Limit Analysis of Beam-Column Connections (I-1)

(Cruciform Connections of Wideflange Members Subjected to Strong-axis Bending)

by Dr. Hisashi TANAKA*, Akio FUKUSHIMA**, Yung Chuan LIN***.
 Members of A.I.J.

§1 Introduction

It has been reported that the ultimate strength of the beam-column connections consisting of wide-flange members depends upon shear-yielding in the connection-web and theoretical analysis gives satisfactory results under the assumptions that the normal stresses caused by bending moment are entirely carried by the flanges and that the shear stresses all carried by the web^{1)~4)}.

However, these assumptions seem to be not always true because the ultimate strength of the connection is largely affected by the ratios of the sectional area of the flange to that of the web, the depth of the column to that of the beam, and the web-thickness of the connection to that of the members^{5), 6)}.

This paper presents a theoretical analysis for the determination of the yield moment of the cruciformly-welded connection consisting of wide-flange members subjected to the anti-symmetrical bending, as shown in Fig.-1, and also clarifies how the yield moment is affected by the proportions of the connection.

Fig.-1 Type and Dimensions of Beam-Column Connection

§2 Assumptions of the analysis

The object of the analysis is to obtain the yield moment of the cruciform connection subjected to the anti-symmetrical bending. The connections consist of the wide-flange members with the beam-depth of $2h_b$, the column-depth of $2h_c$, the sectional area of beam-flange A_{fb} , that of column-flange A_{fc} , and the web-thickness of t . The connection-web is reinforced by the thickness of t_p , as shown in Fig.-1, and the following parameters α , β and λ are defined by

$$\alpha = \frac{h_c}{h_b} = \frac{A_{fc}}{A_{fb}}, \quad \beta = \frac{A_{fb}}{2h_b t}, \quad \lambda = \frac{t_p}{t} \quad \dots\dots\dots (2.1)$$

$$\alpha \geq 1, \quad \beta \geq 0, \quad \lambda \geq 1 \quad \dots\dots\dots (2.2)$$

The following assumptions are used in the analysis.

1. The material is perfectly plastic and obeys to the Tresca's yield condition, that is, the stresses in the webs meet

* Prof., Inst. of Indu. Scie., Univ. of Tokyo

** Assist., Inst. of Indu. Scie., Univ. of Tokyo

*** Graduate Student, Univ. of Tokyo

(昭和42年12月27日日本稿受理・討論期限昭和43年11月末日)

$$|\sigma_1| \leq \sigma_0, |\sigma_2| \leq \sigma_0, |\sigma_1 - \sigma_2| \leq \sigma_0 \quad \dots\dots\dots(2.3)$$

where σ_1 and σ_2 denote the principal stresses and σ_0 the yield stress in simple tension. The specific power of dissipation D is expressed⁷⁾ as

$$D = \frac{1}{2} \cdot \sigma_0 (|\dot{\epsilon}_1| + |\dot{\epsilon}_2| + |\dot{\epsilon}_1 + \dot{\epsilon}_2|) \quad \dots\dots\dots(2.4)$$

where $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$ are principal strain rates.

2. The flanges are treated as "lines" without width and thickness. The yield conditions for the flanges of beam and column are given by

$$\left. \begin{aligned} |F_b| &\leq \sigma_0 A_{fb} (= 2\beta\sigma_0 h_b t) \\ |F_c| &\leq \sigma_0 A_{fc} (= 2\alpha\beta\sigma_0 h_b t) \end{aligned} \right\} \quad \dots\dots\dots(2.5)$$

where F_b and F_c are the axial forces of beam and column, respectively. The specific power of dissipation in the beam-flange is given by

$$D = \sigma_0 A_{fb} \dot{\epsilon} = 2\beta\sigma_0 h_b t \dot{\epsilon} \quad \dots\dots\dots(2.6a)$$

and in the column-flange is given by

$$D = \sigma_0 A_{fc} \dot{\epsilon} = 2\alpha\beta\sigma_0 h_b t \dot{\epsilon} \quad \dots\dots\dots(2.6b)$$

where $\dot{\epsilon}$ denotes the strain rate in the flanges.

