ELASTIC ANALYSIS OF SHEAR WALLS LOADED ANTIMETRICALLY WITH REGARD TO LONGITUDINAL CENTER LINE AND SYMMETRICALLY WITH REGARD TO TRANSVERSAL CENTER LINE PART I

—Airy's Stress Function, Stresses, Displacements, Etc.—

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INTRODUCTION

In this paper, the formulas for the stresses, strains, and deformations of the symmetrical shear walls loaded antimetrically with regard to the longitudinal center line and symmetrically with regard to the transversal center line (see Fig. 4.1) are induced. In general analysis¹⁾²⁾, these were called shear wall III subjected to the forces and moments of type III.

The external forces in the x -direction are not given arbitrarily, but the eq. 4.1 for the equilibrium must be satisfied.

$$\int_0^a (q_{a2} + q_{ae}) dx + H_0 = - \int_0^b w_{b2} dy \dots\dots\dots (4.1)$$

The four corner joints of the boundary frame are assumed as rigid zone (see Fig. 4.1), and the quarter shear wall ($x \geq 0, y \geq 0$) is analysed by considering the antimetric loading conditions with regard to the longitudinal center line and the symmetrical loading conditions with regard to the transversal center line.

Dr. Yoshikatsu Tsuboi already analysed the similar shear walls³⁾. The differences between Dr. Y. Tsuboi's analysis and this analysis were described in other papers¹⁾²⁾, and the details of the comparison will be reported in another paper.

The general analysis was reported in other Transactions¹⁾²⁾ as follows.

§ 1.1 Equilibrium for the External Plane Forces and Moments¹⁾

§ 1.2 Selection of Serial Stress Functions¹⁾

§ 1.3 Elements Not Satisfied only with Serial Stress Function¹⁾

§ 1.4 Derivation of the Equations for the Continuity Conditions on the Boundaries between the Wall and Frame¹⁾

§ 1.5 Equations for Conditions²⁾

§ 1.6 Selection of Algebraic Stress Functions²⁾

§ 1.7 Unit Stresses, Unit Elongations, and Displacements in the Wall²⁾

§ 1.8 Stresses, Displacements, etc. in the Upper Beam and Right Column of the Frame²⁾

SYMBOLS

H_0, V_0 : Horizontal and vertical external forces applied on each corner joint of the frame

M_0 : External moment applied on each corner joint of the frame

q_{a2}, q_{b2} : External axial shearing force (per unit length) distributed on the outside face of the beam, and that of the column

w_{a2}, w_{b2} : Sum of the external normal force (per unit length) distributed on the outside face and the

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external shearing force (per unit length, orthogonal to the axis of the member) distributed on both side faces with regard to the beam, and the similar sum with regard to the column

q_{ae}, q_{be} : Sum of the external axial shearing forces (per unit length and with the same eccentric distance from the center axis of the member) distributed on both side faces with regard to the beam, and the similar sum with regard to the column

e_a, e_b : Eccentric distance (from the center axis of the member) of q_{ae} , and that of q_{be}

t : Thickness of the wall

D_a, D_b : Depth of the beam, and that of the column

A_a, A_b : Cross-sectional area of the beam, and that of the column

I_a, I_b : Geometric moment of inertia of the beam, and that of the column

Z_a, Z_b : Section modulus of the beam, and that of the column

E : Young's modulus of elasticity

ν : Poisson's ratio

Additional symbols used in this paper are defined where they first appear.

§ 4.1 Airy's Stress Function

$$\mathbb{I}F = \sum_m \mathbb{I}F_{sm} + \sum_n \mathbb{I}F_{sn} + \mathbb{I}F_a \quad (4.2)$$

where,

$$\mathbb{I}F_{sm} = (C'_{am} \cosh \eta_m + D'_{am} \eta_m \sinh \eta_m) \sin \xi_m$$

$$\mathbb{I}F_{sn} = (A'_{bn} \sinh \xi_n + B'_{bn} \xi_n \cosh \xi_n) \cos \eta_n$$

$$\mathbb{I}F_a = F_1 \xi_a^3 \eta_b^2 - \frac{1}{5} \lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^5 + F_2 \xi_a \eta_b^4 + F_3 \xi_a^3 + F_4 \xi_a \eta_b^2$$

$\mathbb{I}F$: Stress function

$\mathbb{I}F_{sm}, \mathbb{I}F_{sn}$: Serial stress function (stress function in series)

$\mathbb{I}F_a$: Algebraic stress function

$C'_{am}, D'_{am}, A'_{bn}, B'_{bn}$: Unknown coefficients in $\mathbb{I}F_{sm}$ and $\mathbb{I}F_{sn}$

F_1, F_2, F_3, F_4 : Unknown coefficients in $\mathbb{I}F_a$

m : Odd numbers 1, 3, 5,

n : Even numbers 2, 4, 6,

$$\xi_a = \frac{x}{a}, \quad \xi_b = \frac{y}{b}, \quad \xi_m = \frac{m\pi}{2} \xi_a, \quad \xi_n = \frac{n\pi}{2} \xi_b, \quad \lambda_a = \frac{b}{a}, \quad \alpha_{am} = \frac{m\pi b}{2a},$$

$$\eta_b = \frac{y}{b}, \quad \eta_a = \frac{y}{a}, \quad \eta_n = \frac{n\pi}{2} \eta_b, \quad \eta_m = \frac{m\pi}{2} \eta_a, \quad \lambda_b = \frac{a}{b}, \quad \alpha_{bn} = \frac{n\pi a}{2b}$$

§ 4.2 Unit Stresses, Unit Elongations and Displacements in the Wall

The normal unit stresses σ_x and σ_y parallel to x - and y - axes respectively and the shearing unit stress τ in the rectangular coordinates x and y are given as follows.

