$	$2(s-1)\theta_j$
	C_2	$(s-1)h_2$	$(s-1)(\psi_2 + 2\theta_2)$
	C_1	$(s-1)h_1$	$(s-1)\psi_1$
External column	$\tilde{C}_j$	$2h_j$	$2\theta_j$
	$\tilde{C}_2$	$2h_2$	$2\psi_2 + 4\theta_2$
	$\tilde{C}_1$	$2h_1$	$2\psi_1$

In order that the Foulkes' condition be satisfied, ψ_1, ψ_2, ϕ_1 and θ_j must be given by

$$\left. \begin{aligned} \psi_1 &= h_1 \theta, & \psi_2 &= (L/2s - h_1) \theta, & \phi_1 &= (1/\bar{s})(L/2 - sh_1) \theta, \\ \theta_j &= (1/2)h_j \theta, & \theta_2 &= (1/2)(h_1 + h_2 - L/2s) \theta. \end{aligned} \right\} \dots\dots\dots (20)$$

All the hinge rotations are non-negative for the frames satisfying the conditions

$$2(h_1 + h_2) \geq L/s \geq 2h_1, \quad 2(s + \bar{s})h_1/s \geq L/s \dots\dots\dots (21)$$

These are the restrictions corresponding to (13).

7. Example: 30-story frame of 2 bays.

A 30-story frame of two bays with equal span length carrying equal vertical loads shown in Fig. 9b is designed for the lateral forces distributed as shown in Fig. 9a, where the base shear coefficient

FIG. 9 THE MINIMUM WEIGHT DESIGN OF A 30-STORY FRAME OF TWO EQUAL SPANS

is 0.10. The plastic moments obtained from the general solution (17) are tabulated in Fig. 9. The numbers with asterisks in the plastic moments indicate the modified design of the topmost story members by means of plastic moment distribution. In order to restrict the modification as locally as possible, plastic moment distribution has been carried out under the constraint that the lower end section of the internal column of the topmost story is to be assigned the plastic moment twice as much as that of an external column.

8. Concluding remarks.

The present paper has established the minimum weight design of a broad class of tall multi-story frames of practical interest in an extremely simple and explicit form. The theoretical solution serves to clarify the general features of the minimum weight designs of regular multi-story frames subjected to comparatively large lateral shear. It should be remarked that all the beams except those with the minimum span length are designed to carry the vertical loads in their respective spans, and that the corresponding bays contribute little to supporting the lateral shear. The bays with the minimum span length must then be designed to carry the remaining part of the lateral shear resultant. This result appears to suggest, from economic point of view, the use of shear walls or bracings. It is however noted that in certain cases the present solution results in columns of zero plastic moments. Even if the columns are to be augmented by taking into account the effect of axial forces, this theoretical solution may in some cases appear to be unpractical in view of the elastic-plastic instability just prior to collapse. Hence the paper has explored some special classes of restricted minimum weight designs.

The present design must next be modified for the effect of axial forces within the simple plastic theory. The simple plastic design so constructed must then be augmented against the effect of additional moments induced by the large elastic-plastic deformation under the presence of large axial forces. Then the ultimate strength design within the second order theory may be established. This approach has been suggested by the present authors in Ref.[15] recently and appears to be practically useful in its simplicity and directness.

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大きな水平力をうける1クラスの高層架構の最小重量設計-II

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序 本論(II)は種々の制限条件の下における特別なクラスの最小重量設計の解を提供する。簡潔かつ實際上便利な解が6で得られる。例として、2等長スパンの30層架構が台形分布の水平力に対して設計される。

4. $2h_1 \leq l_m$ の場合の最小重量設計 前節において Foulkes の機構条件が1階柱においてのみ満足されず、 ψ_1 が θ_1 を必要としない程度に小でなければならなかった。従って前記過完全崩壊機構を、最下層で θ_1 のない図6の如き機構に修正する。この場合前の ψ_1 の役割を演ずるように2階柱脚に ψ_2 の sway 用の関節回転角を θ_2 の他に与え、2階以上の節点回転角を $\psi_1 + \psi_2$ とする。対応して1階の l_m 長さの梁の中央にも関節形成を考える必要がある。このように修正すれば、各部材毎の関節回転角の和は、表2で与えられ、条件[A]は(13)式の満足されるクラスの架構について満足させ得る。かような修正機構に対応して、塑性モーメントの割当ても次のように修正されねばならない。第1に前節の一般解形式は1階梁、柱以外についてのみ適用される。第2に、 l_m も含めてすべての1階梁は材端及びスパン中央において関節が形成されるように設計され、更に1階柱は柱脚にのみ関節が形成されるようにかつ層せん断力に対抗し得る限り任意に設計されてよい。このようにして最小重量設計が得られた範囲を図示すると図8の斜線部分となる。但し R_1 は(12)の制限による範囲、 R_2 は(13)による範囲で、 R_2 は鉛直点線で示される制限線で削られる事もある。 h_1, h_2 に対する制限から見て、これ以外のクラスの架構についても類似の修正によって、解の得られる領域は逐次拡張されよう。