3. The statically admissible stress field is divided into regions by the straight discontinuity lines and the stresses in each region are distributed uniformly. The continuity requirements for the stresses of both sides of a discontinuity line are given⁸⁾ by

$$\left. \begin{aligned} \sigma_x^{(i)} \cos \alpha_{ik} + \tau^{(i)} \sin \alpha_{ik} &= \sigma_x^{(k)} \cos \alpha_{ik} + \tau^{(k)} \sin \alpha_{ik} \\ \sigma_y^{(i)} \sin \alpha_{ik} + \tau^{(i)} \cos \alpha_{ik} &= \sigma_y^{(k)} \sin \alpha_{ik} + \tau^{(k)} \cos \alpha_{ik} \end{aligned} \right\} \quad \dots\dots\dots(2.7)$$

Fig. 2 Conditions at Line of Discontinuity

Fig. 3 Discontinuous Stress along Stiffeners

where $\sigma_x^{(i)}$, $\sigma_y^{(i)}$ and $\tau^{(i)}$ denote the stress components belonging to the region (i) , and α_{ik} denotes the angle between x axis and the normal n of the discontinuity line.

4. The axial forces of the flanges are constant in each region or vary linearly and, referring to Fig. 3, the equilibrium conditions can be expressed for the beam as

$$\left. \begin{aligned} \sigma_y^{(m)} t^{(m)} &= \sigma_y^{(n)} t^{(n)} \\ F_b^{(R)} + \tau^{(m)} t^{(m)} l_{LR} &= F_b^{(L)} + \tau^{(n)} t^{(n)} l_{LR} \end{aligned} \right\} \quad \dots\dots\dots(2.8a)$$

and for the column as

$$\left. \begin{aligned} \sigma_x^{(p)} t^{(p)} &= \sigma_x^{(q)} t^{(q)} \\ F_c^{(O)} + \tau^{(q)} t^{(q)} l_{OU} &= F_c^{(U)} + \tau^{(p)} t^{(p)} l_{OU} \end{aligned} \right\} \quad \dots\dots\dots(2.8b)$$

where $F^{(x)}$ is the axial force at the boundary X .

§ 3 Classification of analysis

The statically admissible stress fields are classified into the following six cases, and their graphical relations are shown in Fig. 4, but the case F does not exist for $\alpha \geq 2$.

$$A. \alpha\lambda \leq 4\beta \quad \dots\dots\dots(3.1)$$

$$B. \alpha\lambda \geq 2(1+2\beta), \lambda \geq 2 \quad \dots\dots\dots(3.2)$$

$$C. \alpha\lambda \geq 2(1+2\beta), \lambda \leq 2 \quad \dots\dots\dots(3.3)$$

Fig.-4 Classification of Statically Analyses

Fig.-5 Classification of Kinematically Analyses

$$D. 2(1+2\beta) \geq \alpha\lambda \geq 4\beta, \lambda \geq 2 \quad (3.4)$$

$$E. 2(1+2\beta) \geq \alpha\lambda \geq 4\beta, \alpha\lambda^2 - 2(1+2\beta)\lambda + 8\alpha\beta \geq 0, \lambda \leq 2 \quad (3.5)$$

$$F. 2(1+2\beta) \geq \alpha\lambda \geq 4\beta, \alpha\lambda^2 - 2(1+2\beta)\lambda + 8\alpha\beta \leq 0 \quad (3.6)$$

The kinematically admissible velocity fields are also classified into the following four cases, as shown in Fig.-5, but the case *d* does not exist for $\alpha \geq 2$.

$$a. \alpha\lambda \leq 4\beta \quad (3.7)$$

$$b. \alpha\lambda \geq 2(1+2\beta) \quad (3.8)$$

$$c. 2(1+2\beta) \geq \alpha\lambda \geq 4\beta, \text{ and except case } d \quad (3.9)$$

$$d. \left. \begin{array}{l} 2(1+2\beta) \\ 4\beta + 2(\lambda - 4\alpha\beta) \end{array} \right\} \geq \alpha\lambda \geq \left\{ \begin{array}{l} 4\beta + (\lambda - 4\alpha\beta)/2 \\ 4\beta \end{array} \right\} \quad (3.10)$$

$$2(\lambda - 4\alpha\beta) \geq \{1 - \sqrt{3(\lambda - 1)}\}(\alpha\lambda - 4\beta)$$

§4 Analysis by statically admissible stress field

1. Analysis for the case A.

To obtain a statically admissible stress field, the webs and the flanges are divided into three regions as shown in Fig.-6. Assuming that there exists only the shearing stress

$$\tau^{(3)} = \frac{\sigma_0}{2} \quad (4.1)$$

in the region (3) and no stresses in the other regions of webs, the flange forces F shown in Table-1 are in equilibrium. It is clear that the stress $\tau^{(3)}$ of eq. (4.1) satisfies the yield conditions (2.3) and

Fig.-6 Statically Admissible Stress Field in Case A

Fig.-7 Statically Admissible Stress Field in Cases B and C

the stresses in Table-1 satisfy the yield conditions (2.5) for $\alpha\lambda \leq 4\beta$. Hence, these stresses are statically admissible.