Here the positive sign should be taken as shown in Fig. 4.2.

$$\begin{aligned} \sigma_y = \frac{\partial^2 \mathbb{I}F}{\partial x^2} = & - \sum_m \left(\frac{m\pi}{2a} \right)^2 (C'_{am} \cosh \eta_m + D'_{am} \eta_m \sinh \eta_m) \sin \xi_m \\ & + \sum_n \left(\frac{n\pi}{2b} \right)^2 \{ (A'_{bn} + 2B'_{bn}) \sinh \xi_n + B'_{bn} \xi_n \cosh \xi_n \} \cos \eta_n \\ & + \frac{2}{a^2} \{ 3F_1 \xi_a \eta_b^2 - 2\lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^3 + 3F_3 \xi_a \} \quad \dots\dots\dots (4.3) \end{aligned}$$

Fig. 4.1 The external forces and moments

Fig. 4.2 Unit stresses in the wall

$$\begin{aligned}\sigma_y|_{y=b} = & -\sum_m \left(\frac{m\pi}{2a}\right)^2 (C'_{am} \cosh \alpha_{am} + D'_{am} \alpha_{am} \sinh \alpha_{am}) \sin \xi_m \\ & + \sum_n (-1)^{\frac{n}{2}} \left(\frac{n\pi}{2b}\right)^2 \{(A'_{bn} + 2B'_{bn}) \sinh \xi_n + B'_{bn} \xi_n \cosh \xi_n\} \\ & + \frac{2}{a^2} \{3F_1 \xi_a - 2\lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^3 + 3F_3 \xi_a\} \dots\dots\dots (4.4)\end{aligned}$$

$$\begin{aligned}\sigma_x = \frac{\partial^2 \mathbb{W} F}{\partial x \partial y} = & \sum_m \left(\frac{m\pi}{2a}\right)^2 \{(C'_{am} + 2D'_{am}) \cosh \eta_m + D'_{am} \eta_m \sinh \eta_m\} \sin \xi_m \\ & - \sum_n \left(\frac{n\pi}{2b}\right)^2 (A'_{bn} \sinh \xi_n + B'_{bn} \xi_n \cosh \xi_n) \cos \eta_n + \frac{2}{b^2} \{F_1 \xi_a^3 + 6F_2 \xi_a \eta_b^2 + F_4 \xi_a\} \dots\dots\dots (4.5)\end{aligned}$$

$$\begin{aligned}\sigma_x|_{x=a} = & \sum_m (-1)^{\frac{m-1}{2}} \left(\frac{m\pi}{2a}\right)^2 \{(C'_{am} + 2D'_{am}) \cosh \eta_m + D'_{am} \eta_m \sinh \eta_m\} \\ & - \sum_n \left(\frac{n\pi}{2b}\right)^2 (A'_{bn} \sinh \alpha_{bn} + B'_{bn} \alpha_{bn} \cosh \alpha_{bn}) \cos \eta_n + \frac{2}{b^2} (F_1 + 6F_2 \eta_b^2 + F_4) \dots\dots\dots (4.6)\end{aligned}$$

$$\begin{aligned}\tau = -\frac{\partial^2 \mathbb{W} F}{\partial x \partial y} = & -\sum_m \left(\frac{m\pi}{2a}\right)^2 \{(C'_{am} + D'_{am}) \sinh \eta_m + D'_{am} \eta_m \cosh \eta_m\} \cos \xi_m \\ & + \sum_n \left(\frac{n\pi}{2b}\right)^2 \{(A'_{bn} + B'_{bn}) \cosh \xi_n + B'_{bn} \xi_n \sinh \xi_n\} \sin \eta_n - \frac{2}{ab} (3F_1 \xi_a^2 \eta_b + 2F_2 \eta_b^3 + F_4 \eta_b) \dots\dots (4.7)\end{aligned}$$

$$\tau|_{y=b} = \sum_m \tau_m \cos \xi_m - \frac{2}{ab} (3F_1 \xi_a^2 + 2F_2 + F_4) \dots\dots\dots (4.8)$$

where,

$$\tau_m = -\left(\frac{m\pi}{2a}\right)^2 \{(C'_{am} + D'_{am}) \sinh \alpha_{am} + D'_{am} \alpha_{am} \cosh \alpha_{am}\} \dots\dots\dots (4.9)$$

$$\tau|_{x=a} = \sum_n \tau_n \sin \eta_n - \frac{2}{ab} (3F_1 \eta_b + 2F_2 \eta_b^3 + F_4 \eta_b) \dots\dots\dots (4.10)$$

where,

$$\tau_n = \left(\frac{n\pi}{2b}\right)^2 \{(A'_{bn} + B'_{bn}) \cosh \alpha_{bn} + B'_{bn} \alpha_{bn} \sinh \alpha_{bn}\} \dots\dots\dots (4.11)$$

The unit elongations ε_x and ε_y parallel to x - and y - axes respectively, and the shearing unit strain γ are given as follows from eqs. 4.3, 4.5, and 4.7.

Here the positive sign of γ should be taken if τ is caused by positive τ .

$$\begin{aligned}\varepsilon_x = \frac{\partial u}{\partial x} = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{E} \left[\sum_m \left(\frac{m\pi}{2a}\right)^2 [C'_{am}(1+\nu) \cosh \eta_m + D'_{am} \{2 \cosh \eta_m + (1+\nu) \eta_m \sinh \eta_m\}] \sin \xi_m \right. \\ \left. - \sum_n \left(\frac{n\pi}{2b}\right)^2 [A'_{bn}(1+\nu) \sinh \xi_n + B'_{bn} \{2 \nu \sinh \xi_n + (1+\nu) \xi_n \cosh \xi_n\}] \cos \eta_n \right. \\ \left. + \frac{2}{a^2} [\lambda_b^2 (F_1 \xi_a^3 + 6F_2 \xi_a \eta_b^2 + F_4 \xi_a) - \nu \{3F_1 \xi_a \eta_b^2 - 2\lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^3 + 3F_3 \xi_a\}] \right] \dots\dots\dots (4.12)\end{aligned}$$

$$\begin{aligned}\varepsilon_x|_{y=b} = \frac{1}{E} \left[\sum_m \left(\frac{m\pi}{2a}\right)^2 [C'_{am}(1+\nu) \cosh \alpha_{am} + D'_{am} \{2 \cosh \alpha_{am} + (1+\nu) \alpha_{am} \sinh \alpha_{am}\}] \sin \xi_m \right. \\ \left. - \sum_n (-1)^{\frac{n}{2}} \left(\frac{n\pi}{2b}\right)^2 [A'_{bn}(1+\nu) \sinh \xi_n + B'_{bn} \{2 \nu \sinh \xi_n + (1+\nu) \xi_n \cosh \xi_n\}] \right. \\ \left. + \frac{2}{a^2} [\lambda_b^2 (F_1 \xi_a^3 + 6F_2 \xi_a + F_4 \xi_a) - \nu \{3F_1 \xi_a - 2\lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^3 + 3F_3 \xi_a\}] \right] \dots\dots (4.13)\end{aligned}$$

$$\begin{aligned}\varepsilon_y = \frac{\partial v}{\partial y} = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{E} \left[-\sum_m \left(\frac{m\pi}{2a}\right)^2 [C'_{am}(1+\nu) \cosh \eta_m + D'_{am} \{2 \nu \cosh \eta_m + (1+\nu) \eta_m \sinh \eta_m\}] \sin \xi_m \right. \\ \left. + \sum_n \left(\frac{n\pi}{2b}\right)^2 [A'_{bn}(1+\nu) \sinh \xi_n + B'_{bn} \{2 \sinh \xi_n + (1+\nu) \xi_n \cosh \xi_n\}] \cos \eta_n \right. \\ \left. + \frac{2}{b^2} [\lambda_a^2 \{3F_1 \xi_a \eta_b^2 - 2\lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^3 + 3F_3 \xi_a\} - \nu (F_1 \xi_a^3 + 6F_2 \xi_a \eta_b^2 + F_4 \xi_a)] \right] \dots\dots\dots (4.14)\end{aligned}$$

