LIMIT ANALYSIS OF BEAM-COLUMN CONNECTIONS (V)

—Connections of Box-Column to Wide-flange Beam
Subjected to Anti-symmetrical Bending—

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§ 1. Introduction

In the previous papers, the limit analyses of wide-flange beam-to-column connections subjected to strong-axis bending were presented¹⁾. In this paper, the authors analyze the yield moments of cruciform connections, consisting of a box-column and wide-flange beam and subjected to anti-symmetrical bending as shown in Fig. 1. Assuming that the flanges of beam are strong enough in comparison with the column strength, we may replace the bending moments of beam with concentrated forces F and analyze the simplified model shown in Fig. 2. The dimensions of connection are shown in Fig. 2, namely, the sides of square column are $2d$, the beam-depth $2\alpha d$, the thickness of the column-plates t , and that of the connection-webs λt .

For the convenience of analysis, we develop the column-plates and take the coordinates x and y as shown in Fig. 3, so that the problem can be treated in the same plane.

The following assumptions are adopted in this analysis.

(1) The state of stress is specified by one normal stress σ_y and one shearing stress $\tau(=\tau_{xy})$ in Fig. 3.

(2) The material is rigid-perfectly-plastic and subject to von Mises' yield condition.

From the assumption (1), the equilibrium equations are given by

$$\left. \begin{aligned} \frac{\partial \tau}{\partial y} &= 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau}{\partial x} &= 0 \end{aligned} \right\} \dots\dots\dots (1.1)$$

and from the assumptions (1) and (2), we obtain

the following yield condition and specific power of dissipation D :

$$\sigma_y^2 + 3\tau^2 \leq \sigma_0^2 \dots\dots\dots (1.2)$$

$$D = \sqrt{\dot{\epsilon}_y^2 + \frac{1}{3}\dot{\gamma}^2} \dots\dots\dots (1.3)$$

Fig. 1 Type of Connection

Fig. 2 Simplified Model for Analysis Fig. 3 Development of Column plates

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where σ_0 is the yield stress in simple tension, and $\dot{\epsilon}_y$ and $\dot{\gamma}$ denote normal and shearing strain-rates, respectively.

§ 2. Statically admissible stress fields

1) Case $\lambda \leq 1$

To obtain statically admissible stress fields, the column plates are divided by lines of discontinuity as shown in Fig. 4 and the stresses in Table 1 are assumed²⁾.

Fig. 4 Stress Field for $\lambda \leq 1$

Fig. 5 Stress Distributions for $\lambda \leq 1$

Table 1

Stress Region	σ_y	τ
(1)	0	0
(2)	0	$\frac{\sigma_0}{\sqrt{3}}$
(3)	$\lambda \sigma_0 \sin \frac{\sqrt{3}}{\alpha} \left(\frac{x}{d} - \xi_1 \right)$	0
(4)	$\sigma_0 \frac{y}{\alpha d} \sin \frac{\sqrt{3}}{\alpha} \left(\frac{x}{d} - \xi_1 \right)$	$\frac{\sigma_0}{\sqrt{3}} \cos \frac{\sqrt{3}}{\alpha} \left(\frac{x}{d} - \xi_1 \right)$

These stresses satisfy the equilibrium eqs. (1.1), the discontinuity conditions of stresses on the boundaries of each region, and the boundary condition that shearing stress τ vanishes along the boundary $x=2d$, if we take ξ_1 in Table 1 as

$$\xi_1 = 2 - \frac{\pi}{2} \cdot \frac{\alpha}{\sqrt{3}} \quad \dots \dots \dots (2.1)$$

Moreover, these stresses do not violate the yield condition (1.2) in every region, and hence they are statically admissible. The assumed stress distributions are sketched in Fig. 5.

