Numerical Analysis of Elastic-Plastic Deformation of Simply Supported Rectangular Plates

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1. Introduction

Previous investigations on the analysis of plastic plates may be divided into two categories. The limit analysis determines the stress and velocity fields at plastic collapse together with the collapse load, provided a complete solution is obtained. Number of lower and upper bounds on the collapse load of a rectangular plate have been obtained [1]. The elastic-plastic analysis is concerned with the process of elastic-plastic deformation and with the variation of stress fields as the load is varied. Previous analytical works in the latter category have been restricted to rotationally symmetric problems of plates. The difficulties in the elastic-plastic analysis of a rectangular plate lie not only in the set of nonlinear moment curvature-rate relations, but also in the unknown elastic-plastic interfaces as the load is varied.

This paper presents the result of a numerical analysis of elastic-plastic deformation of simply supported rectangular plates subjected to uniformly distributed load. While the present work has been carried out, Bhaumik and Hanley [2] presented a numerical work for a square plate which does not appear to be based upon the incremental theory. In March 1968 when the present work was completed as the junior author's M. Eng. thesis [6], the numerical result by Ang and Lopez [3] which is based upon the incremental theory came to the authors' attention. Although the numerical method based upon the stiffness matrix concept [4] has been employed with different derivations but essentially in a similar fashion in both works, Ang and Lopez is concerned only with a square plate, whereas the present work deals with rectangular plates.

2. Fundamental Equations in Incremental Form

The following assumptions are made in this paper.

- (i) The moment-curvature relationship for a plate element is governed by the incremental theory.
- (ii) An element of a plate is either elastic or plastic. No transitional state is taken into account. This implies that the plate is assumed to be an idealized sandwich plate with a shear-resistant core.
- (iii) The displacement is so small that membrane forces are negligible.
- (iv) The moment and displacement fields are continuous and have continuous derivatives up to the second order so that the differential equations may be approximated by the corresponding finite difference equations. Theoretically admissible discontinuities may be averaged and smoothed in applying the finite difference procedure.

The dimensionless coordinate axes are chosen as shown in Fig. 1, where. $x=X/L$, $y=Y/L$, L being a side length of a plate. The following dimensionless quantities are employed here.

$$m_x = \frac{M_x}{M_0}, \text{ etc.}; p = \frac{PL^2}{6M_0}, w = \frac{W}{H}, k_x = \frac{K_x L^2}{H}, \text{ etc.} \dots (1)$$

where H denotes the plate thickness and P , the distributed load in the positive W direction.

Fig. 1

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The von Mises yield condition for a plate element may be written as

$$f[m_x, m_y, m_{xy}] = m_x^2 - m_x m_y + m_y^2 + 3 m_{xy}^2 - 1 = 0 \quad (2)$$

It is assumed that the actual curvatures k_x , etc. may be written as the sum of the elastic curvature k_x^e , etc. and the plastic curvature k_x^p , etc. The plastic potential flow law gives

$$\dot{k}_x^p = \dot{\mu} \frac{\partial f}{\partial m_x}, \quad \dot{k}_{xy}^p = \dot{\mu} \frac{\partial f}{\partial m_{xy}}, \quad \dot{k}_y^p = \dot{\mu} \frac{\partial f}{\partial m_y} \quad (3)$$

where

$$\dot{\mu} = 0 \text{ if } f < 0 \text{ and also if } f = 0 \text{ and } \dot{f} < 0$$

$$\dot{\mu} \geq 0 \text{ if } f = 0 \text{ and}$$

$$\dot{f} = (2m_x - m_y)\dot{m}_x + 6m_{xy}\dot{m}_{xy} + (2m_y - m_x)\dot{m}_y = 0 \quad (4)$$

Let “ Δ ” denote a finite difference operator with respect to time or any other parameter which increases monotonically with time. The curvature increments are the sum of the elastic and plastic curvature increments.

$$\left. \begin{aligned} \Delta k_x &= \frac{1}{C(1-\nu^2)} \Delta m_x - \frac{\nu}{C(1-\nu^2)} \Delta m_y + \frac{1}{C(1-\nu)} (2m_x - m_y) \Delta \mu \\ \Delta k_{xy} &= \frac{1}{C(1-\nu)} \Delta m_{xy} + \frac{1}{C(1-\nu)} 6m_{xy} \Delta \mu \\ \Delta k_y &= -\frac{\nu}{C(1-\nu^2)} \Delta m_x + \frac{1}{C(1-\nu^2)} \Delta m_y + \frac{1}{C(1-\nu)} (2m_y - m_x) \Delta \mu \end{aligned} \right\} \quad (5)$$

where

$$C = \frac{DH}{M_0 L^2}$$

The inversion of Eqs. (5) may be found with the help of Eq. (4).