$$\varepsilon_y|_{x=a} = \frac{1}{E} \left[-\sum_m (-1)^{\frac{m-1}{2}} \left(\frac{m\pi}{2a}\right)^2 [C'_{am}(1+\nu) \cosh \eta_m + D'_{am} \{2 \nu \cosh \eta_m + (1+\nu) \eta_m \sinh \eta_m\}] \right]$$

$$\left[+ \sum_n \left(\frac{n\pi}{2b} \right)^2 [A'_{bn}(1+\nu) \sinh \alpha_{bn} + B'_{bn} \{ 2 \sinh \alpha_{bn} + (1+\nu) \alpha_{bn} \cosh \alpha_{bn} \}] \cos \eta_n \right. \\ \left. + \frac{2}{b^2} [\lambda_a^2 \{ 3 F_1 \eta_b^2 - 2 \lambda_b^2 (F_1 + \lambda_b^2 F_2) + 3 F_3 \} - \nu (F_1 + 6 F_2 \eta_b^2 + F_4)] \right] \dots\dots\dots (4.15)$$

$$r = \frac{2(1+\nu)}{E} \tau = \frac{2(1+\nu)}{E} \left[- \sum_m \left(\frac{m\pi}{2a} \right)^2 \{ C'_{am} \sinh \eta_m + D'_{am} (\sinh \eta_m + \eta_m \cosh \eta_m) \} \cos \xi_m \right. \\ \left. + \sum_n \left(\frac{n\pi}{2b} \right)^2 \{ A'_{bn} \cosh \xi_n + B'_{bn} (\cosh \xi_n + \xi_n \sinh \xi_n) \} \sin \eta_n \right. \\ \left. - \frac{2}{ab} (3 F_1 \xi_a^2 \eta_b + 2 F_2 \eta_b^3 + F_4 \eta_b) \right] \dots\dots\dots (4.16)$$

The displacements u and v parallel to x - and y - axes respectively are given as follows. Here the $\pm$ positive sign should be taken if the u and v have the positive directions of the x - and y - axes.

$$u = \int \frac{\partial u}{\partial x} dx = \frac{1}{E} \left[- \sum_m \left(\frac{m\pi}{2a} \right) [C'_{am}(1+\nu) \cosh \eta_m + D'_{am} \{ 2 \cosh \eta_m + (1+\nu) \eta_m \sinh \eta_m \}] \cos \xi_m \right. \\ \left. - \sum_n \left(\frac{n\pi}{2b} \right) [A'_{bn}(1+\nu) \cosh \xi_n + B'_{bn} \{ -(1-\nu) \cosh \xi_n + (1+\nu) \xi_n \sinh \xi_n \}] \cos \eta_n \right. \\ \left. + \frac{1}{a} \left[\lambda_b^2 \left(\frac{1}{2} F_1 \xi_a^4 + 6 F_2 \xi_a^2 \eta_b^2 + F_4 \xi_a^2 \right) - \nu \{ 3 F_1 \xi_a^2 \eta_b^2 - \lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^4 + 3 F_3 \xi_a^2 \} \right] \right. \\ \left. + f_u(y) + u_0 \right] \dots\dots\dots (4.17)$$

$$v = \int \frac{\partial v}{\partial y} dy = \frac{1}{E} \left[- \sum_m \left(\frac{m\pi}{2a} \right) [C'_{am}(1+\nu) \sinh \eta_m + D'_{am} \{ -(1-\nu) \sinh \eta_m + (1+\nu) \eta_m \cosh \eta_m \}] \sin \xi_m \right. \\ \left. + \sum_n \left(\frac{n\pi}{2b} \right) [A'_{bn}(1+\nu) \sinh \xi_n + B'_{bn} \{ 2 \sinh \xi_n + (1+\nu) \xi_n \cosh \xi_n \}] \sin \eta_n \right. \\ \left. + \frac{2}{b} [\lambda_a^2 \{ F_1 \xi_a \eta_b^3 - 2 \lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^3 \eta_b + 3 F_3 \xi_a \eta_b \} \right. \\ \left. - \nu (F_1 \xi_a^3 \eta_b + 2 F_2 \xi_a \eta_b^3 + F_4 \xi_a \eta_b) \right] \dots\dots\dots (4.18)$$

where, $f_u(y) + u_0$ is the integration constant in eq. 4.17, and $f_u(y)$ which is the even function with regard to y (η_b) and the constant u_0 can be determined as follows.

Note : The integration constant in eq. 4.18 is zero by considering $v|_{y=0}=0$ (see eq. 4.57).

$$\frac{\partial v}{\partial x} = \frac{1}{E} \left[- \sum_m \left(\frac{m\pi}{2a} \right)^2 [C'_{am}(1+\nu) \sinh \eta_m + D'_{am} \{ -(1-\nu) \sinh \eta_m + (1+\nu) \eta_m \cosh \eta_m \}] \cos \xi_m \right. \\ \left. + \sum_n \left(\frac{n\pi}{2b} \right)^2 [A'_{bn}(1+\nu) \cosh \xi_n + B'_{bn} \{ (3+\nu) \cosh \xi_n + (1+\nu) \xi_n \sinh \xi_n \}] \sin \eta_n \right. \\ \left. + \frac{2}{ab} [\lambda_a^2 \{ F_1 \eta_b^3 - 6 \lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^2 \eta_b + 3 F_3 \eta_b \} - \nu (3 F_1 \xi_a^2 \eta_b + 2 F_2 \eta_b^3 + F_4 \eta_b)] \right] \dots\dots\dots (4.19)$$

$$\frac{\partial u}{\partial y} = \frac{1}{E} \left[- \sum_m \left(\frac{m\pi}{2a} \right)^2 [C'_{am}(1+\nu) \sinh \eta_m + D'_{am} \{ (3+\nu) \sinh \eta_m + (1+\nu) \eta_m \cosh \eta_m \}] \cos \xi_m \right. \\ \left. + \sum_n \left(\frac{n\pi}{2b} \right)^2 [A'_{bn}(1+\nu) \cosh \xi_n + B'_{bn} \{ -(1-\nu) \cosh \xi_n + (1+\nu) \xi_n \sinh \xi_n \}] \sin \eta_n \right. \\ \left. + \frac{6}{ab} (2 \lambda_b^2 F_2 \xi_a^2 \eta_b - \nu F_1 \xi_a^2 \eta_b) \right] + \frac{df_u(y)}{dy} \dots\dots\dots (4.20)$$