2) Case $\lambda > 1$

In this case, the stresses in Table 2 are assumed in each region shown in Fig. 6, and parameters ξ_1 and ξ_2 in Table 2 are taken as

Fig. 6 Stress Field for $\lambda > 1$

Fig. 7 Stress Distribution for $\lambda > 1$

$$\xi_1 = 2 - \frac{\alpha}{\sqrt{3}} \left(\sqrt{\lambda^2 - 1} + \sin^{-1} \frac{1}{\lambda} \right) \dots\dots\dots (2.2 a)$$

$$\xi_2 = 2 - \frac{1}{\sqrt{3}} \sqrt{\lambda^2 - 1} \dots\dots\dots (2.2 b)$$

Table 2

Stress Region	σ_y	τ
(1)	0	0
(2)	0	$\frac{\sigma_0}{\sqrt{3}}$
(3)	$\lambda \sigma_0 \sin \frac{\sqrt{3}}{\alpha} \left(\frac{x}{d} - \xi_1 \right)$	0
(4)	$\sigma_0 \frac{y}{\alpha d} \sin \frac{\sqrt{3}}{\alpha} \left(\frac{x}{d} - \xi_1 \right)$	$\frac{\sigma_0}{\sqrt{3}} \cos \frac{\sqrt{3}}{\alpha} \left(\frac{x}{d} - \xi_1 \right)$
(5)	σ_0	0
(6)	$\frac{\sigma_0}{\lambda} \frac{y}{\alpha d}$	$\frac{\sigma_0}{\lambda} \frac{1}{\alpha} \left(2 - \frac{x}{d} \right)$

Similarly to the previous case, we can also verify that these stresses are statically admissible. Fig. 7 shows this stress distributions.

§ 3. Lower bounds of the yield moment

The statically admissible stress fields obtained in the preceding paragraph may be classified into nine cases in Table 3, according to the ranges of ξ_1 and ξ_2 . This classification can be also represented by the parameters α and λ as shown in Fig. 8.

Fig. 8 Classification of Statical Analyses**Table 3**

$\xi_1 \backslash \xi_2$	$\xi_1 < 0$	$0 \leq \xi_1 \leq 1$	$1 \leq \xi_1 \leq 2$	λ
$\xi_2 = 2$	Case A ₁	Case A ₂	Case A ₃	$\lambda \leq 1$
$2 \geq \xi_2 \geq 1$	Case B ₁	Case B ₂	Case B ₃	$\lambda > 1$
$1 \geq \xi_2 \geq 0$	Case C ₁	Case C ₂	—	
$0 \geq \xi_2$	Case D ₁	—	—	

In the cases of A₁, A₂ and A₃, the lower bounds of yield moment M are obtained in use of the normal stress in the region (3) in Table 1, as follows :

For the case A₁

$$M=4t\left[\int_0^d x\sigma_y^{(3)}dx+\int_d^{2d} d\sigma_y^{(3)}dx\right]=4\lambda td^2\sigma_0\frac{\alpha^2}{3}\left[\sin\frac{\sqrt{3}}{\alpha}(1-\xi_1)+\sin\frac{\sqrt{3}}{\alpha}\xi_1\right] \quad \dots\dots\dots(3.1)$$

For the case A₂

$$M=4t\left[\int_{\xi_1 d}^d x\sigma_y^{(3)}dx+\int_d^{2d} d\sigma_y^{(3)}dx\right]=4\lambda td^2\sigma_0\frac{\alpha^2}{3}\left[\sin\frac{\sqrt{3}}{\alpha}(1-\xi_1)+\frac{\sqrt{3}}{\alpha}\xi_1\right] \quad \dots\dots\dots(3.2)$$

For the case A₃

$$M=4t\int_{\xi_1 d}^{2d} d\sigma_y^{(3)}dx=4\lambda td^2\sigma_0\frac{\alpha}{\sqrt{3}} \quad \dots\dots\dots(3.3)$$

where superscript (3) refers to the region (3).