The result may be written as

$$[\Delta m_x, \Delta m_{xy}, \Delta m_y, \Delta \mu]^T = [S_{ij}][\Delta k_x, \Delta k_{xy}, \Delta k_y] \quad (6)$$

The fourth row of $[S_{ij}]$ which does not consist of stiffness quantities has been included in $[S_{ij}]$ for convenience.

The stiffness matrices $[S_{ij}]$ are

$$C \begin{bmatrix} 1 & 0 & \nu \\ 0 & 1-\nu & 0 \\ \nu & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \frac{C(1-\nu)}{S} \begin{bmatrix} (1+\nu)F^2 + G^2 & -(E+\nu F)G & -(1+\nu)EF + \nu G^2 \\ -G(E+\nu F) & E^2 + 2\nu EF + F^2 & -G(F+\nu E) \\ -(1+\nu)EF + \nu G^2 & -(F+\nu E)G & (1+\nu)E^2 + G^2 \\ E+\nu F & (1-\nu)G & F+\nu E \end{bmatrix}$$

for an elastic response and a plastic response, respectively, where

$$E = 2m_x - m_y, \quad G = 6m_{xy}, \quad F = 2m_y - m_x$$

and

$$S = E^2 + 2\nu EF + F^2 + (1-\nu)G^2 \quad (7)$$

The incremental equations of equilibrium

$$\frac{\partial^2 \Delta m_x}{\partial x^2} + 2 \frac{\partial^2 \Delta m_{xy}}{\partial x \partial y} + \frac{\partial^2 \Delta m_y}{\partial y^2} + 6 \Delta p = 0 \quad (8)$$

may be reduced to the finite difference equations.

$$\begin{aligned} &\Delta m_x(i-1, j) - 2 \Delta m_x(i, j) + \Delta m_x(i+1, j) \\ &+ \frac{2\alpha}{4} [\Delta m_{xy}(i-1, j-1) - \Delta m_{xy}(i-1, j+1) - \Delta m_{xy}(i+1, j-1) + \Delta m_{xy}(i+1, j+1)] \\ &+ \alpha^2 [\Delta m_y(i, j-1) - 2 \Delta m_y(i, j) + \Delta m_y(i, j+1) + 24 \delta_x^2 \Delta p] = 0 \quad (6) \end{aligned}$$

where $\delta_x = \alpha \cdot \delta_y$, δ_x and δ_y being the mesh lengths in x and y directions, respectively.

The incremental kinematic relations

$$\Delta k_x = -\frac{\partial^2 \Delta w}{\partial x^2}, \quad \Delta k_{xy} = -\frac{\partial^2 \Delta w}{\partial x \partial y}, \quad \Delta k_y = -\frac{\partial^2 \Delta w}{\partial y^2} \quad (10)$$

are also replaced by the finite difference equations:

$$\left. \begin{aligned} \Delta k_x &= -\frac{1}{\delta x^2} [\Delta w(i-1, j) - 2\Delta w(i, j) + \Delta w(i+1, j)] \\ \Delta k_{xy} &= -\frac{\alpha}{4\delta x^2} [\Delta w(i-1, j-1) - \Delta w(i+1, j-1) - \Delta w(i-1, j+1) + \Delta w(i+1, j+1)] \\ \Delta k_y &= -\frac{\alpha^2}{\delta x^2} [\Delta w(i, j-1) - 2\Delta w(i, j) + \Delta w(i, j+1)] \end{aligned} \right\} \dots (11)$$

