Limit Analysis of Beam-Column Connections (VIII-2)

—Cruciform Connections of Wideflange Members Subjected to Strong-axis Bending and Thrust on Column—

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Continued from the statical analysis in the previous transactions, the kinematical analysis will be presented in this paper.

§4 Kinematical Analysis

The kinematical analysis is classified into case (a),(b),(c),(d),(f),(g),(h), and (i).

Case (a) : Assuming a kinematically velocity field shown in Fig. 6, the power of load and dissipation, denoted by W and D respectively, are given by

Table 15 Case D

regions	$\frac{\sigma_x}{\sigma_0}$	$\frac{\sigma_y}{\sigma_0}$	$\frac{\tau}{\sigma_0}$	$\left \frac{\sigma_1}{\sigma_0} \right , \left \frac{\sigma_2}{\sigma_0} \right , \left \frac{\sigma_1 - \sigma_2}{\sigma_0} \right _{\max}$
(1)	0	1	0	1
(2)	0	$-\nu_1$	0	ν_1
(3)	0	-1	0	1
(4)	1	0	0	0
(5)	0	0	0	0
(6)	-1	0	0	1
(7)	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$	$\frac{\mu}{2} - \frac{1}{\lambda}$	$\frac{\mu}{2}$
(8)	$-\frac{1}{\lambda}$	$-\frac{1}{\lambda}$	$\frac{\mu}{2} = \frac{1}{\lambda}$	$\frac{\mu}{2}$
(11), (9)	0	0	$\frac{\mu}{2}$	$\frac{\mu}{2}$
(10)	0	$-\frac{\nu_1}{\lambda}$	$\frac{\mu}{2}$	$* \nu_1 = \lambda \sqrt{1 - \mu^2}$
ξ_1, ξ_2	$\xi_1 = 1 + 2\beta - \frac{\lambda\mu}{2}, \xi_2 = \alpha(1 + 2\beta) - \frac{\lambda\mu}{2}$			
ν_1, ν_2, ν_3	$0 \leq \nu_1 \leq 1, \nu_2 = 0, 0 \leq \nu_3 \leq 1 - \frac{1}{2\beta} \left[\frac{\lambda\mu}{2} (\alpha - 1) + 2\alpha\beta \right]$			
F in $\sigma_0 h t$	$F_c^{(A)} = -F_c^{(A')} = -2\beta\nu_3$ $F_c^{(B)} = \frac{\mu\lambda}{2} \left[\alpha(1 + 2\beta) - \frac{\lambda\mu}{2} \right] - 2\beta\nu_3$ $F_c^{(C)} = F_c^{(D)} = \frac{\lambda\mu}{2} (\alpha - 1) + 2\alpha\beta - 2\beta\nu_3$ $F_c^{(B')} = -\frac{\mu\lambda}{2} \left[\alpha(1 + 2\beta) - \frac{\lambda\mu}{2} \right] - 2\beta\nu_3$ $F_c^{(C')} = F_c^{(D')} = -\frac{\lambda\mu}{2} (\alpha - 1) - 2\alpha\beta - 2\beta\nu_3,$ $F_b^{(F)} = -F_b^{(F')} = \frac{\mu\lambda}{2} \xi_1$ $F_b^{(C)} = F_b^{(G)} = -F_b^{(C')} = -F_b^{(G')} = 2\alpha\beta, F_b^{(E)} = 0$			
* Case D ₁	$\lambda \sqrt{1 - \mu^2} \leq 1$			
Case D ₂	$\lambda \sqrt{1 - \mu^2} \geq 1$			