In the cases of B, C and D, the lower bounds M are obtained from the normal stresses in the regions (3) and (5) in Table 2, as follows :

For the case B₁

$$M=4t\left[\int_0^d x\sigma_y^{(3)}dx+\int_d^{\xi_1 d} d\sigma_y^{(3)}dx+\int_{\xi_2 d}^{2d} d\sigma_y^{(3)}dx\right]=4\lambda td^2\sigma_0\frac{\alpha^2}{3}\left[\sin\frac{\sqrt{3}}{\alpha}(1-\xi_1)+\sin\frac{\sqrt{3}}{\alpha}\xi_1\right] \quad \dots\dots\dots(3.4)$$

For the case B₂

$$M=4t\left[\int_{\xi_1 d}^d x\sigma_y^{(3)}dx+\int_d^{\xi_2 d} d\sigma_y^{(3)}dx+\int_{\xi_2 d}^{2d} d\sigma_y^{(3)}dx\right]=4\lambda td^2\sigma_0\frac{\alpha^2}{3}\left[\sin\frac{\sqrt{3}}{\alpha}(1-\xi_1)+\frac{\sqrt{3}}{\alpha}\xi_1\right] \quad \dots\dots\dots(3.5)$$

For the case B₃

$$M=4t\left[\int_{\xi_1 d}^{\xi_2 d} d\sigma_y^{(3)}dx+\int_{\xi_2 d}^{2d} d\sigma_y^{(5)}dx\right]=4\lambda td^2\sigma_0\frac{\alpha}{\sqrt{3}} \quad \dots\dots\dots(3.6)$$

For the case C₁

$$M=4t\left[\int_0^{\xi_2 d} x\sigma_y^{(3)}dx+\int_{\xi_2 d}^d x\sigma_y^{(5)}dx+\int_d^{2d} d\sigma_y^{(5)}dx\right]=4\lambda td^2\sigma_0\frac{\alpha^2}{3}\left[\frac{\lambda^2+1-3/\alpha^2}{2\lambda}+\sin\frac{\sqrt{3}}{\alpha}\xi_1\right] \quad \dots\dots\dots(3.7)$$

For the case C₂

$$M=4\lambda\left[\int_{\xi_1 d}^{\xi_2 d} x\sigma_y^{(3)}dx+\int_{\xi_2 d}^d x\sigma_y^{(5)}dx+\int_d^{2d} d\sigma_y^{(5)}dx\right]=4\lambda td^2\sigma_0\frac{\alpha^2}{3}\left[\frac{\lambda^2+1-3/\alpha^2}{2\lambda}+\frac{\sqrt{3}}{\alpha}\xi_1\right] \quad \dots\dots\dots(3.8)$$

For the case D₁

$$M=4t\left[\int_0^d x\sigma_y^{(5)}dx+\int_d^{2d} d\sigma_y^{(5)}dx\right]=6td^2\sigma_0 \quad \dots\dots\dots(3.9)$$

where the superscripts of σ_y refer to the names of regions.

§ 4. Upper bounds of the yield moment

Upper bounds of the yield moment can be derived from kinematically admissible velocity fields. In the first place, a kinematically admissible velocity field shown in Fig. 9 is assumed³⁾. As seen in Fig. 9, plastic flow with unit width occurs closely inside of the beam-flange, where a shearing mechanism shown in Fig. (a) and a bending mechanism shown in Fig. (b) form simultaneously. And strain rates $\dot{\epsilon}_y$ and $\dot{\gamma}$ in the flow range are given by

$$\left. \begin{aligned} \dot{\epsilon}_y &= x\dot{\kappa}, \quad \dot{\gamma} = \dot{\phi} \quad (0 \leq x \leq d) \\ \dot{\epsilon}_y &= d\dot{\kappa}, \quad \dot{\gamma} = 0 \quad (d \leq x \leq 2d) \end{aligned} \right\} \quad \dots\dots\dots(4.1)$$

where $\dot{\kappa}$ and $\dot{\phi}$ denote the curvature and shearing velocities, respectively, as shown in Fig. 9 (a) and (b). Substitution of eqs. (4.1) into eq. (1.3) then leads to the following specific power of dissipation

D :