The boundary conditions along a simply supported edge, say $x=\text{const.}$ may be written as

$$\Delta w=0, \text{ and } \Delta m_x=0 \dots\dots\dots (12)$$

At an elastic or unloading boundary point, the latter equation of (12) may be replaced by

$$\Delta k_x=0$$

At a plastic boundary point, the first and second equations of (6) may be written as

$$\Delta m_x = S_{11} \Delta k_x + S_{12} \Delta k_{xy} = 0, \quad \Delta m_y = S_{31} \Delta k_x + S_{32} \Delta k_{xy}$$

Since $m_x = m_y = 0$ in the elastic state, $S_{12} = S_{32} = 0$ and hence, for all the subsequent loading step, $\Delta k_x = 0$ and $\Delta m_y = 0$ except possibly at a point where $S_{11} = 0$, implying $m_{xy} = 0$. Because of the assumed continuity in Δk_x , $\Delta k_x = 0$ for all the plastic boundary points. Therefore the finite difference form of the boundary conditions may be written as

$$\Delta w(b, j) = 0, \quad \Delta w(b+1, j) + \Delta w(b-1, j) = 0 \dots\dots\dots (13)$$

where “ b ” denotes a boundary mesh point. At a corner point the condition

$$\Delta w(b+1, b+1) = \Delta w(b-1, b-1) \dots\dots\dots (14)$$

is postulated by assuming a checker-board displacement pattern for the imaginary exterior plate region.

Eqs. (6), (9) and (11) together with (13) and (14) constitute a system of simultaneous linear algebraic equations for $\Delta w(i, j)$ if the preceding state of moments is known. If a successive forward integration procedure is repeated with respect to some appropriate load increments, then the elastic-plastic response of a plate may be numerically traced by a piecewise linear approximation.

3. Computational Procedure

As the load intensity is increased beyond the elastic limit, the number of plastic grid points will increase gradually. The elastic-plastic interfaces at different load levels will form different contours which must be found as a part of the solution.

A plastic grid point must have experienced a transition step in which the previously elastic point becomes plastic for the first time. Where the response to a predetermined load increment is elastic throughout the increment or plastic partially is not known a priori. Hence it seems convenient to choose a load increment such that no new transition occurs during that increment and such that at least one previously elastic point will have just satisfied the yield condition at the end of the increment. Let $m_x(i, j, n)$, etc. be the moments at the end of n -th step. Let $\Delta m_x(i, j, n+1)$, etc. denote the moment increments for a unit load increment obtained from the new stiffness matrix for the $(n+1)$ -st step. Then the load increment $\Delta p(n+1)$ for the $(n+1)$ -st step may be determined as the minimum of $\Delta p(i, j, n+1)$ which satisfies

$$\begin{aligned} f[m_x(i, j, n) + \Delta \bar{p}(i, j, n+1) \Delta m_x(i, j, n+1), m_{xy}(i, j, n) \\ + \Delta \bar{p}(i, j, n+1) \Delta m_{xy}(i, j, n+1), m_y(i, j, n) + \Delta \bar{p}(i, j, n+1) \Delta m_y(i, j, n+1)] = 0 \dots (15) \end{aligned}$$

A forward integration step consists of the following computational substeps.

- (i) Determine the stiffness matrix for each grid point from the previous values of moments.
- (ii) Compute the coefficients of the finite difference equations for $\Delta w(i, j)$.
- (iii) Solve the system of simultaneous linear algebraic equations for $\Delta p=1$ and obtain $\Delta w(i, j)$.
- (iv) Compute $\Delta \mu(i, j)$ for all the grid points. If $\Delta \mu(i, j) < 0$ for some grid points, then replace the corresponding plastic stiffness matrix by the elastic one for that grid point at which $\Delta \mu(i, j)$ is the smallest negative. Then return to (ii).
- (v) If $\Delta \mu(i, j) > 0$ for all the grid points, then determine the actual load increment $\Delta p(n+1)$ in

accordance with Eq. (15).

- (vi) Compute the moment increments corresponding to $\Delta p(n+1)$ and add them respectively to the previous moment values, and then return to (i)

In order to avoid the circumstance in which the coefficient matrix of step (iii) becomes nearly singular as plastic grid points prevail, the solution for a prescribed displacement $\Delta w(0,0)=1$ is obtained instead of step (iii).