$$r = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \frac{1}{E} \left[- 2(1+\nu) \sum_m \left(\frac{m\pi}{2a} \right)^2 \{ C'_{am} \sinh \eta_m + D'_{am} (\sinh \eta_m + \eta_m \cosh \eta_m) \} \cos \xi_m \right. \\ \left. + 2(1+\nu) \sum_n \left(\frac{n\pi}{2b} \right)^2 \{ A'_{bn} \cosh \xi_n + B'_{bn} (\cosh \xi_n + \xi_n \sinh \xi_n) \} \sin \eta_n \right. \\ \left. + \frac{2}{ab} [\lambda_a^2 \{ F_1 \eta_b^3 - 6 \lambda_b^2 (F_1 + \lambda_b^2 F_2) \xi_a^2 \eta_b + 3 F_3 \eta_b \} + 6 \lambda_b^2 F_2 \xi_a^2 \eta_b \right. \\ \left. - \nu (6 F_1 \xi_a^2 \eta_b + 2 F_2 \eta_b^3 + F_4 \eta_b) \right] + \frac{df_u(y)}{dy} \dots\dots\dots (4.21)$$

By making eq. 4.21 equal to eq. 4.16, eq. 4.22 is obtained.

$$\frac{df_u(y)}{dy} = -\frac{2}{Eab} \{ (2+\nu)(2F_2\eta b^3 + F_4\eta b) + \lambda_a^2(F_1\eta b^3 + 3F_3\eta b) \} \dots\dots\dots(4.22)$$

By integrating eq. 4.22, eq. 4.23 for $f_u(y)$ is obtained.

$$f_u(y) = \int \frac{df_u(y)}{dy} dy = -\frac{1}{Ea} \left\{ (2+\nu)(F_2\eta b^4 + F_4\eta b^2) + \lambda_a^2 \left(\frac{1}{2} F_1\eta b^4 + 3F_3\eta b^2 \right) \right\} \dots\dots\dots(4.23)$$

The constant term u_0 can be obtained from eq. 4.17 by considering $u|_{x=0} = 0$ (see eq. 4.56).

$$u_0 = \frac{1}{E} \left[\sum_m \left(\frac{m\pi}{2a} \right) \{ C'_{am}(1+\nu) + 2D'_{am} \} + \sum_n \left(\frac{n\pi}{2b} \right) \{ A'_{bn}(1+\nu) - B'_{bn}(1-\nu) \} \right] \dots\dots\dots(4.24)$$

By putting $y=b$ in eq. 4.18, eq. 4.25 is obtained.

$$\therefore v|_{y=b} = \sum_m v_m \sin \xi_m + \frac{2}{Eb} [\lambda_a^2 \{ F_1\xi_a - 2\lambda_b^2(F_1 + \lambda_b^2 F_2)\xi_a^3 + 3F_3\xi_a \} - \nu(F_1\xi_a^3 + 2F_2\xi_a + F_4\xi_a)] \dots\dots(4.25)$$

where,

$$v_m = -\frac{1}{E} \left(\frac{m\pi}{2a} \right) [C'_{am}(1+\nu) \sinh \alpha_{am} + D'_{am} \{ -(1-\nu) \sinh \alpha_{am} + (1+\nu) \alpha_{am} \cosh \alpha_{am} \}] \dots\dots\dots(4.26)$$

By substituting eq. 4.23 in eq. 4.17 and putting $x=a$, eq. 4.27 is obtained.

$$\begin{aligned} u|_{x=a} = \sum_n u_n \cos \eta_n + \frac{1}{Ea} \left[\lambda_b^2 \left(\frac{1}{2} F_1 + 6F_2\eta b^2 + F_4 \right) - \nu \{ 3F_1\eta b^2 - \lambda_b^2(F_1 + \lambda_b^2 F_2) + 3F_3 \} \right. \\ \left. - (2+\nu)(F_2\eta b^4 + F_4\eta b^2) - \lambda_a^2 \left(\frac{1}{2} F_1\eta b^4 + 3F_3\eta b^2 \right) \right] + \frac{1}{E} \left[\sum_m \left(\frac{m\pi}{2a} \right) \{ C'_{am}(1+\nu) + 2D'_{am} \} \right. \\ \left. + \sum_n \left(\frac{n\pi}{2b} \right) \{ A'_{bn}(1+\nu) - B'_{bn}(1-\nu) \} \right] \dots\dots\dots(4.27) \end{aligned}$$

$$u_n = -\frac{1}{E} \left(\frac{n\pi}{2b} \right) [A'_{bn}(1+\nu) \cosh \alpha_{bn} + B'_{bn} \{ -(1-\nu) \cosh \alpha_{bn} + (1+\nu) \alpha_{bn} \sinh \alpha_{bn} \}] \dots\dots\dots(4.28)$$

$$\begin{aligned} u|_{x=a} = \sum_n (-1)^{\frac{n}{2}} u_n + \frac{1}{Ea} \left[\lambda_b^2 \left(\frac{1}{2} F_1 + 6F_2 + F_4 \right) - \nu \{ 3F_1 - \lambda_b^2(F_1 + \lambda_b^2 F_2) + 3F_3 \} \right. \\ \left. - (2+\nu)(F_2 + F_4) - \lambda_a^2 \left(\frac{1}{2} F_1 + 3F_3 \right) \right] + \frac{1}{E} \left[\sum_m \left(\frac{m\pi}{2a} \right) \{ C'_{am}(1+\nu) + 2D'_{am} \} \right. \\ \left. + \sum_n \left(\frac{n\pi}{2b} \right) \{ A'_{bn}(1+\nu) - B'_{bn}(1-\nu) \} \right] \dots\dots\dots(4.29) \end{aligned}$$

§4.3 Stresses, Displacements, etc. in the Upper Beam and Right Column of the Frame

By making the deflection v_a of the beam equal to that $v|_{y=b}$ of the wall, the relationship (eq. 4.31) between the bending moment and the curvature of the beam is obtained and the other stresses, unit elongations, and displacements of the beam are obtained as follows.