Table-1

Regions of flanges	F
A~B	$\alpha\lambda\sigma_0 h_b t/2 \sim \alpha\lambda\sigma_0 h_b t/2$
B~C	$\alpha\lambda\sigma_0 h_b t/2 \sim 0$
E~B	$\lambda\sigma_0 h_b t/2 \sim \lambda\sigma_0 h_b t/2$
B~D	$\lambda\sigma_0 h_b t/2 \sim 0$

The bending moment M associated with the assumed stress field is obtained as

$$M = \alpha\lambda\sigma_0 h_b^2 t \quad (4.2)$$

Since the fully plastic moment M_0 of the beam is

$$M_0 = (1 + 4\beta)\sigma_0 h_b^2 t \quad (4.3)$$

the reduction ratio of yield moment is given by

$$M/M_0 = \alpha\lambda/(1 + 4\beta) \quad (4.4)$$

2. Analysis for the case B.

To obtain a statically admissible stress field, we divide the webs and flanges by the discontinuity lines as shown in Fig.-7. From the assumption of the stress components shown in Table-2 and 3, referring to eqs. (2.7) and (2.8), the following continuity requirements for the stresses are obtained:

Table-2

Regions of webs	σ_x	σ_y	τ
(1)	$\sigma_x^{(1)}$	0	0
(2)	0	$\sigma_y^{(2)}$	0
(3)	0	0	0
(4)	$\sigma_x^{(4)}$	$\sigma_y^{(4)}$	$\tau^{(4)}$
(5)	0	0	$\tau^{(5)}$

Table-3

Regions of flanges	F
A~B	$F_1 \sim F_1$
B~C	$F_1 \sim F_2$
C~D	$F_2 \sim 0$
F~B	$F_3 \sim F_3$
B~E	$F_3 \sim 0$

$$\text{for the boundary (1)~(4)} \quad \sigma_x^{(1)} = \lambda\sigma_x^{(4)}, \quad F_3 = \tau^{(4)}\lambda h_b t \quad (4.5)$$

$$\text{for the boundary (2)~(4)} \quad \sigma_y^{(2)} = \lambda\sigma_y^{(4)}, \quad F_1 - F_2 = \tau^{(4)}\lambda(\alpha - \xi)h_b t \quad (4.6)$$

$$\text{for the boundary (3)~(5)} \quad F_2 = \tau^{(5)}\lambda\xi h_b t \quad (4.7)$$

$$\text{for the boundary (4)~(5)} \quad \left. \begin{aligned} \sigma_x^{(4)} + \tau^{(4)}(\alpha - \xi) &= \tau^{(5)}(\alpha - \xi) \\ \sigma_y^{(4)}(\alpha - \xi) + \tau^{(4)} &= \tau^{(5)} \end{aligned} \right\} \quad (4.8)$$

where the requirements automatically satisfied are omitted.

The above equations are satisfied, if

$$\xi = \alpha - 1 \quad \dots\dots\dots(4.9)$$

and if the stresses in Table-4 are taken for the webs, and the axial force in Table-5 for the flanges.

Table-4

Regions of webs	σ_x	σ_y	τ
(1)	σ_0	0	0
(2)	0	σ_0	0
(3)	0	0	0
(4)	$\frac{1}{\lambda} \cdot \sigma_0$	$\frac{1}{\lambda} \cdot \sigma_0$	$\frac{1+2\beta-\alpha}{\alpha\lambda} \cdot \sigma_0$
(6)	0	0	$\frac{1+2\beta}{\alpha\lambda} \cdot \sigma_0$