4. Numerical Results

Numerical solutions have been obtained for rectangular plates with the side length ratio or span

Fig. 2

Fig. 3

length ratio $1/\alpha=1, 2$ and 4 for $C=1.0$ and $\nu=0.3$. The mesh sizes for plates with $1/\alpha=1, 2$ and 4 were chosen respectively as $(\delta_x, \delta_y)=(0.2, 0.2)$, $(0.2, 0.4)$ and $(0.2, 0.8)$. Fig. 2 shows the load-displacement curve for several mesh points for plate of $1/\alpha=2$. The broken lines I and II indicate the corresponding lower bounds on the collapse loads obtained by the present authors by means of the linear programming technique (proposed by Koopman and Lance [5]) for the case of linearized Tresca polyhedron consisting of 16 planes. The dotted line III indicates the collapse load of a beam of unit width. Fig. 3 shows the displacement patterns along the axes, for uniformly distributed unit load. It is observed that the displacement becomes greater for the same load as the plastic mesh points increase.

Fig. 4 shows the order of yielding of mesh points. No unloading response was obtained for plates with $1/\alpha=2$ and 4 . For a square plate, however, unloading responses occurred in the 6th step at $(0, 0)$ and in the 9th and 10th step at $(1, 0)$ and $(0, 1)$. Although the computational outputs show unloading responses under compatible stiffness matrices, the resulting values of the yield function have never become less than -0.01 which is the computational tolerance limit. The plastic region for a square plate is developed from the corners and then from the center, whereas the plastic region for rectangular plates $1/\alpha=2$ is first developed from the center parallel to the longer side and then also from the corners. It has been pointed out by Ang and Lopez that overall stiffness of a square plate, as indicated by the slope of the load-displacement curve, decreases considerably when a pseudo-mechanism is for-

Fig. 4

Fig. 5

Fig. 6

med by the plastic region. In a rectangular plate, however, the overall stiffness decreases considerably already when the oblong plastic region is developed at the center of the plate as may be also seen from Fig. 3.

Fig. 5 shows the stress surfaces at $p=0.473$ (the elastic limit) and at $p=0.591$ in a (m_x, m_{xy}, m_y) -space for the plate with $1/\alpha=2$. The broken lines indicate the intersection of the yield surface with the coordinate planes and with $m_x=m_y$.

The plastic mesh points are indicated by * and the hatched portions show the plastic regions. These stress surfaces will give some idea how the stress surface is inflated, expanded and approaches the yield surface from inside as the plastic region grows greater. Fig. 6 and 7 show some stress point paths for those mesh points which are known a priori to remain in $m_{xy}=0$ and $m_x=m_y$ for the plates with $1/\alpha=1$ and 2, respectively.

In order to investigate the influence of mesh size on the numerical result, the three different solutions based upon the mesh sizes $(\delta_x, \delta_y)=(0.25, 0.50)$, $(0.2, 0.4)$ and $(0.167, 0.333)$ are compared for the plate with $1/\alpha=2$. Fig. 8 shows the load-displacement curves for the mesh point $(0,0)$. It is noted that no significant di-

Fig. 7

Fig. 8

Fig. 9

ference is observed in these curves. The plastic regions at $p=0.55$ and $p=0.60$ are also compared for these three cases in Fig. 9. Fig. 8 and 9 and the numerical results for these three cases provide a numerical assurance on the convergence of the present numerical analysis.

5. Conclusion

The following conclusions may be drawn from the result.

- (i) A simple forward integration procedure with respect to incremental load on the spatial finite difference equations furnishes numerical solutions with numerical assurance on the convergence of the procedure.
- (ii) The plastic region in a rectangular plate with $1/\alpha=2$ is developed first from the center parallel to the longer side and then also from the corners, whereas the plastic region in a square plate is developed, first from the corners and then from the center along the diagonals. The central portion in a narrow rectangular plate behaves like a very wide beam with the shorter span length. The well developed plastic region in a narrower rectangular plate which may be called to have formed a pseudo-mechanism, has a narrow band parallel to the longer side.
- (iii) The slope of the load-displacement curve for a rectangular plate decreases considerably already when the oblong plastic region is developed at the center. The increment of load from the elastic limit to the state of well-developed plastic region decreases as $1/\alpha$ becomes longer.
- (iv) The stress point for a plastic mesh point travels in an (m_x, m_{xy}, m_y) space, on the Mises yield surface. This implies that the state of moments changes after yielding. The redistribution of moments at a plastic grid point in a square plate occurs in such a way that m_{xy} decreases whereas m_x+m_y increases. In a rectangular plate, on the other hand, the stress points within the narrower triangle with respect to the diagonals move toward the yield surface comparatively rapidly and travel after yielding, on the yield surface in such a way that m_{xy} decreases whereas m_x+m_y increases. The stress points within the wider triangles approach the yield surface rather slowly. The inflation or expansion of the stress surface inside the yield surface is quite complicated as the plastic region is developed.