The slope of the beam :

$$\frac{dv_a}{dx} = \frac{\partial v}{\partial x} \Big|_{y=b} = \sum_m \left(\frac{m\pi}{2a} \right) v_m \cos \xi_m + \frac{2}{Eab} [\lambda_a^2 \{ F_1 - 6\lambda_b^2(F_1 + \lambda_b^2 F_2)\xi_a^2 + 3F_3 \} - \nu \{ 3F_1\xi_a^2 + 2F_2 + F_4 \}] \dots\dots\dots(4.30)$$

The bending moment of the beam :

$$\begin{aligned} M_a = EI_a \frac{d^2 v_a}{dx^2} = -EI_a \left[\sum_m \left(\frac{m\pi}{2a} \right)^2 v_m \sin \xi_m + \frac{12}{Ea^2 b} \right. \\ \left. \cdot \{ 2(F_1 + \lambda_b^2 F_2)\xi_a + \nu F_1\xi_a \} \right] \dots\dots\dots(4.31) \end{aligned}$$

If the external forces (per unit length) on the beam are given in the Fourier series as follows (see Fig. 4.3 a) :

$$q_{a2} = q_{a20} + \sum_m q_{a2m} \cos \xi_m \dots\dots\dots(4.32)$$

$$q_{ae} = q_{ae0} + \sum_m q_{aem} \cos \xi_m \dots\dots\dots(4.33)$$

$$w_{a2} = \sum_m w_{a2m} \sin \xi_m \dots\dots\dots(4.34),$$

the stresses, unit elongations, and displacements of the beam can be obtained as follows.

The shearing force on the cross section of the beam :

Fig. 4.3 a The forces and moments on the segment $D_a \times dx$ of the upper beam

References of Fig. 4.3 a :

- i The forces and moments have a positive sign.
- ii The forces shown with dotted line are the given external forces per unit length.
- iii The hatched part is the in-filled panel wall.

$$\begin{aligned}
Q_a &= \frac{dM_a}{dx} - \frac{D_a}{2} (t\tau|_{y=b+q_{a2}}) - e_a q_{ae} \\
&= -\sum_m \left\{ EI_a \left(\frac{m\pi}{2a} \right)^3 v_m + \frac{D_a}{2} (t\tau_m + q_{a2m}) + e_a q_{aem} \right\} \cos \xi_m \\
&\quad - \frac{12 I_a}{a^3 b} \{ 2(F_1 + \lambda_b^2 F_2) + \nu F_1 \} + \frac{t D_a}{ab} (3 F_1 \xi_a^2 + 2 F_2 + F_4) - \left(\frac{D_a}{2} q_{a20} + e_a q_{ae0} \right) \dots \dots \dots (4.35)
\end{aligned}$$

The normal force (per unit length) distributed on the inside face (boundary adjacent to the wall) of the beam :

$$\begin{aligned}
w_{a1} &= -\frac{dQ_a}{dx} + w_{a2} \\
&= -\sum_m \left\{ EI_a \left(\frac{m\pi}{2a} \right)^4 v_m + \left(\frac{m\pi}{2a} \right) \left(\frac{t D_a}{2} \tau_m + \frac{D_a}{2} q_{aem} + e_a q_{aem} \right) - w_{a2m} \right\} \sin \xi_m - \frac{6 t D_a}{a^2 b} F_1 \xi_a \\
&\quad \dots \dots \dots (4.36)
\end{aligned}$$

The axial tensile force on the cross section of the beam :

$$\begin{aligned}
N_a &= \int t \tau|_{y=b} dx - \int (q_{a2} + q_{ae}) dx \\
&= \sum_m \left(\frac{2a}{m\pi} \right) (t \tau_m - q_{a2m} - q_{aem}) \sin \xi_m - \frac{2t}{b} (F_1 \xi_a^3 + 2 F_2 \xi_a + F_4 \xi_a) - a (q_{a20} + q_{ae0}) \xi_a \\
&\quad \dots \dots \dots (4.37)
\end{aligned}$$

Note : The integration constant is zero by assuming $N_a|_{x=0}=0$.

The axial unit elongation on the inside face (boundary adjacent to the wall) of the beam :

$$\begin{aligned}
\varepsilon_{a1} &= \frac{1}{E} \left(\frac{M_a}{Z_a} + \frac{N_a}{A_a} \right) \\
&= \frac{1}{E} \left[\sum_m \left(\frac{2a}{m\pi} \right) \left\{ -\frac{EI_a}{Z_a} \left(\frac{m\pi}{2a} \right)^3 v_m + \frac{t}{A_a} \tau_m - \frac{1}{A_a} (q_{a2m} + q_{aem}) \right\} \sin \xi_m \right. \\
&\quad \left. - \frac{12 I_a}{Z_a a^2 b} \{ 2(F_1 + \lambda_b^2 F_2) \xi_a + \nu F_1 \xi_a \} - \frac{2t}{A_a b} (F_1 \xi_a^3 + 2 F_2 \xi_a + F_4 \xi_a) - \frac{a}{A_a} (q_{a20} + q_{ae0}) \xi_a \right] \\
&\quad \dots \dots \dots (4.38)
\end{aligned}$$

The axial displacement on the inside face (boundary adjacent to the wall) of the beam :

$$\begin{aligned}
u_{a1} &= \int \varepsilon_{a1} dx = \sum_m \left(\frac{2a}{m\pi} \right)^2 \left\{ \frac{I_a}{Z_a} \left(\frac{m\pi}{2a} \right)^3 v_m - \frac{t}{EA_a} \tau_m + \frac{1}{EA_a} (q_{a2m} + q_{aem}) \right\} \cos \xi_m \\
&\quad - \frac{6 I_a}{EZ_a ab} \{ 2(F_1 + \lambda_b^2 F_2) \xi_a^2 + \nu F_1 \xi_a^2 \} - \frac{\lambda_b t}{EA_a} \left(\frac{1}{2} F_1 \xi_a^4 + 2 F_2 \xi_a^2 + F_4 \xi_a^2 \right) \\
&\quad - \frac{a^2}{2 EA_a} (q_{a20} + q_{ae0}) \xi_a^2 + u_{a10} \dots \dots \dots (4.39)
\end{aligned}$$

$$\begin{aligned}
u_{a1}|_{x=a} &= -\frac{6 I_a}{EZ_a ab} \{ 2(F_1 + \lambda_b^2 F_2) + \nu F_1 \} - \frac{\lambda_b t}{EA_a} \left(\frac{1}{2} F_1 + 2 F_2 + F_4 \right) - \frac{a^2}{2 EA_a} (q_{a20} + q_{ae0}) + u_{a10} \\
&\quad \dots \dots \dots (4.40)
\end{aligned}$$

By making $u_{a1}|_{x=a}$ equal to the displacement $u|_{x=a, y=b}$ (parallel to x -axis, see eq. 4.29) of the corner ($x=a, y=b$) of the wall, the integration constant u_{a10} can be determined as follows.