Table-5

Regions of flanges	F
A~B	$2\beta\sigma_0 h_b t \sim 2\beta\sigma_0 h_b t$
B~C	$2\beta\sigma_0 h_b t \sim \frac{(1+2\beta)(\alpha-1)}{\alpha} \cdot \sigma_0 h_b t$
C~D	$\frac{(1+2\beta)(\alpha-1)}{\alpha} \cdot \sigma_0 h_b t \sim 0$
F~B	$\frac{1+2\beta-\alpha}{\alpha} \cdot \sigma_0 h_b t \sim \frac{1+2\beta-\alpha}{\alpha} \cdot \sigma_0 h_b t$
B~E	$\frac{1+2\beta-\alpha}{\alpha} \cdot \sigma_0 h_b t \sim 0$

Now, we need confirm whether these stresses satisfy the yield conditions or not. For the stress in the region (5) to satisfy eq. (2.3), the condition

$$2(1+2\beta) \leq \alpha\lambda \quad \dots\dots\dots(4.10)$$

is necessary and for the axial force F_2 to satisfy eq. (2.5), the condition

$$1+2\beta \geq \alpha \quad \dots\dots\dots(4.11)$$

is also necessary. These two conditions coincide with eqs. (3.2). In the region (4) the principal stresses are

$$\left. \begin{aligned} \sigma_1 &= (1+2\beta)\sigma_0/\alpha\lambda \\ \sigma_2 &= (2/\lambda) - (1+2\beta)/\alpha\lambda \end{aligned} \right\} \quad \dots\dots\dots(4.12)$$

and these stresses lead to

$$\sigma_0/2 \geq \sigma_1 \geq 0, \quad \sigma_0/\lambda \geq \sigma_2 \geq -\sigma_0/2 \quad \dots\dots\dots(4.13)$$

for eq. (3.2). For the flange force F_3 to satisfy eq. (2.5), the condition

$$\left. \begin{aligned} \frac{1+2\beta-\alpha}{\alpha} &\leq 2\alpha\beta \\ \text{or} \quad \frac{1+2\beta}{1+2\alpha\beta} &\leq \alpha \end{aligned} \right\} \quad \dots\dots\dots(4.14)$$

is necessary, and since $(1+2\beta)$ is not larger than $(1+2\alpha\beta)$ for $\alpha \geq 1$, eq. (4.14) is satisfied.

The bending moment M relating to this stress field is

$$M = \sigma_x^{(1)} h_b^2 t + 2 F_1 h_b t = (1+4\beta)\sigma_0 h_b^2 t \quad \dots\dots\dots(4.15)$$

and the reduction ratio of yield moment is

$$M/M_0 = 1 \quad \dots\dots\dots(4.16)$$

3. Analysis for the case C.

In this case, a part of the stresses in Table-2 and the forces in Table-3 is changed as

$$\tau^{(4)} = 0, \quad F_2 = F_1, \quad F_3 = 0 \quad \dots\dots\dots(4.17)$$

Requirements (4.5)~(4.8) can be satisfied if

$$\xi = 2\alpha\beta/(1+2\beta) \quad \dots\dots\dots(4.18)$$

$$F_1 = 2\beta\sigma_0 h_b t \quad \dots\dots\dots(4.19)$$

and if the stresses in the webs are assumed as shown in Table-6. For the stress in the web region (5) to satisfy the yield condition (2.3), the necessary condition is

$$2(1+2\beta) \leq \alpha\lambda \quad \dots\dots\dots(4.20)$$

and for the stress in the web region (2) the condition is

$$1+2\beta \leq \alpha \quad \dots\dots\dots(4.21)$$

Both of these equations are satisfied for eq. (3.3). Since the stresses in the web region (1) and (4), and the axial force of the flange F_1 satisfy the yield conditions, the assumed stress field is statically admissible. The bending moment M is given by eq. (4.15), so that

$$M/M_0=1 \quad \dots\dots\dots(4.22)$$

can be obtained.

Table-6

Regions of webs	σ_x	σ_y	τ
(1)	σ_0	0	0
(2)	0	$\frac{(1+2\beta)^2}{\alpha^2} \cdot \sigma_0$	0
(3)	0	0	0
(4)	$\frac{1}{2} \cdot \sigma_0$	$\frac{(1+2\beta)^2}{\alpha^2\lambda} \cdot \sigma_0$	0
(5)	0	0	$\frac{1+2\beta}{\alpha\lambda} \cdot \sigma_0$

(Continued to Part 1-2)

References :