Fig. 9 Kinematically Admissible Velocity Field

$$D = \begin{cases} \sigma_0 \dot{\kappa} \sqrt{x^2 + \nu^2 d^2} & (0 \leq x \leq d) \\ d \sigma_0 \dot{\kappa} & (d \leq x \leq 2d) \end{cases} \quad (4.2a)$$

$$(4.2b)$$

where ν is defined by

$$\nu = \frac{\dot{\phi}}{\sqrt{3} \dot{\kappa} d} \quad (4.3)$$

The total power of dissipation D_i in this velocity field is obtained from eqs. (4.2), as follows :

$$D_i = 8 \lambda t \int_0^{2d} D dx = 4 \lambda t d^2 \sigma_0 \dot{\kappa} \left[\sqrt{1 + \nu^2} + \nu^2 \sinh^{-1} \frac{1}{\nu} + 2 \right] \quad (4.4)$$

Referring to Fig. 9, and using eq. (4.3) and the relation

$$F = M / \alpha d \quad (4.5)$$

the power W of external loads M and F is obtained as :

$$W = 2 M \dot{\kappa} + 2 M \dot{\phi} = 2 M \dot{\kappa} \left[1 + \frac{\sqrt{3}}{\alpha} \nu \right] \quad (4.6)$$

Equating eq. (4.4) to eq. (4.6), the bending moment M is given by

$$M = \frac{\sqrt{1 + \nu^2} + \nu^2 \sinh^{-1} \frac{1}{\nu} + 2}{1 + \frac{\sqrt{3}}{\alpha} \nu} 2 \lambda t d^2 \sigma_0 \quad (4.7)$$

The bending moment M in the above equation is minimized when

$$\frac{\alpha}{\sqrt{3}} = \frac{1}{2} \frac{\sqrt{1 + \nu^2} - \nu^2 \sinh^{-1} \frac{1}{\nu} + 2}{\nu \sinh^{-1} \frac{1}{\nu}} \quad (4.8)$$

and its minimum value is

$$M = 4 \lambda t d^2 \sigma_0 \frac{\alpha}{\sqrt{3}} \nu \sinh^{-1} \frac{1}{\nu} \quad (4.9)$$

which may be an upper bound of the yield moment.

To simplify the expressions of M and α , introducing a new parameter ω defined by

$$\omega = \sinh^{-1} \frac{1}{\nu} \quad (4.10)$$

then eq. (4.9) and eq. (4.8) yield

$$M = 4 \lambda t d^2 \sigma_0 \frac{\alpha}{\sqrt{3}} \frac{\omega}{\sinh \omega} \quad (4.11a)$$

$$\frac{\alpha}{\sqrt{3}} = \frac{\sinh 2\omega - 2\omega}{4\omega \sinh \omega} + \frac{\sinh \omega}{\omega} \quad (4.11b)$$

Next, we assume another kinematically admissible velocity field shown in Fig. 10. In this figure, the region OABC deforms in shear with angular velocity $\dot{\phi}$, and BD and DF make discontinuous lines with relative velocity $\dot{e}_y$ in the direction of y -axis, and all the other parts remain rigid. In order to get better solutions, we place the discontinuous lines BD and DF inside of the beam-flange in the case that $\lambda \leq 1$ and outside in the case that $\lambda > 1$. Referring to Fig. 10, the shear-strain-rate $\dot{\gamma}$ in the region OABC and relative velocities $\dot{e}_y$ on the lines BD and DF are given by

$$\dot{\gamma} = \dot{\phi} \quad (4.12)$$

$$\dot{e}_y = \begin{cases} (x - \xi d) \dot{\phi} & \text{for } \xi d \leq x \leq d \\ (d - \xi d) \dot{\phi} & \text{for } d \leq x \leq 2d \end{cases} \quad (4.13)$$

From eq. (1.3), the power of dissipation D_1 in the region OABC is obtained as

$$D_1 = \frac{\sigma_0}{\sqrt{3}} \dot{\phi} \alpha \lambda \xi t d^2 \quad (4.14)$$