$$\begin{aligned}
u_{a10} &= \sum_n (-1)^{\frac{n}{2}} u_n + \frac{1}{Ea} \left[\lambda_b^2 \left(\frac{1}{2} F_1 + 6 F_2 + F_4 \right) - \nu \{ 3 F_1 - \lambda_b^2 (F_1 + \lambda_b^2 F_2) + 3 F_3 \} - 2(2 + \nu) (F_2 + F_4) \right. \\
&\quad \left. - \lambda_a^2 \left(\frac{1}{2} F_1 + 3 F_3 \right) \right] + \frac{1}{E} \left[\sum_m \left(\frac{m\pi}{2a} \right) \{ C'_{am} (1 + \nu) + 2 D'_{am} \} + \sum_n \left(\frac{n\pi}{2b} \right) \{ A'_{bn} (1 + \nu) - B'_{bn} (1 - \nu) \} \right] \\
&\quad + \frac{6 I_a}{EZ_a ab} \{ 2(F_1 + \lambda_b^2 F_2) + \nu F_1 \} + \frac{\lambda_b t}{EA_a} \left(\frac{1}{2} F_1 + 2 F_2 + F_4 \right) + \frac{a^2}{2 EA_a} (q_{a20} + q_{ae0}) \\
&\quad \dots \dots \dots (4.41)
\end{aligned}$$

The axial displacement on the center line of the beam :

$$\begin{aligned}
u_{ac} &= \int \frac{N_a}{EA_a} dx = \frac{1}{EA_a} \left[\sum_m \left(\frac{2a}{m\pi} \right)^2 (-t \tau_m + q_{a2m} + q_{aem}) \cos \xi_m - \lambda_b t \left(\frac{1}{2} F_1 \xi_a^4 + 2 F_2 \xi_a^2 + F_4 \xi_a^2 \right) \right. \\
&\quad \left. - \frac{a^2}{2} (q_{a20} + q_{ae0}) \xi_a^2 \right] + u_{ac0} \dots \dots \dots (4.42)
\end{aligned}$$

$$u_{ac}|_{x=a} = -\frac{\lambda_b t}{EA_a} \left(\frac{1}{2} F_1 + 2 F_2 + F_4 \right) - \frac{a^2}{2EA_a} (q_{a20} + q_{ae0}) + u_{aco} \dots (4.43)$$

The u_{aco} is determined from the condition $u_{ac}|_{x=a} = u_{a1}|_{x=a} - \frac{D_a}{2} \frac{dv_a}{dx} \Big|_{x=a}$, where $u_{a1}|_{x=a}$ and $\frac{dv_a}{dx}$ are given by eqs. 4.40 and 4.30.

$$\begin{aligned} u_{aco} = & \sum_n (-1)^{\frac{n}{2}} u_n + \frac{1}{EA} \left[\lambda_b^2 \left(\frac{1}{2} F_1 + 6 F_2 + F_4 \right) - \nu \{ 3 F_1 - \lambda_b^2 (F_1 + \lambda_b^2 F_2) + 3 F_3 \} - 2(2+\nu)(F_2 + F_4) \right. \\ & \left. - \lambda_a^2 \left(\frac{1}{2} F_1 + 3 F_3 \right) \right] + \frac{1}{E} \left[\sum_m \left(\frac{m\pi}{2a} \right) \{ C'_{am}(1+\nu) + 2 D'_{am} \} + \sum_n \left(\frac{n\pi}{2b} \right) \{ A'_{an}(1+\nu) - B'_{bn}(1-\nu) \} \right] \\ & + \frac{\lambda_b t}{EA_a} \left(\frac{1}{2} F_1 + 2 F_2 + F_4 \right) + \frac{a^2}{2EA_a} (q_{a20} + q_{ae0}) + \frac{\lambda_a D_a}{EA^2} \{ F_1 - 6 \lambda_b^2 (F_1 + \lambda_b^2 F_2) + 3 F_3 \} \dots (4.44) \end{aligned}$$

By making the deflection u_b of the column equal to that $u|_{x=a}$ of the wall, the relationship (eq. 4.46) between the bending moment and curvature of the column is obtained and the other stresses, unit elongations, and displacements of the column are obtained as follows.

The slope of the column :

$$\frac{du_b}{dy} = \frac{\partial u}{\partial y} \Big|_{x=a} = -\sum_n \left(\frac{n\pi}{2b} \right) u_n \sin \eta_n - \frac{2}{Eab} \{ 3\nu F_1 \eta_b - 6 \lambda_b^2 F_2 \eta_b + \lambda_a^2 (F_1 \eta_b^3 + 3 F_3 \eta_b) + (2+\nu)(2 F_2 \eta_b^3 + F_4 \eta_b) \} \dots (4.45)$$

The bending moment of the column :

$$\begin{aligned} M_b = EI_b \frac{d^2 u_b}{dy^2} = EI_b \left[-\sum_n \left(\frac{n\pi}{2b} \right)^2 u_n \cos \eta_n - \frac{2}{Eab^2} \{ 3\nu F_1 - 6 \lambda_b^2 F_2 + 3 \lambda_a^2 (F_1 \eta_b^2 + F_3) \right. \\ \left. + (2+\nu)(6 F_2 \eta_b^2 + F_4) \} \right] \dots (4.46) \end{aligned}$$

If the external forces (per unit length) on the column are given in the Fourier series as follows (see Fig. 4.3 b) :

$$q_{b2} = \sum_n q_{b2n} \sin \eta_n \dots (4.47)$$

$$q_{be} = \sum_n q_{ben} \sin \eta_n \dots (4.48)$$

$$w_{b2} = w_{b20} + \sum_n w_{b2n} \cos \eta_n \dots (4.49)$$

the stresses, unit elongations, and displacements of the beam can be obtained as follows.

The shearing force on the cross section of the column :

$$\begin{aligned} Q_b = & \frac{dM_b}{dy} - \frac{D_b}{2} (t\tau|_{x=a} + q_{b2}) - e_b q_{be} \\ = & \sum_n \left\{ EI_b \left(\frac{n\pi}{2b} \right)^3 u_n - \frac{tD_b}{2} \tau_n - \left(\frac{D_b}{2} q_{b2n} + e_b q_{ben} \right) \right\} \sin \eta_n \\ & - \frac{12 I_b}{ab^3} \{ \lambda_a^2 F_1 \eta_b + 2(2+\nu) F_2 \eta_b \} + \frac{tD_b}{ab} (3 F_1 \eta_b + 2 F_2 \eta_b^3 \\ & + F_4 \eta_b) \dots (4.50) \end{aligned}$$