- 1) A.A. Topractsoglou, L.S. Beedle, B.G. Johnston; Connections for Welded Continuous Portal Frames, Part I, II, III, Welding Journal, July 1951, August 1951, November 1952.
- 2) T. Naka, H. Saito; Investigation of the Strength of Welded Beam-Column Connections, Part 15 (in Japanese), Trans. A.I.J. 66, Oct. 1960.
- 3) S. Igarashi, J. Sakamoto; Experimental Research for Plastic Connections of Steel Members (in Japanese), Trans. A.I.J. 66 Oct. 1960.
- 4) T. Naka, B. Kato, M. Yuasa, T. Sasaki; The Experimental Research on the Behavior of Steel Beam to Column Connection Effected by Lateral Force, No. 3, (in Japanese), Trans. A.I.J. 104, Oct. 1964.
- 5) H. Tanaka; Limit Analysis of Beam Column Connections Subject to Anti-symmetrical Bending, Recent Researches of Structural Mechanics, Prof. Tsuboi Anniversary Volume, Uno-shoten, to be published.
- 6) H. Tanaka; Limit Analysis of Cruciform Beam-Column Connections (in Japanese), Prof. Naka Anniversary Volume, to be published.
- 7) H. Tanaka; Limit Analysis of Structures, p. 43, (in Japanese), Shokoku-sha, July 1966.
- 8) p. 53, the same as 7).
- 9) Appendix of 6).
- 10) A.S.C.E.; Commentary on Plastic Design in Steel, 1961.

柱はり接合部の極限解析(I)(梗概)

—強軸曲げをうける十字形接合部—

正 会 員	田	中	尚*
”	福	島	男**
”	林	永	全***

§1 はしき

H形断面の柱, はり接合部の終局強度は接合部ウェブのせん断降伏によって定まるとは各種の実験によって確かめられており, 解析に際しては, 接合部ウェブのせん断力は均等分布で, 曲げモーメントはフランジのみによって伝達されると仮定して良好な結果が得られることが報告されている^{1)~4)}。しかしながら, 柱はりフランジの断面積とウェブのそれとの比や, 接合部ウェブの厚みや, はり柱のせいの比などによって必ずしもこの仮定が成立するとは限らない^{5), 6)}。

この研究は図—1に示すようなH形断面材よりなる十字形接合部が逆対称曲げをうける場合の降伏モーメントの値を解析したものである。

§2 解析上の仮定

図—1に示すようなH形断面材よりなる接合部において, はりせい $2h_b$, 柱幅 $2h_c$, ウェブ厚は共に t , はり柱のフランジ断面積はそれぞれ A_{fb} , A_{fc} , 接合部のウェブは補強されて厚さ t_p で, 式 (2.1), (2.2) の成立する場合を取扱うことにする。

解析にあたっては次の仮定を用いる。

1. 材料は完全剛塑性体とし, 降伏条件は Tresca のそれを用いる。すなわちウェブの降伏条件は式 (2.3) であり, エネルギー散逸率 D は式 (2.4) で計算される⁷⁾。
2. フランジは厚さと幅をもたない線材として取扱う。すなわち, はり柱フランジの軸方向力を F_b, F_c としたとき降伏条件として式 (2.5) を用い, エネルギー散逸率 D はフランジ軸方向ひずみを $\dot{\epsilon}$ としたとき式 (2.6 a), (2.6 b) で計算される。
3. 静的許容応力場は直線の不連続線で区切られ, 一つの領域の中では一様な応力分布と仮定する。つりあい条件は図—2のような不連続線において式 (2.7) が満足されることである⁸⁾。
4. フランジの軸方向力は一つの領域で一定又は直線的に変化すると仮定する。図—3 からわかるようにフランジのつりあい条件は, 式 (2.8 a), (2.8 b) である。

§3 解析の分類

静的許容応力場による解析は, 図—4 に示すような範囲 A, B, C, D, E, F に分けて行う。式 (3.1)~(3.6) にその範囲を示す。

運動的許容速度場による解析は 図—5 に示すような範囲 a, b, c, d に分けて行う。式 (3.7)~(3.10) にその範囲を示す。

§4 静的許容応力場による解析

1. 解 析 A

静的許容応力場を 図—6 に示すような領域にわけ, 領域 (3) にせん断応力度 $\sigma_0/2$ のみが存在し, 他の領域 (1), (2) は無応力と仮定するとフランジの応力 F は 表—1 の値でつりあい状態になる。式 (4.1) は降伏条件を満足し, 又 表—1 の応力も式 (3.1) の範囲で降伏条件式 (2.5) を満足するから, 仮定した応力分布は静的許容応力場である。