Fig. 10 Kinematically Admissible Velocity Field

and the powers of dissipation D_2 on the line BD and D_3 on the line DF are given by

$$D_2 = \begin{cases} 4 \lambda t d^2 \sigma_0 \dot{\phi} (1-\xi)^2 & \text{for } \lambda \leq 1 \\ 4 t d^2 \sigma_0 \dot{\phi} (1-\xi)^2 & \text{for } \lambda > 1 \end{cases} \quad (4.15 \text{ a})$$

and

$$D_3 = \begin{cases} 8 \lambda t d^2 \sigma_0 \dot{\phi} (1-\xi) & \text{for } \lambda \leq 1 \\ 8 t d^2 \sigma_0 \dot{\phi} (1-\xi) & \text{for } \lambda > 1 \end{cases} \quad (4.16 \text{ a})$$

Therefore, the total power of dissipation D_i in this velocity field, being the sum of D_1 , D_2 and D_3 , we obtain

$$D_i = D_1 + D_2 + D_3 = \begin{cases} 8 \lambda t d^2 \sigma_0 \dot{\phi} \left[\frac{\alpha}{\sqrt{3}} \xi + \frac{1}{2} (1-\xi)^2 + (1-\xi) \right] & \text{for } \lambda \leq 1 \\ 8 t d^2 \sigma_0 \dot{\phi} \left[\frac{\lambda \alpha}{\sqrt{3}} \xi + \frac{1}{2} (1-\xi)^2 + (1-\xi) \right] & \text{for } \lambda > 1 \end{cases} \quad (4.17 \text{ a})$$

Referring to Fig. 10 and using eq. (4.5), the power W of external loads is

$$W = 2 F \alpha d \dot{\phi} = 2 M \dot{\phi} \quad (4.18)$$

Equating eq. (4.17) to eq. (4.18), we obtain

$$M = \begin{cases} 2 \lambda t d^2 \sigma_0 \left[\left\{ \xi - \left(2 - \frac{\alpha}{\sqrt{3}} \right) \right\}^2 + 3 - \left(2 - \frac{\alpha}{\sqrt{3}} \right)^2 \right] & \text{for } \lambda \leq 1 \\ 2 t d^2 \sigma_0 \left[\left\{ \xi - \left(2 - \frac{\lambda \alpha}{\sqrt{3}} \right) \right\}^2 + 3 - \left(2 - \frac{\lambda \alpha}{\sqrt{3}} \right)^2 \right] & \text{for } \lambda > 1 \end{cases} \quad (4.19 \text{ a})$$

in which

$$0 \leq \xi \leq 1 \quad (4.20)$$

Hereon, it is necessary to obtain the minimum value of eq. (4.19) under the condition of eq. (4.20). Fig. 11 shows the relation between ξ and M expressed in eq. (4.19 a), and referring to this figure we obtain the minimum value of M for $\lambda \leq 1$ as follows :

$$M = \begin{cases} 6 \lambda t d^2 \sigma_0 & \text{for } \frac{\alpha}{\sqrt{3}} \geq 2 \\ 2 \lambda t d^2 \sigma_0 \left[3 - \left(2 - \frac{\alpha}{\sqrt{3}} \right)^2 \right] & \text{for } 2 \geq \frac{\alpha}{\sqrt{3}} \geq 1 \\ 4 \lambda t d^2 \sigma_0 \frac{\alpha}{\sqrt{3}} & \text{for } 1 \geq \frac{\alpha}{\sqrt{3}} \end{cases} \quad (4.21)$$

Similarly, for $\lambda > 1$ we obtain

$$M = \begin{cases} 6 t d^2 \sigma_0 & \text{for } \frac{\alpha}{\sqrt{3}} \geq \frac{2}{\lambda} \\ 2 t d^2 \sigma_0 \left[3 - \left(2 - \frac{\lambda \alpha}{\sqrt{3}} \right)^2 \right] & \text{for } \frac{2}{\lambda} \geq \frac{\alpha}{\sqrt{3}} \geq \frac{1}{\lambda} \\ 4 \lambda t d^2 \sigma_0 \frac{\alpha}{\sqrt{3}} & \text{for } \frac{1}{\lambda} \geq \frac{\alpha}{\sqrt{3}} \end{cases} \quad (4.22)$$