5. 等張間の架構 すべての k について $l_k = l$ であるような特別な架構を考える。この時 G は(14a)式で表現され、(3)を(14a)に代入すると(14b)式のように、重量が(3)のみで決定されてしまう結果を得る。これは次のような幾何学的意味をもつ。即ち fs -空間において、重量超平面は(3)で定義される f 個の超平面の共通部分に平行であって、制約条件(4)はこの共通部分の許容範囲を定義する。 G はこの共通部分に一致する時に最小値に達する。この共通部分の許容範囲が点である場合以外は、その中の点で表現されるいかなる設計も同一の最小重量を与える。対応する過完全崩壊機構は図5c

の梁型関節を含む必要はない。Foulkes の機構条件は $2h_1 \geq l, 2h_1 \leq l$ のいずれの場合も容易に検討され得る。その設計の一つの特別な場合として各階毎に構モーメントが等しい場合がある。但し(4)は満足されねばならない。この時、各階毎に梁と内柱はそれぞれ同一の塑性モーメントの割当となり、外柱は内柱1/2の塑性モーメントの割当となる。

6. 各階毎に構モーメントが等しく割当てられた架構等張間でなくてかつ各階毎に構モーメントが等しいという制限条件を満足する架構を考える。これはとりもなおさず、前記の如く、各階毎に梁と、内柱とが、それぞれ同一の塑性モーメント、外柱は内柱の1/2の塑性モーメントを割当てるとして、重量最小化を考えることを意味する。即ち Foulkes の機構条件検討の際には、梁長は $L = \sum_{k=1}^s l_k$ であり、内柱は $(s-1)h_j$ 、外柱は $2h_j$ の長さを有するとして取扱わねばならないことになる。この場合、(16)式のように表現できる層方程式を(5)に代入すると、5節と同様、重量は、(4a)の満足されている荷重の組に対して、(16)のみで一意に決定される。対応する塑性モーメントは(17)与えられる。

各階について上記の如く3個の異なるスパンがあるから、 $3f$ 個のスパン長に対する $3f$ 個の関節回転角総和の比が Foulkes の機構条件を満足していなければならない。これらは表3に示される如くであって、(19)式を満足する架構については、(18)式のように条件[A]を満足させ得る。前記一般の場合に対する制限(12)に対応する場合の制限が(19)式である。

(19)式の満足されない架構では、4節で図6の修正機構を考えたのと同様に、この場合図7に示す修正機構を考える。しかし、1階の各梁は異なる鉛直荷重を支持せねばならず、従って塑性モーメントの限界最小値も異なる。所で1階の梁がすべて同一の塑性モーメントを割当てられるとすると、実際の最小値は、上記の1階各梁の限界最小値の中の最大値で与えられることになる。もし1階梁の等しい塑性モーメントが丁度この限界値に減少せしめられると、少くとも1個の梁のスパン中央に関節が形成されてよい状態になる。今このようにスパン中央に関節形成され得る1階梁スパン数が s 個あるとし、その関節回転角を $2\phi_1$ で表わせば、各部材毎の関節回転角和は表4で与えられる。従って(21)式で与えられる条件を満足する架構については(20)式の如くに Foulkes 条件

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を満足させ得る。この制限(21)は一般の場合の制限(13)と対応するものである。

7. 例：等スパン 30 層架構 図9に示す水平力と鉛直荷重を支持する2等スパン長の30層架構の最小重量設計を考える。前述の如く、一般解(10)式が軸力の考慮と弾塑性安定性の観点から必ずしも現実に受入れ得るものではないので、この場合に対して(17)式の解を採用すれば、図9右側に示すような塑性モーメントの割当てが得られる。*印を附した数値は、最上階で前記制限条件(4)が乱されるので、それより下層の修正を伴わないように塑性モーメント分配法で修正したものである。単純理論の範囲で、軸力を考慮して上の設計を修正する方法は、文献[15]にも述べたように、上記崩壊時のM分布に対応する軸力を求めて、各部材について曲げモーメントと軸力の両者が作用する時の降伏条件式に代入してその断面の塑性モーメントを新たに決定することである。