The normal force (per unit length) distributed on the inside face (boundary adjacent to the wall) of the column :

$$\begin{aligned} w_{b1} = & -\frac{dQ_b}{dy} + w_{b2} = \sum_n \left\{ -EI_b \left(\frac{n\pi}{2b} \right)^4 u_n + \frac{tD_b}{2} \left(\frac{n\pi}{2b} \right) \tau_n \right. \\ & \left. + \left(\frac{n\pi}{2b} \right) \left(\frac{D_b}{2} q_{b2n} + e_b q_{ben} \right) + w_{b2n} \right\} \cos \eta_n \\ & + \frac{12 I_b}{ab^4} \{ \lambda_a^2 F_1 + 2(2+\nu) F_2 \} - \frac{tD_b}{ab^2} (3 F_1 + 6 F_2 \eta_b^2 + F_4) + w_{b20} \dots (4.51) \end{aligned}$$

The axial tensile force on the cross section of the column :

$$\begin{aligned} N_b = & t \int_{x=a} \tau dy - \int (q_{b2} + q_{be}) dy \\ = & \sum_n \left(\frac{2a}{n\pi} \right) (-t \tau_n + q_{b2n} + q_{ben}) \cos \eta_n - \frac{t}{a} (3 F_1 \eta_b^2 + F_2 \eta_b^4 + F_4 \eta_b^2) + N_{b0} \dots (4.52) \end{aligned}$$

Fig. 4.3 b The forces and moments on the segment $D_b \times dy$ of the right column

References of Fig. 4.3 b :

- The forces and moments have a positive sign.
- The forces shown with dotted line are the given external forces per unit length.
- The hatched part is the in-filled panel wall.

where, N_{bo} is the integration constant.

The N_{bo} is the unknown factor in the eq. 4.77 for the continuity of the unit elongations parallel to and on the boundary between the wall and column (see eq. 4.53), and be determined together with many unknown coefficients in the stress function $\mathfrak{M}F$ (see eq. 4.2), by solving the simultaneous equations of the conditions mentioned in §4.4.

The axial unit elongation on the inside face (boundary adjacent to the wall) of the column :

$$\begin{aligned}\varepsilon_{b1} &= \frac{1}{E} \left(\frac{M_b}{Z_b} + \frac{N_b}{A_b} \right) \\ &= \frac{1}{E} \sum_n \left(\frac{2b}{n\pi} \right) \left\{ -\frac{EI_b}{Z_b} \left(\frac{n\pi}{2b} \right)^3 u_n - \frac{t}{A_b} \tau_n + \frac{1}{A_b} (q_{b2n} + q_{ben}) \right\} \cos \eta_n \\ &\quad - \frac{2I_b}{Z_b a b^2} \{ 3\nu F_1 - 6\lambda_b^2 F_2 + (2+\nu)(6F_2\eta_b^2 + F_4) + 3\lambda_a^2 (F_1\eta_b^2 + F_3) \} \\ &\quad - \frac{1}{A_b} \left\{ \frac{t}{a} (3F_1\eta_b^2 + F_2\eta_b^4 + F_4\eta_b^2) - N_{bo} \right\} \dots\dots\dots (4.53)\end{aligned}$$

The axial displacement on the inside face (boundary adjacent to the wall) of the column :

$$\begin{aligned}v_{b1} &= \int \varepsilon_{b1} dy = \sum_n \left(\frac{2b}{n\pi} \right)^2 \left\{ -\frac{I_b}{Z_b} \left(\frac{n\pi}{2b} \right)^3 u_n - \frac{t}{EA_b} \tau_n + \frac{1}{EA_b} (q_{b2n} + q_{ben}) \right\} \sin \eta_n \\ &\quad - \frac{2I_b}{EZ_b a b} \{ 3\nu F_1\eta_b - 6\lambda_b^2 F_2\eta_b + (2+\nu)(2F_2\eta_b^3 + F_4\eta_b) + \lambda_a^2 (F_1\eta_b^3 + 3F_3\eta_b) \} \\ &\quad - \frac{b}{EA_b} \left\{ \frac{t}{a} \left(F_1\eta_b^3 + \frac{1}{5} F_2\eta_b^5 + \frac{1}{3} F_4\eta_b^3 \right) - N_{bo}\eta_b \right\} \dots\dots\dots (4.54)\end{aligned}$$

Note : The integration constant is zero by assuming $v_{b1}|_{y=0}=0$.

The axial displacement on the center line of the column :

$$\begin{aligned}v_{bc} &= \int \frac{N_b}{EA_b} dy = \frac{1}{EA_b} \sum_n \left(\frac{2b}{n\pi} \right)^2 (-t\tau_n + q_{b2n} + q_{ben}) \sin \eta_n \\ &\quad - \frac{b}{EA_b} \left\{ \frac{t}{a} \left(F_1\eta_b^3 + \frac{1}{5} F_2\eta_b^5 + \frac{1}{3} F_4\eta_b^3 \right) - N_{bo}\eta_b \right\} : \dots\dots\dots (4.55)\end{aligned}$$

Note : The integration constant is zero by assuming $v_{bc}|_{y=0}=0$.

NOTE

As the pages are limited,

§4.4 Equations for Condition which are Necessary to Define the Unknown Coefficients and Unknown Factor

§4.5 Numerical Results of an Example

ACKNOWLEDGEMENT

REFERENCES

will be reported in the following Transactions as Part II and Part III.

縦軸に関して逆対称、横軸に関して対称な外力を 受ける耐震壁の弾性解析 第1報

(エアリーの応力関数、応力度、変位、そのほか)

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序

本論は、壁板と付帯ラーメンが一体に作られ、縦横軸に関してそれぞれ対称な形状および断面を有する鉄筋コンクリート造の単独耐震壁に、縦軸に関し逆対称、横軸に関して対称の既知面内外力が付帯ラーメンに作用した場合の平面応力、ひずみ度、変位などの式を誘導したものである。 x 方向の面内外力は任意に与えることができないで、面内外力に関する釣合式(4.1式)を満たすように与えなければならない。付帯ラーメンの四隅は剛域とみなし(4.1図のハッチした部分)、外力が縦軸に関して逆対称、横軸に関して対称の条件を考慮して、耐震壁の4分の1の部分($x \geq 0, y \geq 0$)について解析を行なった。

すでに坪井善勝博士は、縦軸に関し逆対称、横軸に関して対称な集中外力が付帯ラーメンの節点に作用した耐震壁の解析を行なっている³⁾。坪井博士の解析法と本論の解析法のちがいはすでに述べたが^{1), 2)}、比較の詳細についてはさらに追って報告の予定である。

本解析に関する一般論は、次の項目に従ってすでに発表してある^{1), 2)}ので、それらを参照されたい。

- §1.1 外力の釣合条件¹⁾
- §1.2 級数型応力関数の選定¹⁾
- §1.3 級数型応力関数だけでは条件が満足されない要素¹⁾
- §1.4 壁板と付帯ラーメンの境界条件式のたて方¹⁾
- §1.5 未知係数および未知積分定数を求めるための条件式(連立方程式)²⁾
- §1.6 代数型応力関数の選定²⁾
- §1.7 壁板の応力度、ひずみ度、変位²⁾
- §1.8 上ばりおよび右柱の応力、変形、そのほか²⁾