Comparing the results given by eq. (4.11), eq. (4.21) and eq. (4.22) with each other, it is found that the following give the best upper bounds of the yield moment for the regions (a) to (d) shown in Fig. 12.

$$(a) \quad M = 4 \lambda t d^2 \sigma_0 \frac{\alpha}{\sqrt{3}} \frac{\omega}{\sinh \omega} \quad (4.23)$$

$$(b) \quad M = 4 \lambda t d^2 \sigma_0 \frac{\alpha}{\sqrt{3}} \quad (4.24)$$

Fig. 11 Relation between ξ and M (Case $\lambda < 1$)

Fig. 12 Classification of Kinematical Analyses

$$(c) \quad M = 2 t d^2 \sigma_0 \left[3 - \left(2 - \frac{\lambda \alpha}{\sqrt{3}} \right)^2 \right] \dots\dots\dots (4.25)$$

$$(d) \quad M = 6 t d^2 \sigma_0 \dots\dots\dots (4.26)$$

§ 5. Conclusion

The lower and upper bounds obtained in § 3 and § 4 are graphically shown in Fig. 13 and Fig. 14, where M_0 is the full plastic moment of the column defined by

$$M_0 = 6 t d^2 \sigma_0 \dots\dots\dots (5.1)$$

Fig. 13 Reduction-ratio of Yield Moment

Fig. 14 Reduction-ratio of Yield Moment

It is concluded from these figures that the connections cannot sufficiently transmit the full plastic moment of column to the beam without reinforcement of the connection-webs, especially when the beam-depth is comparatively small.

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柱はり接合部の極限解析(V) (梗概)

——箱型柱とH形はりの十字形接合部——

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§1. はしがき

前報までに、ワイドフランジ材の柱はり接合部が強軸曲げを受ける場合の降伏モーメントを解析した¹⁾。本報では、図—1に示すような箱形断面柱とH形断面はりの十字形接合部が逆対称曲げを受ける場合の極限解析を行ない、接合部の降伏モーメントを求める。解析にあたり、はりフランジは十分な強度を持つものとして、はりの曲げモーメントを図—2に示すような集中力 F に置き換えたモデルによって考察する。接合部の寸法を図—2に示す。すなわち、柱断面は一辺が $2 \cdot d$ の正方形、はりせいは $2 \cdot \alpha \cdot d$ 、柱の板厚は t 、接合部ウェブの板厚は $\lambda \cdot t$ とする。

解析の便宜上、図—2において柱フランジの中心線で切り離し、展開して、図—3のような x - y 座標を設定し、問題が同一平面内で取扱えるようにする。また本解析においては次の二つの仮定を設ける。

(1) 応力状態は、図—3の座標で、一つの垂直応力度 σ_y と一つのせん断応力度 $\tau(=\tau_{xy})$ のみによってあらわされる。

(2) 材料は完全剛塑性体で、von Mises の降伏条件に従う。

仮定(1)より、つりあい条件は式(1.1)のように得られ、仮定(1)と(2)より、降伏条件および塑性流れが起っているときのエネルギー散逸率 D はそれぞれ式(1.2)および式(1.3)で表わされる。ここに σ_0 は単純引張り時の降伏点であり、 $\dot{\epsilon}_y$ と $\dot{\gamma}$ はそれぞれ垂直ひずみ速度およびせん断ひずみ速度である。

§2. 静的許容応力場

1) $\lambda \leq 1$ の場合

静的許容応力場を求めるために、応力の不連続線により、図—4に示すような領域に分け、各領域に対し表—1に示すような応力分布を仮定する²⁾。 ϵ_1 を式(2.1)のように与えると、表—1の応力はつりあい式(1.1)と各領域の境界における応力の不連続条件および $x=2d$ で $\tau=0$ の条件を満足する。さらにすべての領域において、降伏条件式(1.2)を侵かさないことが確かめられるから、