Table 16 Case E

regions	$\frac{\sigma_x}{\sigma_0}$	$\frac{\sigma_y}{\sigma_0}$	$\frac{\tau}{\sigma_0}$	$\left \frac{\sigma_1}{\sigma_0} \right , \left \frac{\sigma_2}{\sigma_0} \right , \left \frac{\sigma_1 - \sigma_2}{\sigma_0} \right _{\max}$
(1)	0	$\frac{\mu^2\lambda^2}{4} - \nu_2$	0	$\frac{\mu^2\lambda^2}{4} - \nu_2$
(2)	0	$-\nu_1$	0	ν_1
(3)	0	$-\frac{\mu^2\lambda^2}{4} - \nu_2$	0	$\frac{\mu^2\lambda^2}{4} + \nu_2$
(4)	1	0	0	0
(5)	0	0	0	0
(6)	-1	0	0	1
(7)	$\frac{1}{\lambda}$	$\frac{\mu^2\lambda}{4} - \frac{\nu_2}{\lambda}$	0	$\frac{1}{\lambda}$
(8)	$-\frac{1}{\lambda}$	$-\frac{\mu^2\lambda}{4} - \frac{\nu_2}{\lambda}$	0	$\frac{1}{\lambda}$
(11), (9)	0	$-\frac{\nu_2}{\lambda}$	$\frac{\mu}{2}$	$* \nu_2 = \lambda \sqrt{1 - \mu^2}$
(10)	0	$-\frac{\nu_1}{\lambda}$	$\frac{\mu}{2}$	$* \nu_1 = \lambda \sqrt{1 - \mu^2}$
ξ_1, ξ_2	$\xi_1 = \frac{4\alpha\beta}{\mu\lambda}, \xi_2 = 2\alpha\beta + \alpha - \frac{\mu\lambda}{2}$			
ν_1, ν_2, ν_3	$0 \leq \nu_1 \leq 1, 0 \leq \nu_2 \leq 1 - \frac{\mu^2\lambda^2}{4},$ $0 \leq \nu_3 \leq 1 - \frac{\mu\lambda}{4\beta} \left(2\alpha\beta + \alpha - \frac{\mu\lambda}{2} \right)$			
F in $\sigma_0 h t$	$F_c^{(A)} = -F_c^{(A')} = 2\beta\nu_3,$ $F_c^{(B)} = F_c^{(C)} = F_c^{(D)} = \frac{\mu\lambda}{2} \left(2\alpha\beta + \alpha - \frac{\mu\lambda}{2} \right) - 2\beta\nu_3$ $F_c^{(B')} = F_c^{(C')} = F_c^{(D')} = -\frac{\mu\lambda}{2} \left(2\alpha\beta + \alpha - \frac{\mu\lambda}{2} \right) - 2\beta\nu_2$ $F_b^{(R)} = 0,$ $F_b^{(F)} = -F_b^{(F')} = 2\alpha\beta$ $F_b^{(C)} = F_b^{(G)} = -F_b^{(C')} = -F_b^{(G')} = 2\alpha\beta$			
* Case E ₁	$\lambda \sqrt{1 - \mu^2} \leq 1, \lambda \sqrt{1 - \mu^2} \leq 1 - \frac{\mu^2\lambda^2}{4}$			
Case E ₂	$\lambda \sqrt{1 - \mu^2} \leq 1, \lambda \sqrt{1 - \mu^2} \geq 1 - \frac{\mu^2\lambda^2}{4}$			
Case E ₃	$\lambda \sqrt{1 - \mu^2} \geq 1, \lambda \sqrt{1 - \mu^2} \geq 1 - \frac{\mu^2\lambda^2}{4}$			

Table 17 Case F_1

regions	$\frac{\sigma_x}{\sigma_0}$	$\frac{\sigma_y}{\sigma_0}$	$\frac{\tau}{\sigma_0}$	$\left \frac{\sigma_1}{\sigma_0} \right , \left \frac{\sigma_2}{\sigma_0} \right , \left \frac{\sigma_1 - \sigma_2}{\sigma_0} \right _{\max}$
(1)	0	$\frac{\mu^2 \lambda_1^2}{4} - \nu_2$	0	$\frac{\mu^2 \lambda_1^2}{4} - \nu_2$
(2)	0	$-\nu_1$	0	ν_1
(3)	0	$-\frac{\mu^2 \lambda_1^2}{4} - \nu_2$	0	$\frac{\mu^2 \lambda_1^2}{4} + \nu_2$
(4)	1	0	0	1
(5)	0	0	0	0
(6)	-1	0	0	1
(7)	$\frac{1}{\lambda}$	$\frac{\mu^2 \lambda_1}{4} - \frac{\nu_2}{\lambda_1}$	0	$\frac{1}{\lambda}$
(8)	$-\frac{1}{\lambda}$	$-\frac{\mu^2 \lambda_1}{4} - \frac{\nu_2}{\lambda_1}$	0	$\frac{1}{\lambda}$
(11), (9)	0	$-\frac{\nu_2}{\lambda_1}$	$\frac{\mu}{2}$	* $\nu_2 = \mu_1 \sqrt{1 - \mu^2}$
(10)	0	$-\frac{\nu_1}{\lambda_1}$	$\frac{\mu}{2}$	* $\nu_1 = \lambda_1 \sqrt{1 - \mu^2}$
ξ_1, ξ_2	$\xi_1 = \frac{4\alpha\beta_1}{\mu\lambda_1}, \xi_2 = 2\alpha\beta_1 + \alpha - \frac{\mu\lambda_1}{2}$			
ν_1, ν_2, ν_3	$0 \leq \nu_1 \leq 1, 0 \leq \nu_2 \leq 1 - \frac{\mu^2 \lambda_1^2}{4},$ $0 \leq \nu_3 \leq 1 - \frac{\mu\lambda_1}{4\beta_1} \left(2\alpha\beta_1 + \alpha - \frac{\mu\lambda_1}{2} \right)$			
F in $\sigma_0 h t_1$	$F_c^{(A)} = -F_c^{(A')} = 2\beta_1\nu_3$ $F_c^{(B)} = F_c^{(C)} = F_c^{(D)} = \frac{\mu\lambda_1}{2} \left(2\alpha\beta_1 + \alpha - \frac{\mu\lambda_1}{2} \right) - 2\beta_1\nu_3$ $F_c^{(B')} = F_c^{(C')} = F_c^{(D')} = -\frac{\mu\lambda_1}{2} \left(2\alpha\beta_1 + \alpha - \frac{\mu\lambda_1}{2} \right) - 2\beta_1\nu_3$ $F_b^{(E)} = 0$ $F_b^{(F)} = -F_b^{(F')} = 2\alpha\beta_1$ $F_b^{(C)} = F_b^{(G)} = -F_b^{(C')} = -F_b^{(G')} = 2\alpha\beta_1$			
* Case F_{1-1}	$\lambda_1 \sqrt{1 - \mu^2} \leq 1, \lambda_1 \sqrt{1 - \mu^2} \leq 1 - \frac{\mu^2 \lambda_1^2}{4}$			
Case F_{1-2}	$\lambda_1 \sqrt{1 - \mu^2} \leq 1, \lambda_1 \sqrt{1 - \mu^2} \geq 1 - \frac{\mu^2 \lambda_1^2}{4}$			
Case F_{1-3}	$\lambda_1 \sqrt{1 - \mu^2} \geq 1, \lambda_1 \sqrt{1 - \mu^2} \geq 1 - \frac{\mu^2 \lambda_1^2}{4}$			