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単純支持弾塑性平板の数値解析¹⁾(梗概)

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1. 緒 言

塑性平板の研究は、大きく2種類に別けられる。極限解析においては、完全解は崩壊荷重と、崩壊時のモーメント場および変位速度場からなる。これに対して弾塑性解析は、荷重の変動に伴うモーメント場と変位場の変化を追跡するものである。

本論は、単純支持長方形板の弾塑性変形状を数値解析によって追跡解明しようとするものである。非軸対称板の弾塑性解析における最大の困難は、未知弾塑性境界と基礎方程式の非線型性にある。それ故に既往の研究は円板に限られていたが、本研究遂行中に、正方形板について Bhaumick and Hanley [2] の数値解析結果が発表され、続いて本研究終了時に Ang and Lopez [3] の数値解析結果が提出された。[2] は歪増分理論による解析かどうか不明であるのに対し、[3] は板要素をモデル化して歪増分理論を適用している。本論では基礎微分方程式系を直接差分式に置換し歪増分理論を適用しているので誘導過程は [3] と異なり、また [3] のモデル化のパラメーターのために、その数値解析結果と本研究の結果を直接比較する事は困難である。更に [3] が正方形板のみを扱うのに対し、本論は長方形板について、その弾塑性性状が如何に変化するかを解明するものである。

2. 増分形式の基礎方程式と対応する差分方程式

次の仮定に基いて解析を行なう。

- (i) モーメント曲率関係は歪増分理論に基く。
- (ii) 断面は降伏しているか、弾性域にあるかのいずれかである。これは、サンドイッチ断面を考えることに相当する。
- (iii) 微小変形とし、面内力は無視する。
- (iv) モーメント場、変位場は2階連続微分可能であるとし、また理論的に可能な塑性域でのモーメントの不連続については、平均化、平滑化して近似され得るものとする。

座標系は図-1 に示されるように選ぶ。式(1)で定義される無次元化量を用いる。ミーゼスの降伏条件は式(2)で与えられる。曲率 k_x 等は、弾性歪 k_x^e 等と塑性歪 k_x^p 等の和として表わされる。塑性法則は条件式

(4) のもとで、式(3)で表わされる。以下“ Δ ”は各量の増分を表わすものとする。歪増分は、式(5)のよう表現され得る。式(5)の逆変換は式(4)を用いて、式(6)のように表わされる。“ $\Delta\mu$ ”はモーメントとは異なった量であるが、便宜上この列をも stiffness matrix $[S_{ij}]$ に含めることとする。 $[S_{ij}]$ は弾性応答、塑性応答各々に対して、式(7)の内容を有する。

増分形式で表わされた釣合式(8)を階差式表示したものが式(9)である。ここに $\delta_x = \alpha \delta_y$ であり、 δ_x , δ_y は各々 x, y 方向の格子間隔を表わす。歪変位関係の増分形式(10)を階差式表示すると式(11)を得る。単純支持辺の場合の境界条件は式(12)で与えられる。これは降伏以前、以後ともに $x = \text{const.}$ で $\Delta w = 0$, $\Delta k_x = 0$ となる事が証明される。故に境界条件の階差式は式(13)で与えられる。隅点に対しては、近傍での checker board 型変形を仮定して式(14)を用いる。

式(6), (9), (11)は、境界条件式(13), (14)とともに、ある状態のモーメント分布に基いて、これに近い次の荷重状態に対する $\Delta w(i, j)$ に関する連立一次方程式系を構成する。故に、適当な荷重増分に関する数値積分が繰り返されれば、板の弾塑性変形が追跡されることとなる。