これらの応力状態は静的許容である。この応力分布の概略を図—5に示す。

2) $\lambda > 1$ の場合

図—6に示す領域に対し、応力を表—2に示すように仮定する。ここで ϵ_1 と ϵ_2 を式(2.2)で与えれば、1)の場合と同様にして、表—2の応力が静的許容であることが確かめられる。この応力分布を図—7に示す。

§3. 降伏モーメントの下界

前項で求めた静的許容応力場は ϵ_1 と ϵ_2 の大きさにより表—3に示すような9個の場合に分類される。各場合の α と λ の関係を図—8に示す。Case A₁, A₂, A₃における曲げモーメントは表—1の領域(3)の垂直応力度 $\sigma_y^{(3)}$ により、式(3.1)、式(3.2)、式(3.3)のように求められる。同様にして、Case B, C, Dでの曲げモーメントは表—2の領域(3)、(5)の垂直応力度 $\sigma_y^{(3)}$ および $\sigma_y^{(5)}$ を用いて式(3.4)から式(3.9)のように求められる。

§4. 降伏モーメントの上界

降伏モーメントの上界は運動的許容速度場から求められる。まず図—9に示すような運動的許容速度場を仮定する³⁾。すなわち、はりフランジの内側でこれの近傍に単位巾の領域をとり、そこにせん断機構図—9(a)と曲げ機構同図(b)が同時に起ると仮定する。この速度場で塑性流れが起っている部分でのひずみ速度 $\dot{\epsilon}_y$ および $\dot{\gamma}$ は式(4.1)で与えられる。同式中 $\dot{\phi}$ および $\dot{\kappa}$ はそれぞれ図—9(a)、(b)に示されたせん断変形角速度および曲率速度を表わす。単位体積あたりのエネルギー散逸率 D は、式(4.1)を式(1.3)に代入して式(4.2)のように得られる。同式で ν は式(4.3)とする。よってこの速度場における全エネルギー散逸率 D_i は式(4.2)を用いて式(4.4)のように求められる。また外力 M と F による仕事率 W は図—9を参照し、式(4.3)と式(4.5)を用いて、式(4.6)となる。曲げモーメントは式(4.4)と式(4.6)を等置して、式(4.7)となり、これはパラメーター ν が式(4.8)を満たすとき最小値式(4.9)を取る。式(4.9)は一つの上界である。いま式(4.10)の置き換えにより式(4.9)、式(4.8)の表現を簡単にすると式(4.11a)、式(4.11b)となる。

次に図—10に示す運動的許容速度場を仮定する。す

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なわち、同図 (b) に示すように、領域 $OABC$ はせん断変形角速度 $\dot{\phi}$ を持ち、線分 BD , DF は変形の不連続線となって y 方向の相対速度 $\dot{e}_y$ を持ち、その他の部分はすべて剛であるとする。またより良い解が得られるように、不連続線 BD と DF は $\lambda \leq 1$ の場合には、はりフランジの内側に、 $\lambda > 1$ の場合には外側に起ると仮定する。図—10 を参照して領域 $OABC$ 内のせん断ひずみ速度 $\dot{\gamma}$ および不連続線 BD , DF 上の相対速度 $\dot{e}_y$ はそれぞれ式 (4.12), 式 (4.13) のようになる。

領域 $OABC$ 内のエネルギー散逸率 D_1 は式 (1.3) を用いて式 (4.14), 不連続線 BD , DF 上の散逸率 D_2 , D_3 はそれぞれ式 (4.15), 式 (4.16) のようになる。これらよりこの速度場の全エネルギー散逸率 D_i は、 D_1 , D_2 , D_3 の総和をとって式 (4.17) となる。また図—10 を参照して、外力のなす仕事率 W は式 (4.18) で表わされるので、これと D_i を等置して曲げモーメントは式 (4.19) のように得られる。但し ε には式 (4.20) の制限がある。