Table 18 Necessary Conditions to Satisfy Yield Conditions

Case	Conditions for $\left \frac{\sigma_1}{\sigma_0} \right , \left \frac{\sigma_2}{\sigma_0} \right , \left \frac{\sigma_1 - \sigma_2}{\sigma_0} \right \leq 1$	Conditions for $F_b \leq 2\alpha\beta\sigma_0 h t$ $F_c \leq 2\beta\sigma_0 h t$
A	A ₁ (3.1), (3.9)	(3.11)
	A ₂ (3.1), (3.10)	(3.11)
B	B ₁ (3.4), (3.9), (3.5)	(3.21) from $F_c^{(C')} \leq -2\beta$
	B ₂ (3.4), (3.10), (3.5)	(3.21)
C	C ₁ (3.4), (3.6), (3.9), (3.12)	(3.22) from $F_c^{(C')} \leq -2\beta$
	C ₂ (3.4), (3.6), (3.9), (3.13)	(3.22)
	C ₃ (3.4), (3.6), (3.10), (3.13)	(3.22)
D	D ₁ (3.5), (3.9)	(3.23) from $F_c^{(C')} \leq -2\beta$
	D ₂ (3.5), (3.10)	(3.23)
E	E ₁ (3.6), (3.9), (3.14)	(3.7) from $F_c^{(B)} \leq 2\beta\alpha^2$
	E ₂ (3.6), (3.9), (3.15)	(3.7)
	E ₃ (3.6), (3.10), (3.15)	(3.7)
F ₁	F ₁₋₁ (3.24), (3.16)	(3.8) from $F_c^{(B)} \leq 2\beta\alpha^2$
	F ₁₋₂ (3.25), (3.17)	(3.8)
	F ₁₋₃ (3.25), (3.17)	(3.8)
F ₂	F ₂₋₁ (3.18), (3.19)	—
	F ₂₋₂ (3.20)	—

Fig. 8 Kinematically Admissible Velocity Field for Case (d)

$$W = 2N\xi h_c \dot{\theta} + 2M\dot{\theta} \quad \dots\dots\dots(4.1)$$

$$D = 2 \left[2\xi(\xi + 2\beta) + (1 - \xi)\alpha\lambda\sqrt{4\left(\frac{\xi}{\alpha}\right)^2 + 1} \right] \dot{\theta} h_c^2 t \sigma_0 \quad \dots\dots\dots(4.2)$$

Equating W and D , gives

$$\frac{M}{M_0} = \frac{1}{1 + 4\beta} \left[2\xi(\xi + 2\beta) + \lambda(1 - \xi)\sqrt{4\xi^2 + \alpha^2} - 2\xi(1 + 2\beta) \left(\frac{N}{N_0} \right) \right] \quad \dots\dots\dots(4.3)$$

Where M_0 and N_0 denote the fully plastic moment and axial force of column, given by

$$M_0 = (1 + 4\beta)\sigma_0 h_c^2 t \quad \text{and} \quad N_0 = 2(1 + 2\beta)\sigma_0 h_c t$$

respectively.

The minimum value of Eq. (4.3) is obtained for

$$\frac{N}{N_0} = \frac{4(\xi + \beta) + \frac{\{4\xi(1 - 2\xi) - \alpha^2\}\lambda}{\sqrt{4\xi^2 + \alpha^2}}}{2(1 + 2\beta)} \quad \dots\dots\dots(4.4)$$

as

$$\frac{M}{M_0} = \frac{1}{1 + 4\beta} \left[\frac{\lambda(4\xi^3 + \alpha^2)}{\sqrt{4\xi^2 + \alpha^2}} - 2\xi^2 \right] \quad \dots\dots\dots(4.5)$$