3. 数 値 計 算 法

荷重の上昇に伴い、塑性格子点数は徐々に増加する。即ち、荷重変化にともなう塑性域の拡大が、数値計算解の一部として得られる。

各格子点はそれぞれ異なる荷重値において弾性から塑性へ移る。前以て定められた荷重増分に対する各格子点の応答が、その荷重増分全量に対して弾性応答となるか、部分的に塑性応答となるかは、あらかじめ解っていない。故に、部分的な塑性応答の起ることがなく、しかも前段階の弾性格子点が少なくとも1ケ丁度降伏するように荷重増分を定めることとする。 $m_x(i, j, n)$ 等を n 段階最終のモーメントとし、 $\Delta m_x(x, j, n+1)$ 等を $n+1$ 段階での単位荷重増分に対するモーメント増分とすると、格子点 (i, j) が降伏するのに必要な荷重増分 $\Delta p(i, j, n+1)$ は(15)式より求められ、 $\Delta p(n+1)$ は $\Delta p(i, j, n+1)$ の最小値として定められる。

数値計算の1段階は以下より成る。

- (i) n 段階のモーメント最終値により各格子点の

1) 昭和43年5月近畿支部研究会で発表したものである

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stiffness matrix を定める。

(ii) $4w(i, j)$ に関する階差式の係数を定める。

(iii) $4p=1$ に対して連立一次方程式を解き $4w(i, j)$ を求める。

(iv) 各格子点について $4\mu(i, j)$ を求め、 $4\mu(i, j) < 0$ となる格子点があれば、 $4\mu(i, j)$ が最小値となる点の塑性 stiffness matrix を弾性のものに改め、(ii) にもどる。

(v) すべての格子点に対して $4\mu(i, j) \geq 0$ であれば、式 (15) に従い $4p(n+1)$ を定める。

(vi) $4p(n+1)$ に対するモーメント増分を求め、これを n 段階最終モーメントに加え、 $n+1$ 段階最終モーメントを得る。

4. 計算結果

$C=1.0$, $\nu=0.3$ とし、辺長比 $1/\alpha=1, 2, 4$ の場合について計算を行なった。格子はいずれの場合も 5 等分である。図—2 は若干の格子点についての荷重変位曲線である。破線 I, II は、[5] により提案されたリニヤプログラミングを適用する方法により得られた、トレスカの降伏条件に従う、対応する長方形板の崩壊荷重の下界である。図—3 は $1/\alpha=2$ の板の単位荷重による、 x 軸、 y 軸上の点の撓み曲線である。塑性格子点の増加にともなう板全体の曲げ剛性の低下が明らかである。

図—4 は格子点の降伏の順序を示す。 $1/\alpha=2, 4$ の場合は部分的除荷応答は結果として生じなかった。だが正方形板において、6 段階目 $(0, 0)$ 点で、9, 10 段階目 $(0, 1)$, $(1, 0)$ 点でそれぞれ部分的除荷応答が結果として得られた。この場合降伏関数は計算上の許容範囲としておいた -0.01 以下となることはなかった。正方形板において塑性域は隅点から、ついで中央点から対角線上へと拡がるが、 $1/\alpha=2, 4$ の長方形板においては中央点から長手方向へ拡がり、ついで隅点から拡がってくる塑性域とつながる。[3] によると、正方形板においては対角線上に塑性域がつながり、pseudo-mechanism が形成される時、板全体の曲げ剛性が急速に低下するということであるが、図—2, 図—3 からみられるとおり、長方形板においては中央部に帯状の塑性域が形成された段階で既に、板全体の曲げ剛性はかなり低下する。

格子点のモーメント状態を (m_x, m_{xy}, m_y) 空間の像としてみた時、これを応力点と称する。図—5 は $1/\alpha=2$ の場合の $p=0.473$ (弾性限), $p=0.591$ における応力点を連ねた図、応力図を示す。荷重の増大にともない応力図が膨張し降伏曲面に接近する様子がみられる。図—6, 図—7 は各々 $1/\alpha=1, 2$ の場合の、常に $m_x=m_y$ もしくは $m_{xy}=0$ が成立する格子点の stress-path を示す。