8. 結語 本論によって、實際上興味あるクラスの高層架構の最小重量計の一般解が非常に簡潔かつ explicit な形式で求められ、その性格、特徴が明らかにされ得た。注目すべきは、最も一般的な場合の解(10)の形式においては、最小スパン長以外の梁はそれぞれのスパンの鉛直荷重を主として支持するように、また最小スパン

長の梁とこれらに連結する柱は層せん断力の残りの部分を全部支持するように設計されることが要求される点にある。この結果は、水平力が各柱梁で支持されるよりは、集中的な水平力支持体を作る方が経済的であることを示していると思われる。しかしながらその結果、ある場合には塑性モーメント割当量が零の柱が、解として得られるし、一般に柱の設計が極端に不釣合になるようである。たとえ、単純理論の範囲内で軸方向力に対して修正するとしても、なお極端に異なる柱設計は非実際的と見られる。そこで本論のⅡでは、このような不釣合な設計とならないような制限付最小重量設計の解(17)式を求めたのである。これに対する計算例としての30層架構の設計には従って上記の困難点は現れていない。

実際の設計上は、(17)式の解も、前述の如く、まず第1に単純理論の範囲内で、降伏条件に軸方向力を考慮して修正されねばならない。更にこのような単純理論による設計は、崩壊点近傍における大きな層変位と、大きな軸方向力によって生ずる附加モーメントの影響に対しても十分増強されねばならない。かくて初めて二次理論における終局強度設計が確立され得る。このようなアプローチは、既に筆者が文献[15]で最近提唱した所である。

curvature integration is readily accomplished by electronic digital computer and be applicable to any practical section (Proceedings)

U.D.C. 624.042.7 : 624.97

DYNAMIC CHARACTERISTICS OF STACK-LIKE STRUCTURES

by HIROYOSHI KOBAYASHI, Assistant Prof., Tokyo Institute of Technology Member of A.I.J., HAJIME HIRAKAWA, Graduate Student, Tokyo Institute of Technology Member of A.I.J.

Authors studied a calculation method on tapered stack, to obtain the natural frequencies and mode shapes of bending vibration, of which cross section varies freely in height, and this paper is the first report of a series of the dynamic response analysis of stack-like structures due to earthquake.

They carried out the analysis, regarding the stack as a continuous bending vibration system, not as a concentrated mass system, by numerical computation. To simplify the analysis, they assumed as follows;

- The stack is resisted only by the bending moment and there are no effects of gravity and shearing deformation.
- The stack is rigidly fixed at base.
- The stack has a constant cross section in each divided part.

With these assumptions, the equations of natural frequencies and mode shapes of a stack are defined by the following equations;

$$G \cdot \bar{H} = \bar{H} / w^2$$

where G , $\bar{H}$ and w represent the matrix of coefficients, the column matrices of mode shape and the natural frequency respectively.

These results are compared with the results of Stodola's method, Mononobe's method, Naruoka's method and Ishizaki's experiments. All these results (shown in Table 4) show that natural frequencies of stacks will be computed well approximately, for practical purposes, by this method.

In this study they used the digital computer FACOM 222, Tokyo Institute of Technology.

U.D.C. 624.042.7 : 624.043

THE MINIMUM WEIGHT DESIGN OF A CLASS OF TALL MULTI-STORY FRAMES SUBJECTED TO LARGE LATERAL LOADS (II) (See Page 37)

by Dr. RYO TANABASHI, Prof. of Architecture Kyoto University, Kyoto, TSUNEYOSHI NAKAMURA, Assistant Prof. of Architecture, Kyoto University Members of A.I.J.

U.D.C. 711.433

STUDIES ON THE SIZE OF CITY (4)

by TAKAMASA YOSIZAKA, Prof. of Waseda Univ., KOITI TONUMA Research Student of Waseda Univ. Members of A.I.J.

This report is the fourth part of the series of studies on the size of city.

In this report, we take up the third Block which is enclosed by main trunk line of road, and public space (school, hospital, green, government office, factory, traffic, commercial zone)

Through the studies on the size of city (1), (2), (3), (4), finally we have classified each physical structure of cities Japan, and made model type of them basing on potentiality or strength of city.

U.D.C. 628.1.001 : 711.11 : 333.32

DISTRIBUTIONS OF DWELLING-UNIT-SPACE (3)

—On distribution type of living spaces and its changing trend—

by TSUNESHI MIWA Member of J.H.C., Member of A.I.J.

By the data from census and housing survey, the distribution type of living spaces well fits in Gibrat distribution (lognormal distribution).

This report is about the "r" (deviation) and the "m" (mean) of Gibrat distribution on the number of tatami (living space) in dwellings and households.