Table 19 Yield Moment M and Axial Thrust N of Each Case

Case		M	N
A	A ₁	$\mu\alpha\lambda\sigma_0h^2t$	$[2(1+2\beta)-\mu\alpha\lambda]\sigma_0ht$
	A ₂	$\mu\alpha\lambda\sigma_0h^2t$	$[2\lambda\sqrt{1-\mu^2}+4\beta-\mu\alpha\lambda]\sigma_0ht$
B	B ₁	$\alpha(\lambda\mu-\alpha)\sigma_0h^2t$	$\left\{4\beta\left[1-\frac{\alpha\lambda}{2\beta}\left(\frac{\mu}{2}-\frac{1}{\lambda}\right)\right]+2\lambda\sqrt{1-\mu^2}\times(1-\alpha)\right\}\sigma_0ht$
	B ₂	$\alpha(\lambda\mu-\alpha)\sigma_0h^2t$	$(4\beta-\alpha\lambda\mu+2)\sigma_0ht$
C	C ₁	$\frac{\alpha\lambda\mu(1+4\beta)}{2(1+2\beta)}\sigma_0h^2t$	$(4\beta+2\lambda\sqrt{1-\mu^2})\sigma_0ht$
	C ₂	$\frac{\alpha\lambda\mu(1+4\beta)}{2(1-2\beta)}\sigma_0h^2t$	$\left\{2(1+2\beta)-\alpha\lambda\mu+\frac{4\beta}{1+2\beta}\times(\lambda\sqrt{1-\mu^2}-1)\right\}\sigma_0ht$
	C ₃	$\frac{\alpha\lambda\mu(1+4\beta)}{2(1+2\beta)}\sigma_0h^2t$	$[2(1+2\beta)-\alpha\lambda\mu]\sigma_0ht$
D	D ₁	$\left[\alpha\lambda\mu(1+2\beta)+4\alpha^2\beta^2-\frac{\lambda^2\mu^2}{4}\right]\sigma_0h^2t$	$(4\beta-\lambda\mu\alpha+2)\sigma_0ht$
	D ₂	$\left[\alpha\lambda\mu(1+2\beta)+4\alpha^2\beta^2-\frac{\lambda^2\mu^2}{4}\right]\sigma_0h^2t$	$\{4\beta-\lambda\mu(\alpha-1)-4\alpha\beta+(2+4\alpha\beta+(2+4\alpha\beta-\lambda\mu)\times\lambda\sqrt{1-\mu^2})\}\sigma_0ht$
E	E ₁	$\left\{\mu\lambda\left(2\alpha\beta+\alpha-\frac{\mu\lambda}{2}\right)+\frac{\mu^2\lambda^2}{4}\left[1-\left(\frac{4\alpha\beta}{\mu\lambda}\right)^2\right]\right\}\sigma_0h^2t$	$\left\{4\beta-\mu\lambda\left[\alpha(1+2\beta)-\frac{\mu\lambda}{2}\right]+2\lambda\sqrt{1-\mu^2}\right\}\sigma_0ht$
	E ₂	$\left\{\mu\lambda\left(2\alpha\beta+\alpha-\frac{\mu\lambda}{2}\right)+\frac{\mu^2\lambda^2}{4}\left[1-\left(\frac{4\alpha\beta}{\mu\lambda}\right)^2\right]\right\}\sigma_0h^2t$	$\left\{2(1+2\beta)-\mu\lambda\alpha+\frac{8\alpha\beta}{\mu\lambda}(\lambda\sqrt{1-\mu^2}-1)\right\}\sigma_0ht$
	E ₃	$\left\{\mu\lambda\left(2\alpha\beta+\alpha-\frac{\mu\lambda}{2}\right)+\frac{\mu^2\lambda^2}{4}\left[1-\left(\frac{4\alpha\beta}{\mu\lambda}\right)^2\right]\right\}\sigma_0h^2t$	$(2(1+2\beta)-\mu\alpha\lambda)\sigma_0ht$
F ₁	F ₁₋₁	$M_1=\left\{\mu\lambda_1\left(2\alpha\beta_1+\alpha-\frac{\mu\lambda_1}{2}\right)+\frac{\mu^2\lambda_1^2}{4}\left[1-\left(\frac{4\alpha\beta_1}{\mu\lambda_1}\right)^2\right]\right\}\sigma_0h^2t_1$	$N_1=\left\{4\beta_1-\mu\lambda_1\left[\alpha(1+2\beta)-\frac{\mu\lambda_1}{2}\right]+2\lambda_1\sqrt{1-\mu^2}\right\}\sigma_0ht_1$
	F ₁₋₂	$M_1=\left\{\mu\lambda_1\left(2\alpha\beta_1+\alpha-\frac{\mu\lambda_1}{2}\right)+\frac{\mu^2\lambda_1^2}{4}\left[1-\left(\frac{4\alpha\beta_1}{\mu\lambda_1}\right)^2\right]\right\}\sigma_0h^2t_1$	$N_1=\left\{2(1+2\beta)-\mu\lambda_1\alpha+\frac{8\alpha\beta_1}{\mu\lambda_1}\times(\lambda_1\sqrt{1-\mu^2}-1)\right\}\sigma_0ht_1$
	F ₁₋₃	$M_1=\left\{\mu\lambda_1\left(2\alpha\beta_1+\alpha-\frac{\mu\lambda_1}{2}\right)+\frac{\mu^2\lambda_1^2}{4}\left[1-\left(\frac{4\alpha\beta_1}{\mu\lambda_1}\right)^2\right]\right\}\sigma_0h^2t_1$	$N_1=(2(1+2\beta_1)-\mu\alpha\lambda_1)\sigma_0ht_1$
F ₂	F ₂₋₁	$M_2=\frac{\alpha\mu}{2}\sigma_0h^2t_2$	$N_2=(2-\alpha\mu)\sigma_0ht_2$
	F ₂₋₂	$M_2=\frac{(1+\xi)\alpha}{2}\mu\sigma_0h^2t_2$	$N_2=2\sqrt{1-\mu^2}\sigma_0ht_2$

where $M=M_1+M_2$, $N=N_1+N_2$, $t_1=\frac{\lambda-1}{\lambda_1-1}t$, $t_2=1-t_1$, $\lambda_1=\frac{2}{\alpha\mu}$,