さてここで式 (4.19) をより良い解とするために、式

(4.20) の制限のもとで式 (4.19) の最小値を求める。図 11 は $\lambda \leq 1$ の場合すなわち式 (4.19 a) の M と ε の関係を示したものであるが、同図を参照してこの場合の M の最小値は式 (4.21) のように得られる。同様にして $\lambda > 1$ の場合には最小値は式 (4.22) となる。以上降伏モーメントの上界として式 (4.11), 式 (4.21), 式 (4.22) が求められたが、これらを比較して、最良の上界 M は図—12 に示す各領域 ($\alpha - \lambda$ の範囲) に対してそれぞれ式 (4.23), 式 (4.24), 式 (4.25), 式 (4.26) で与えられる。

§5. むすび

前二項で得られた下界と上界を図示すると図—13, 図—14 のようになる。図は式 (5.1) で与えられる柱の全塑性モーメント M_0 で上, 下界の M を除した形 M/M_0 (接合部の曲げモーメント低下率) で示してある。これらの図から次のことが広える。はりせいが比較的小さい場合には、接合部の板厚を厚くして補強しないと、柱の全塑性モーメントを十分はりに伝達することができない。

admixture, light weight aggregates, crushed stone and different size of sand, and which are varied mixing proportion in large extent.

Reference

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EVALUATION OF EARTHQUAKE INTENSITY BASED ON DAMAGE OF WOODEN BUILDINGS

By Dr. HIROYOSHI KOBAYASHI*, Prof. of Tokyo Institute of Technology. and SUMIO NAGAHASHI**, Assistant of Tokyo Institute of Technology, Members of A.I.J.

From the point of view of structural engineering, a scale of evaluating intensity of earthquake motions is desirable to represent exactly what degree the earthquake motions cause damage to structures.

Usually, as this kind of scale, maximum magnitude of acceleration and response spectrum are used, but there were several cases in which little damage to structures are inconsistent with large value of maximum magnitude of acceleration and response spectrum. For example, maximum magnitude of acceleration of the earthquake motions recorded at Kushiro in 1962 is 513 gal, and its response spectrum shows remarkable peak at $T=0.33$ second (Fig. 2), that is, 2.2 g for 5 percents damping and 4.8 g for 1 percent damping. But little structures suffered damage.

Then authors proposed to evaluate intensity of earthquake motions by means of another kind of scale, that is, damage ratio of wooden buildings due to earthquake motions.

Damage to a wooden building is defined by elasto-plastic response of one mass system due to earthquake motions, and damage ratio is the total of damage to each group of structures (the structures of a group are different in the period from those of other groups), and is represented by idealised normal distribution curve in relation to maximum magnitude of acceleration of earthquake motions. Here, earthquake motions are used by modifying geometrically the acceleration amplitudes of the of the recorded earthquake motions, so the frequency characteristics are not changed.

By this scale, the intensity of the earthquake motions whose response spectrum has a remarkable peak, for instance, the above-mentioned earthquake motions at Kushiro, is not so large even for structure of the peak period, and is rather small for structure of other period. On the other hand, the intensity of the earthquake motions whose response spectrum has several peaks is generally large.

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ELASTIC ANALYSIS OF SHEAR WALLS LOADED SYMMETRICALLY WITH REGARD TO THEIR LONGITUDINAL AND TRANSVERSAL CENTER LINE —Part II—

—Equations for Conditions which are Necessary to Define the Unknown
Coefficients, and Numerical Results of an Example (See Page 29)

By Dr. MASAhide TOMII Prof. of Kyushu Univ. and IKUO TOKUHIRO, Assistant of Kyushu Univ. Members of A.I.J.

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LIMIT ANALYSIS OF BEAM-COLUMN CONNECTION (V)

—Connections of Box-Column to Wide-flange Beam Subjected
to Anti-Symmetrical Bending— (See Page 43)

By Dr. HISASHI TANAKA Prof., Inst. of Indu. Science, Univ. of Tokyo and TSUNEO SHIGENOBU Assist., Inst. of Indu. Sci., Univ. of Tokyo, Members of A.I.J.