外力、耐震壁の寸法および断面に関する記号は、4.1図を参照されたい。本論で使用している諸記号のうちはじめに解説したもの以外のものは、その記号が始めて出てくる所で説明した。

§4.1 エアリーの応力関数

4.2式に示すように、エアリーの応力関数として級数

型応力関数と代数型応力関数を併用した。級数型応力関数と代数型応力関数を併用して解析を行なう利点や、必要性などについては、一般論 §1.2¹⁾ で論じた。

§4.2 壁板の応力度、ひずみ度、変位

x, y 軸にそれぞれ平行な垂直応力度 σ_x, σ_y 、とせん断応力度 τ はそれぞれ 4.5式、4.3式、4.7式に示してある。符号は4.2図に示す方向を正の方向とする。

x, y 軸方向の伸縮ひずみ度 ϵ_x, ϵ_y および x, y 軸間のせん断ひずみ度 r は σ_y (4.3式)、 σ_x (4.5式)、 τ (4.7式) から誘導し、それぞれ 4.12式、4.14式、4.16式に示した。符号は正の応力度によって生じたひずみ度を正とする。

x, y 軸にそれぞれ平行な変位 u, v はそれぞれ ϵ_x, ϵ_y を積分することによって求められる(4.17式、4.18式参照)。 u の積分定数 $f_u(y) + u_0$ のうち、 $y(\eta_b)$ の偶関数で与えられる $f_u(y)$ はひずみ度の適合条件から求めたせん断ひずみ度 r (4.21式) とせん断応力度から剛性係数を考慮して求めたせん断ひずみ度 r (4.16式) とを等しく置くことによって、4.23式で与えられ、また定数項 u_0 は壁板の中心 ($x=0, y=0$) で $u=0$ という条件(4.56式) から 4.24式で与えられる。 v の積分定数は $v|_{y=0}=0$ という条件(4.57式) から零となる。

§4.3 上ばりと右柱の応力、変位、そのほか

壁板とはりの境界において、はりの垂直変位 v_a をフーリエ級数と代数関数の和で与えられている壁板とはりの境界上の壁板の垂直変位 $v|_{y=b}$ (4.25式) と等しく置き、 v_a を微分してはりの傾斜角を求め(4.30式)、さらに傾斜角を微分して、曲げ剛性を考慮して求められる曲率と曲げモーメントの関係から、はりの曲げモーメントを求めると(4.31式参照)、これらもフーリエ級数と代数関数の和で表わされる。

はりの軸方向の分布外力 q_{a2}, q_{ae} および軸に垂直な分布外力 w_{a2} (4.3a図参照) をそれぞれ 4.32式、4.33式、4.34式に示すようなフーリエ級数で与え、これらと 4.8式で与えられる壁板周辺のせん断応力度 $\tau|_{y=b}$ (注: フーリエ級数と代数関数の和で表わされている) に壁厚 t を乗じた壁板周辺のせん断応力 $t\tau|_{y=b}$ をはりに与えると、

1. はりのせん断力 Q_a (4.35式)

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2. はりに分布する垂直分布荷重 w_{a1} (4.36 式)
3. はりの軸方向力 N_a (4.37 式)
4. 壁板との境界上のはりの縁ひずみ度

$$\varepsilon_{a1} \quad (4.38 \text{ 式})$$

5. 壁板との境界上のはりの軸方向変位

$$u_{a1} \quad (4.39 \text{ 式})$$

注： u_{a1} の積分定数 u_{a10} は，はり端の軸方向変位 $u_{a1}|_{x=a}$ を壁板の四隅の x 方向変位 $u|_{x=a, y=b}$ (4.29 式) と等しく置くことによって求められる (4.41 式参照)。

6. はりの中心線上の軸方向変位 u_{ac} (4.42 式)

注： u_{ac} の積分定数 u_{ac0} は， $u_{a1}|_{x=a}$ に付帯ラメンの節点 (剛域) の回転角 (4.30 式に $x=a$ を代入すると得られる) を考慮して求められる (4.44 式参照)。

などもフーリエ級数と代数関数の和で表わされる。

壁板と柱の境界において，柱の垂直変位 u_b をフーリエ級数と代数関数の和で表わされている壁板と柱の境界上の壁板の垂直変位 $u|_{x=a}$ (4.27 式) と等しく置き， u_b を微分して柱の傾斜角を求め (4.45 式参照)，さらに傾斜角を微分して，曲げ剛性を考慮して求められる曲率と曲げモーメントの関係から，柱の曲げモーメントを求めると (4.46 式参照)，これらもフーリエ級数と代数関数の和で表わされる。

柱の軸方向の分布外力 q_{b2} ， q_{be} および軸に垂直な分布外力 w_{b2} (4.3 b 図参照) をそれぞれ 4.47 式，4.48 式，4.49 式に示すようなフーリエ級数で与え，これらと 4.10 式で与えられる壁板周辺のせん断応力度 $\tau|_{x=a}$ (注：フーリエ級数と代数関数の和で与えられている) に壁厚

t を乗じた壁板周辺のせん断応力 $t\tau|_{x=a}$ を柱に与えると，

1. 柱のせん断力 Q_b (4.50 式)
2. 柱に分布する垂直分布荷重 w_{b1} (4.51 式)
3. 柱の軸方向力 N_b (4.52 式)

注： N_b の積分定数 N_{b0} は，壁板と柱の境界上の境界線方向の両者の伸縮ひずみ度の連続条件式 (4.77 式) 中に含まれる未知量であって (4.53 式参照)，応力関数の未知係数とともに，§ 4.4 で述べる各条件式の連立方程式を解くことによって求められる。

4. 壁板との境界上の柱の縁ひずみ度 ε_{b1} (4.53 式)
5. 壁板との境界上の柱の軸方向変位 v_{b1} (4.54 式)

注： v_{b1} の積分定数は，柱の中央部の軸方向変位が $v_{b1}|_{y=0}=0$ であるから零である。

6. 柱の中心線上の軸方向変位 v_{bc} (4.55 式)

注： v_{bc} の積分定数は，柱の中央部の軸方向変位が $v_{bc}|_{y=0}=0$ であるから零である。

などもフーリエ級数と代数関数の和で表わされる。

おことわり

紙面の都合で次の項目は第 2 報および第 3 報として続報する。

§ 4.4 未知係数および未知積分定数を求めるための条件式 (連立方程式)

§ 4.5 数値計算例

謝 辞

参 考 文 献

注：文中の式および図は英論文を参照されたい。