Case F is given by superposing Case F₁ on F₂ $\beta_1=\beta\frac{t}{t_1}$

Fig. 9 Kinematically Admissible Velocity Field for Case (f)

where ξ is a parameter for $0 \leq \xi \leq 1$

Case (b) : From a velocity field shown in Fig. 7

W and D are given by

$$W=2N\xi h_c\dot{\theta}+2M\dot{\theta} \quad (4.6)$$

$$D=2[(1+\xi^2)+4\beta]\dot{\theta}h_c^2t\sigma_0 \quad (4.7)$$

Equating W and D , and the minimum value of

$\frac{M}{M_0}$ is obtained for

$$\xi=(1+2\beta)\cdot\frac{N}{N_0} \quad (4.8)$$

as

$$\frac{M}{M_0}=1-\frac{(1+2\beta)^2}{1+4\beta}\cdot\left(\frac{N}{N_0}\right)^2 \quad (4.9)$$

The assumed velocity field is established for $0 \leq \xi \leq 1$ so that the condition

$$0 \leq \frac{N}{N_0} \leq \frac{1}{1+2\beta} \quad (4.10)$$

must be established, according to Eq. (4.8).

Case (c) : Taking $\xi=1$ in the case (b), we have $\frac{M}{M_0}$ as

$$\frac{M}{M_0} = \frac{2(1+2\beta)}{1+4\beta} \left(1 - \frac{N}{N_0}\right) \dots\dots\dots (4.11)$$

The assumed velocity field is established for $1 \leq \xi \leq 1+2\beta$, so that the condition

$$1 \geq \frac{N}{N_0} \geq \frac{1}{1+2\beta} \dots\dots\dots (4.12)$$

must be established in this case, according to Eq. (4.8)

Case (d) : From the assumption of a velocity field shown in Fig. 8, the power of dissipation D are obtained as

$$W = 2N\xi_1 h_c \dot{\theta} + 2M\dot{\theta} \dots\dots\dots (4.13)$$

$$D = 2 \left[\xi_2 \alpha \lambda \sqrt{4 \left(\frac{\xi_1}{\alpha} \right)^2 + 1} + \xi_1^2 + (1 - \xi_2) \times (1 - \xi_2 + 4\beta) \right] \dot{\theta} h_c^2 t \sigma_0 \dots\dots\dots (4.14)$$

Equating W and D , we have

$$\frac{M}{M_0} = \frac{1}{1+4\beta} \left[\xi_2 \lambda \sqrt{4 \xi_1^2 + \alpha^2} + \xi_1^2 + (1 - \xi_2)(1 - \xi_2 + 4\beta) - 2\xi_1(1+2\beta) \left(\frac{N}{N_0} \right) \right] \dots\dots\dots (4.15)$$

the minimum value of $\frac{M}{M_0}$ is obtained by equations

$$\frac{\partial \left(\frac{M}{M_0} \right)}{\partial \xi_1} = 0, \quad \frac{\partial \left(\frac{M}{M_0} \right)}{\partial \xi_2} = 0$$

which give

$$\frac{N}{N_0} = \frac{1}{1+2\beta} \left[\xi_1 + \frac{2\lambda\xi_1\xi_2}{\sqrt{4\xi_1^2 + \alpha^2}} \right] \dots\dots\dots (4.16)$$

$$\xi_2 = 1 + 2\beta - \frac{\lambda}{2} \sqrt{4\xi_1^2 + \alpha^2} \dots\dots\dots (4.17)$$

finally we have

$$\frac{M}{M_0} = \frac{1}{1+4\beta} \left[\frac{\xi_2 \lambda (2\xi_1^2 + \alpha^2)}{\sqrt{4\xi_1^2 + \alpha^2}} - \xi_1^2 + (1 - \xi_2)(1 - \xi_2 + 4\beta) \right] \dots\dots\dots (4.18)$$

where ξ_1 and ξ_2 are variables under the conditions of

$$0 \leq \xi_1 \leq 1 \text{ and } 0 \leq \xi_2 \leq 1 \dots\dots\dots (4.19)$$

and

$$0 \leq \xi_1 + \xi_2 \leq 1 \dots\dots\dots (4.20)$$

Case (f) : From the velocity field of Fig. 9, W and D are given by

$$W = 2N\xi_1 h_c \dot{\theta} + 2M\dot{\theta} \dots\dots\dots (4.21)$$

$$D = 2 \left[\xi_2 \eta \lambda \sqrt{4 \left(\frac{\xi_1}{\eta} \right)^2 + 1} + \xi_1^2 \lambda + (1 - \xi_2) \{ (1 - \xi_2) \lambda + 4\beta \} + (\alpha - \eta) \{ (\alpha - \eta) \lambda + 4\alpha\beta \} \right] \dot{\theta} h_c^2 t \sigma_0 \dots\dots\dots (4.22)$$