格子分割の影響を調べるために、 $1/\alpha=2$ の場合を対象とし、4, 5, 6 等分格子に対する数値解が得られた。図は中央点の荷重変位曲線の、図—9 は定荷重値における塑性域の格子分割による差異が比較されている。大きな差異のないことがみられる。

5. 結 論

(i) 板の弾塑性変形の逐次数値解析方法が、十分良好な収束性を持って、示された。

(ii) 正方形板においては塑性域はまず隅点から、その後中央点からも拡がり、対角線上に進んでいく。ところが長方形板ではまず中央から長手方向に塑性域は拡がり、あるところで隅点方向に向い、隅点から拡がってきた塑性域とつながる。

(iii) 長方形板の板全体としての曲げ剛性は、中央点長手方向に帯状の塑性域ができた段階でかなり低下する。弾性限から、板全体の曲げ剛性が十分低下するまでの荷重増分は、板の形状が細長くなるに従い小さくなる。

(iv) 正方形板においては、各応力点は降伏後 m_{xy} が減少し、 m_x+m_y が増加する方向に降伏曲面上を移動する。長方形板においては、対角線に関して短辺側三角形内の応力点は比較的急速に降伏曲面に達し、降伏後 m_{xy} が減少、 m_x+m_y が増加する方向に降伏曲面上を移動するが、長辺側三角形領域内の応力点は比較的緩慢に降伏曲面に接近する。

6. 謝 辞

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SYSTEMATICAL ANALYSIS AND NUMERICAL EXAMPLES ON THE PLASTIC COLLAPSE AND THE BUCKLING COLLAPSE OF ELASTIC PLASTIC FRAMES (In the Case of no unloading) (1)

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One of the present authors has been studied numerically the interaction curves taking account of the buckling (bending under the beam-column effect) on the plastic collapse and the buckling collapse by M. Kuranishi in the case of the unbraced multi-story frames with the initial plastic hinges to be formed at the base of columns. On the other side, the buckling behavior of unbraced frames research has been so developed that plastic design method was prepared as the lecture note at the Lehigh Univ. and others. The most of them were presented as the approximate design methods without buckling collapse that presumed ultimate load by the iterative procedure or compatibility analysis etc. However, in this paper, present authors proposed the systematical analysis method under the assumption to given virtual loads and fixed critical sections at a given place of yield hinge in order to treated the ultimate lateral loads including the buckling collapse as a whole of frames. Then, the main points of our procedure were much the same as the compatibility analysis to expected that newly considered the criterion called the Feimberg's Stability Condition. Finally, the unbraced 1 bay 2 story frames were investigated as numerical examples.

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NUMERICAL ANALYSIS OF ELASTIC-PLASTIC DEFORMATION OF SIMPLY SUPPORTED RECTANGULAR PLATE (See Page 27)

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AN ANALYSIS OF THE SPACE TRUSS BY STIFFNESS MATRIX USED LINEAR AND NON-LINEAR METHOD

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In this papers, we analyse stress and deflection (strain), and behavior of buckling and vibration of spaced trussed members by finite deflection method.

The basic equation of this method is simply expressed by stiffness matrix. In the basic equation of stress and deflection (strain), we have taken up next three methods, and compared each other.

1. Non-linear equation by finite deflection method.
2. Linear equation by finite deflection method.
3. Linear equation by general method.

For obtaining the basic equation of buckling and vibration by finite deflection method, we use linear method for simplification. In approaching this problem, we consider two cases, for spaced members with strain loaded and other without strain.

To add, in order to afford better understanding about several basic equations, we analyse a few simple examples.

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SEASONAL FLUCTUATION OF BUILDING STARTS BY TYPE OF OWNERS

by RYOICHI SHIMADA, Assistant of Tokyo Metropolitan
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Pattern and stability of seasonal fluctuation of building starts are various by type of owners. Besides, the seasonal fluctuation pattern itself seems to change gradually. The pattern of "individual" is the most stable and "companies and other corporations" is the least. The pattern of individuals is significant. The peak comes in the summer and it does not change over the years. The pattern of "governments" is also significant and the peak comes at the end of the year. The peak shifts during the former half of the period of analysis. The pattern of "companies" is not significant because of strong irregular factors. It also seems to have some tendency to shift to the opposite direction. The government and others are in a complementary relation imperfectly with each