降伏モーメント M は式 (4.2) であり, はりの全塑性モーメント M_0 は式 (4.3) であるから, 降伏モーメントの低下率は式 (4.4) で求まる。

2. 解 析 B

静的許容応力場を 図—7 に示すような領域にわけ, 応力の成分を 表—4, 5 のように仮定すると不連続条件を満足し, 式 (4.14) の範囲で静的許容である。

降伏モーメント M は式 (4.15) であるから, M/M_0 は式 (4.16) を得る。

3. 解 析 C

表—3 において, 式 (4.17) (4.19) とし, 表 6 の応力分布を仮定すれば, 不連続条件を満足し, 式 (3.3) の範囲で静的許容であり, 降伏モーメントの低下は $M/M_0 = 1$ となる。

* 東京大学助教授・生産技術研究所・工博

** 東京大学助手・生産技術研究所

*** 東京大学・大学院学生・博士課程

(昭和 42 年 12 月 27 日稿受理・討論期限昭和 43 年 11 月末日)

U.D.C. 550.346 : 624.131 : 624.012.4

EARTHQUAKE DAMAGE TO REINFORCED CONCRETE BUILDINGS AND SUBSOIL CONDITION

—Review of Damage by Kanto Earthquake 1923 (See page 40)—

By Dr YORHIKO OHSAKI and KEIKO KUNIYASU Building Research
Institute, Ministry of Construction. Members of A.I.J.

U.D.C. 624.042.41

CALCULATION METHOD OF WIND PRESSURE ON ROOF AND WALL OF BUILDINGS (part I)

By SHOJI KOBAYASHI, Institute of Construction Technology,
Kajima Construction Co., Ltd. Member of A.I.J.

Though the flow pattern is affected by the roughness of the ground surface, the turbulence of the natural wind, and the configuration of the building, the author tries to get a two-dimensional solution of the pressure distribution on the wall and roof of buildings subjected to a uniform stream, such as factories with or without monitor, by the image method of the potential theory, on the assumption that the air is an ideal fluid, that is, inviscid and incompressible without temperature change, and flows along the wall and roof surfaces, with separation starting at the roof ridge.

The purpose of this paper is to provide useful information on the design of the building under the wind pressure, which is easily calculated with the help of the table of modified elliptic functions attached to the end of the 2nd Part of the series of the papers to be published in the following month, though the derivation of the equations seems rather difficult due to the elliptic functions involved.

The appropriateness of this solution is verified by the experiments as presented in the successive parts of the series in special cases, although it looks reasonable at the first glance.

In conclusion, the distribution of the wind pressure on the surfaces of the building, which has had so far recourse to the experiment in the wind tunnel, can be fairly estimated by this calculation method in some cases.

U.D.C. 624.078.072.2 : 624.043

LIMIT ANALYSIS OF BEAM-COLUMN CONNECTIONS (I)

—Cruciform Connections of Wideflange Members
Subjected to Strong-Axis Bending— (See page 49)

By Dr. HISASHI TANAKA, AKIO FUKUSHIMA and
YUNG CHUAN LIN. Tokyo Univ, members of A.I.J.

U.D.C. 69.022 : 69.003.123

ON THE COST OF THE EXTERIOR SKIN (WALL & ROOF) IN MULTI-STORIED OFFICE BUILDING.

—On Application of Quantitative Study on B.E. —IV

By FUJIO SHIMA, member of Kajima construction
Co. Ltd. member of A.I.J.

The total cost of the exterior skin of the building is discussed in this report, relating with building forms.

The quantitative study on the building elements made out the relation between exterior skin area and building forms in the following formula.

$$R+E=\frac{A}{n}+2h\sqrt{An}\left(\sqrt{k}+\frac{1}{\sqrt{k}}\right)$$

R : roof area (m^2), E : exterior wall area (m^2), A : total floor area, n : number of stories

h : floor height, k : ratio of sides in typical floor

This relation is shown in Fig. 5 by graph. Then the unit cost of wall and roof were introduced this formula. When "A" is fixed, "n" that makes exterior skin cost minimum is given hereunder,

$$n=\sqrt[3]{\frac{A}{h^2\left(\sqrt{k}+\frac{1}{\sqrt{k}}\right)^2}\left(\frac{C_r}{C_e}\right)^2}$$