Equating W and D gives $\frac{M}{M_0}$ as

$$\begin{aligned} \frac{M}{M_0} = \frac{1}{1+4\beta} \left[\xi_2 \eta \sqrt{4 \xi_1^2 + \eta^2} + \lambda \xi_1^2 + (1 - \xi_2) \{ (1 - \xi_2) \lambda + 4\beta \} + (\alpha - \eta) \{ (\alpha - \eta) \lambda \right. \\ \left. + 4\alpha\beta \} - 2\xi_1(1+2\beta) \left(\frac{N}{N_0} \right) \right] \dots\dots\dots (4.23) \end{aligned}$$

the minimum value of $\frac{M}{M_0}$ can be obtained by solutions

$$\frac{\partial \left(\frac{M}{M_0} \right)}{\partial \xi_1} = 0, \quad \frac{\partial \left(\frac{M}{M_0} \right)}{\partial \xi_2} = 0 \quad \text{and} \quad \frac{\partial \left(\frac{M}{M_0} \right)}{\partial \eta} = 0$$

which give

$$\frac{N}{N_0} = \frac{1}{1+2\beta} \left[\lambda \xi_1 + \frac{2\lambda\xi_1\xi_2}{\sqrt{4\xi_1^2 + \eta^2}} \right] \dots\dots\dots (4.24)$$

Fig. 10 Kinematically Admissible Velocity Field for Case (g)

Fig. 11 Kinematically Admissible Velocity Field for Case (h)

Fig. 12 Kinematically Admissible Velocity Field for Case (i)

$$\xi_1 = \frac{1}{2} \sqrt{\left[\frac{2\left(1 + \frac{2\beta}{\lambda}\right)}{1 + \frac{4}{\eta} \left\{ \alpha \left(1 + \frac{2\beta}{\lambda}\right) - \eta \right\}} \right]^2 - \eta^2} \dots\dots\dots (4.25)$$

$$\xi_2 = \frac{\frac{4}{\eta} \left\{ \alpha \left(1 + \frac{2\beta}{\lambda}\right) - \eta \right\} \left(1 + \frac{2\beta}{\lambda}\right)}{1 + \frac{4}{\eta} \left\{ \alpha \left(1 + \frac{2\beta}{\lambda}\right) - \eta \right\}} \dots\dots\dots (4.26)$$

and finally we have

$$\frac{M}{M_0} = \frac{1}{1 + 4\beta} \left[\frac{\lambda \xi_2 \eta^2}{\sqrt{4\xi_1^2 + \eta^2}} - \lambda \xi_1^2 + (1 - \xi_2) \{ (1 - \xi_2) \lambda + 4\beta \} + (\alpha - \eta) \{ (\alpha - \eta) \lambda + 4\alpha\beta \} \right] \dots\dots\dots (4.27)$$

The conditions

$$0 \leq \xi_1 \leq 1, \quad 0 \leq \xi_2 \leq 1, \quad 0 \leq \xi_1 + \xi_2 \leq 1, \quad 0 \leq \eta \leq \alpha \dots\dots\dots (4.28)$$

must be satisfied in this case (f).

The following additional cases (g), (h), and (i) must take into consideration in case of $\alpha \leq 1$.

Case (g) : From the velocity field of Fig. 10, W and D are given by

$$W = 2N\xi h_c \dot{\theta} + 2M\dot{\theta} \dots\dots\dots (4.29)$$

$$D = \left[2(1 - \xi) \eta \lambda \sqrt{4\left(\frac{\xi}{\eta}\right)^2 + 1} + 2\xi(2\xi\lambda + 4\beta) + (\alpha - \eta)^2(\lambda + 1) + 8\alpha\beta(\alpha - \eta) \right] \dot{\theta} h_c^2 t \sigma_0 \dots\dots\dots (4.30)$$

Equating W and D gives $\frac{M}{M_0}$ as

$$\frac{M}{M_0} = \frac{1}{2(1 + 4\beta)} \left[2(1 - \xi) \lambda \sqrt{4\xi^2 + \eta^2} + 2\xi(2\xi\lambda + 4\beta) + (\alpha - \eta)^2(1 + \lambda) + 8\alpha\beta(\alpha - \eta) - 4\xi(1 + 2\beta) \left(\frac{N}{N_0} \right) \right] \dots\dots\dots (4.31)$$

the minimum value of $\frac{M}{M_0}$ can be obtained by solution of equations

$$\frac{\partial \left(\frac{M}{M_0} \right)}{\partial \xi} = 0, \quad \frac{\partial \left(\frac{M}{M_0} \right)}{\partial \eta} = 0$$

which give

$$\frac{N}{N_0} = \frac{4(\xi\lambda + \beta) + \frac{\lambda[4\xi(1 - 2\xi) - \eta^2]}{\sqrt{4\xi^2 + \eta^2}}}{2(1 + 2\beta)} \dots\dots\dots (4.32)$$

$$\xi = \frac{\lambda^2 \eta^2 + \{ \alpha(1 + \lambda + 4\beta) - \eta(1 + \lambda) \} \cdot \eta \sqrt{\lambda^2(4 + \eta^2) - 4\{ \alpha(1 + \lambda + 4\beta) - \eta(1 + \lambda) \}^2}}{\lambda^2 \eta^2 - 4\{ \alpha(1 + \lambda + 4\beta) - \eta(1 + \lambda) \}^2} \dots\dots\dots (4.33)$$

and finally we have

$$\frac{M}{M_0} = \frac{1}{2(1 + 4\beta)} \left[\frac{2\lambda(4\xi^2 + \eta^2)}{\sqrt{4\xi^2 + \eta^2}} - 4\xi^2\lambda + (\alpha - \eta)^2(1 + \lambda) + 8\alpha\beta(\alpha - \eta) \right] \dots\dots\dots (4.34)$$

Fig. 13 (a) Moment Thrust Interaction Curves

Fig. 14 (a) Moment Thrust Interaction Curves

Fig. 13 (b) Moment Thrust Interaction Curves

Fig. 14 (b) Moment Thrust Interaction Curves

Fig. 13 (c) Moment Thrust Interaction Curves

Fig. 14 (c) Moment Thrust Interaction Curves

Fig. 15(a) Moment Thrust Interaction Curves

Fig. 15(b) Moment Thrust Interaction Curves

The conditions

$$0 \leq \xi \leq 1, \quad 0 \leq \eta \leq \alpha \quad (4.35)$$

must be satisfied in this case (g).

Case (h) : From assumed velocity field as Fig. 11, we obtained W and D as

$$W = 2M\dot{\theta} \quad (4.26)$$

$$D = 2[\lambda\eta + (\alpha - \eta)(\alpha - \eta + 4\alpha\beta)] \dot{\theta} h_c^2 t \sigma_0 \quad (4.37)$$

Equating W and D gives $\frac{M}{M_0}$ as

$$\frac{M}{M_0} = \frac{\eta^2 - 2\left(\alpha + 2\alpha\beta - \frac{\lambda}{2}\right)\eta + \alpha^2(1 + 4\beta)}{1 + 4\beta} \quad (4.38)$$

the minimum value of $\frac{M}{M_0}$ is obtained for

$$\eta = \alpha + 2\alpha\beta - \frac{\lambda}{2} \quad (4.39)$$

as

$$\frac{M}{M_0} = \alpha^2 - \frac{\left(\alpha + 2\alpha\beta - \frac{\lambda}{2}\right)^2}{1 + 4\beta} \quad (4.40)$$

The assumed velocity field is established for $0 \leq \eta \leq \alpha$, so that the condition

$$2\alpha(1 + 2\beta) \geq \lambda \geq 4\alpha\beta \quad (4.41)$$

must be established, according to Eq. (4.39) Case (i) : From the assumed velocity field Fig. 12, W and D are given by

$$W = 2M\dot{\theta} \quad (4.42)$$

$$D = 2\alpha^2(1 + 4\beta)h_c^2 t \dot{\theta} \sigma_0 \quad (4.43)$$

Equating W and D , the value of $\frac{M}{M_0}$ is obtained as

$$\frac{M}{M_0} = \alpha^2 \quad (4.44)$$

The above equation is established for

Fig. 15(c) Moment Thrust Interaction Curves

$$2\alpha(1+2\beta) \leq \lambda \dots\dots\dots(4.45)$$

§5 Conclusion

The lower and upper bounds of yield moments and axial thrust of cruciform connections are obtained in §2, §3 and §4. The numerical results of the interaction curves of yield moment and thrust are shown in Fig. 13(a)~(c), Fig. 14(a)~(c), and Fig. 15(a)~(c), in which M_0 and N_0 denote yield moment and axial thrust of column, and M_0' denotes yield moment of beam.

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References

- (1) H. Tanaka, A. Fukushima, Yung Chuan. Lin : Limit Analysis of Beam-Column Connections (I-1), (I-2), Trans. of A.I.J. No. 148, 149, Jun, July, 1968.
- (2) H. Tanaka : Limit Analysis of Cruciform Connections according to Von Mises Yield Condition, Kozokogaku Kenkyu, Todai Shuppan-Kai, 1968.

柱はり接合部の極限解析 (VIII-2) 梗概

——強軸曲げと柱方向に軸力を受ける十字形接合部——

正 会 員 田 中 尚*

大 会 員 林 永 全**

§ 4. 運動的解析

解析を (a), (b), (c), (d), (f), (g), (h) と (i) の 8 場合に分けて行なう。解析 (a) は図—6 に示す運動的許容速度場を仮定すると、荷重のなる仕事率 W と、エネルギー散逸率 D は式 (4.1) と (4.2) である。これらを等置して式 (4.3) が得られる。 $\frac{M}{M_0}$ の最小値は式 (4.4) において式 (4.5) である。解析 (b) は図—7 の許容速度場から W と D を式 (4.6) と (4.7) のように求められ、 $\frac{M}{M_0}$ の最小値は式 (4.8) において、式 (4.9) である。式 (4.9) は式 (4.10) において成立する。解析 (c), 解析 (b) に $\xi=1$ とおいて解くと $\frac{M}{M_0}$ は式 (4.11) になる。上式は式 (4.12) の範囲において成立する。解析 (d), 図—8 の許容速度場から W と D を求めると、式 (4.13) と (4.14) になる。上の両式を等置すると式 (4.15) が得られる。式 (4.15) の最小値は式 (4.16) と (4.17) から求めると式 (4.18) になる。式 (4.18) は式 (4.19) と (4.20) の範囲において成立する。解析 (5) : 図—9 の許容速度場から W と D を求めると式 (4.21) と (4.22) が得られる。 W と D を等置して式 (4.23) が得られる。

式 (4.23) の最小値は式 (4.24), (4.25), (4.26) から式 (4.27) のように求められる。上式は式 (4.28) の範囲で成立する。(g), (h), (i) の場合は $\alpha \leq 1$ のときに考慮する必要がある。解析 (g) において 図—10 から W と D を式 (4.29) と (4.30) のように求められる。 W と D を等置して整理すると式 (4.31) が得られる。式 (4.31) の最小値は式 (4.32) と (4.33) から式 (4.34) のように求められる。式 (4.34) は式 (4.35) の範囲で成立す。解析 (h) では、図—11 の速度場により、 W と D は式 (4.36) と (4.37) になる。それらを等置して式 (4.38) が得られ、 $\frac{M}{M_0}$ の最小値は式 (4.39) によって、式 (4.40) になる。式 (4.40) は式 (4.41) の範囲において成立する。解析 (i) : 図—12 の速度場から W と D を式 (4.42) と式 (4.43) のように求められる。 $\frac{M}{M_0}$ は式 (4.44) で、上式は式 (4.45) の範囲において成立する。

§ 5. 結 論

2, 3 と 4 節で求めた降伏モーメントと軸力の下界と上界を図—13 (a)~(c), 図—14 (a)~(c) と 図—15 (a)~(c) に示す。ここに M_0 と N_0 はそれぞれ柱の降伏モーメントと降伏軸力で、 M_0' は、はりの降伏モーメントである。最後に日本大学講師福島暁男さんの有益な助言と協力に心から感謝の意を表します。

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ON THE STRENGTH AND STIFFNESS OF BEAM-TO-COLUMN CONNECTIONS IN STEEL PIPE COLUMNS

(The Reinforcement for Shear Force in the Connections)

by SUGURU SAKAMOTO, Central Research Laboratories, Sumitomo Metal Industries, Ltd. and Dr. NOBUYOSHI HISAMITSU, Central Research Laboratories, Sumitomo Metal Industries, Ltd.; Members of A.I.J.

It is not desirable that yielding due to shear force occurs in the beam-to-column connections below the flexural yield load in pipe columns. A calculation shows that "shear yielding" will often occur prior to yielding by flexure. Such yielding may cause large deformations of the frame.

This paper is concerned with the study into the reinforcement of connections fabricated of steel pipe columns and H-shape beams. The specimens, having various reinforcements and different dimension in steel pipes and stiffening plates, were tested. The most efficient reinforcement was found to be the doubler stiffening plates, Fig. 4 (c). The stress and deformation of the so reinforced connections can be predicted by the ordinary beam theory. An equation was obtained to determine the desirable thickness of stiffening plates, Eq. (10).

U.D.C. 624.078.072.2 : 624.043

LIMIT ANALYSIS OF BEAM-COLUMN CONNECTIONS (VIII-2)

—Cruciform Connections of Wide-flange Members Subjected
to Strong-axis Bending and Thrust on Column—
(See Page 59)

by Dr. HISASHI TANAKA, Prof., Inst. of Indu. Scie. Univ. of Tokyo and YUNG CHUAN SIN, Graduate Student Univ. of Tokyo, Members of A.I.J.

U.D.C. 624.072.2 : 624.041.62

EXPERIMENTAL STUDY OF THE INELASTIC STEEL BEAM (1)

—Beam under Uniform Moment—

by Dr. TOSHIRO SUZUKI, Assistant Prof. Nagoya Institute of Tech. and TETSURO ONO, Graduate Student. Nagoya Institute of Tech., Members of A.I.J.

This paper is an experimental study of the inelastic steel beam under uniform moment. We know that it is useful to brace the inelastic steel beam in plastic design. But, the behavior after lateral-torsional buckling has not been explained. Then in this paper, in order to obtain the support space-versus-the rotation capacity relationship and the influence of side span under moment gradient, we try to experiment. We experimented eleven test pieces; these test pieces have different length of span, various numbers of span and different moment distribution. We note the outline of experiments, and the result of C-2 type which show typical behavior. Further, we express the results of each type. In general, the beam under uniform moment collapses by lateral-torsional buckling.

But, the beam that is braced sufficiently closely to allow attainment M_p holds constant moment after lateral-torsional buckling until the local buckling occurs on the compression flange. The comparison between test results and the theoretical result of M.G. Lay are shown in Fig. 19. From the results, we know that equation (1-2) is useful within the limit of the application of simple plastic design theory. And, from the experimental results, the value of the effective length factor, k , may be taken as $0.52 < k < 0.56$, however, if the side spans are also under full plastic moment, k should be taken as $0.82 < k < 0.84$.