STRESS ANALYSIS OF⁺ TYPE BEAM-COLUMN CONNECTIONS (II)

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§ 4 The axial load

If the external forces act on each side as in fig. 3, the next analytic function are considered ;

$$\begin{aligned} g(z) &= \sum c_m z^{2m}, \\ h(z) &= \sum d_m z^{2m}, \end{aligned} \quad (m=0, 1, 2, \dots) \dots\dots\dots (4.1)$$

where c_m and d_m are unknown real constants which are determined from the external forces and their stress distribution on each side.

For $m=0$, it yields from (2.1) that

$$\left. \begin{aligned} \sigma_x^{(0)} &= c_0 + 0.5 d_0, \\ \sigma_y^{(0)} &= c_0 - 0.5 d_0, \\ \tau_{xy}^{(0)} &= 0, \end{aligned} \right\} \dots\dots\dots (4.2)$$

where two unknown constants are determined only by the external forces on each side; the axial force N_1 acts on the side AB , then

$$N_1 = \int_{-b}^b \sigma_x^{(0)}|_{x=a} \cdot dy$$

so that

$$\sigma_x^{(0)}|_{x=a} = \frac{N_1}{2} \frac{1}{b}, \quad \dots\dots\dots (4.3)$$

and the axial force N_2 acts on BC , then

$$N_2 = \int_{-a}^a \sigma_y^{(0)}|_{y=b} \cdot dx,$$

so that

$$\sigma_y^{(0)}|_{y=b} = \frac{N_2}{2} \frac{1}{a}. \quad \dots\dots\dots (4.4)$$

Substituting (4.3) and (4.4) into (4.2), c_0 and d_0 are determined.

This solution which is obtained only by the external forces is called the fundamental solution, or the 0-th approximate solution.

The n -th approximate solution is obtained in the similar method as § 3.

For $m=n$ in (4.1), it yields form (2.1) that

$$\left. \begin{aligned} \sigma_x^{(n)} &= \sum_{m=0}^n \sum_{r=0}^m (-r)^r \left\{ \left({}_{2m}C_{2r} - 2m \times {}_{2m-1}C_{2r+1} \times \frac{y^2}{x^2} \right) c_m + \frac{{}_{2m}C_{2r}}{2} d_m \right\} x^{2m-2r} y^{2r}, \\ \sigma_y^{(n)} &= \sum_{m=0}^n \sum_{r=0}^m (-1)^r \left\{ \left({}_{2m}C_{2r} + 2m \times {}_{2m-1}C_{2r+1} \times \frac{y^2}{x^2} \right) c_m - \frac{{}_{2m}C_{2r}}{2} d_m \right\} x^{2m-2r} y^{2r}, \\ \tau_{xy}^{(n)} &= \sum_{m=0}^n \sum_{r=0}^m (-1)^{r+1} \left\{ 2m \times {}_{2m-1}C_{2r} c_m + \frac{{}_{2m}C_{2r+1}}{2} d_m \right\} x^{2m-2r-1} y^{2r+1}. \end{aligned} \right\} \dots\dots\dots (4.5)$$

The $2n+2$ constants are determined by not only the external forces but also the distribution of the normal stress on each side. $E_q(4.5)$ yields two solutions depending on the latter.

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Fig. 3

(1) If the axial force N_1 acts on the side AB , the distribution of the normal stress on AB is selected to take $2n$ -th power of y ;

$$N_1 = \int_{-b}^b \sigma_x^{(n)}|_{x=a} dy,$$

so that

$$\sigma_x^{(n)}|_{x=a} = \frac{N_1}{2} \frac{2n+1}{b^{2n+1}} y^{2n}. \quad (4.6)$$

And if the axial force N_2 acts on the side BC , it is selected to be linear of y .

$$N_2 = \int_{-a}^a \sigma_y^{(n)}|_{y=b} dx,$$

so that

$$\sigma_y^{(n)}|_{y=b} = \frac{N_2}{2} \frac{1}{a} \quad (4.7)$$

Substituting (4.6) and (4.7) into (4.5), then $c_0, d_0, \dots, c_n$ and d_n are determined.

(2) The distribution of the normal stress on the side AB and BC are interchanged with each other: for N_1 on AB , it is selected to be linear

$$\sigma_x^{(n)}|_{x=a} = \frac{N_1}{2} \frac{1}{b}, \quad (4.8)$$

and for N_2 on BC , it is $2n$ -th power degree with respect to x ;

$$\sigma_y^{(n)}|_{y=b} = \frac{N_2}{2} \frac{2n+1}{a^{2n+1}} x^{2n} \quad (4.9)$$

For the case (1) the stresses (4.5) are represented by

$$\sigma_x^{(n)-X}, \sigma_y^{(n)-X}, \tau_{xy}^{(n)-X},$$

and for the case (2)

$$\sigma_x^{(n)-Y}, \sigma_y^{(n)-Y}, \tau_{xy}^{(n)-Y}.$$

We consider the following stresses:

$$\sigma_x^{(n)-X} - \sigma_x^{(n-1)-X}, \sigma_y^{(n)-X} - \sigma_y^{(n-1)-X}, \tau_{xy}^{(n)-X} - \tau_{xy}^{(n-1)-X}, \quad (4.10)$$

and

$$\sigma_x^{(n)-Y} - \sigma_x^{(n-1)-Y}, \sigma_y^{(n)-Y} - \sigma_y^{(n-1)-Y}, \tau_{xy}^{(n)-Y} - \tau_{xy}^{(n-1)-Y}. \quad (4.11)$$

The expressions (4.10) and (4.11) have self-equilibrium of force, namely the external forces on each side are zero. The normal stress of (4.10) is zero on the side BC , and takes $(2n-2)$ -th and $2n$ -th power of y on AB . But the normal stress of (4.11) is zero on the side AB , and takes $(2n-2)$ -th and $2n$ -th power of x on BC .

Finally

$$\left. \begin{aligned} \sigma_x &= \sigma_x^{(0)} + \sum_{n=1}^n \lambda_n (\sigma_x^{(n)-X} - \sigma_x^{(n-1)-X}) + \sum_{n=1}^n \mu_n (\sigma_x^{(n)-Y} - \sigma_x^{(n-1)-Y}), \\ \sigma_y &= \sigma_y^{(0)} + \sum_{n=1}^n \lambda_n (\sigma_y^{(n)-X} - \sigma_y^{(n-1)-X}) + \sum_{n=1}^n \mu_n (\sigma_y^{(n)-Y} - \sigma_y^{(n-1)-Y}), \\ \tau_{xy} &= \tau_{xy}^{(0)} + \sum_{n=1}^n \lambda_n (\tau_{xy}^{(n)-X} - \tau_{xy}^{(n-1)-X}) + \sum_{n=1}^n \mu_n (\tau_{xy}^{(n)-Y} - \tau_{xy}^{(n-1)-Y}), \end{aligned} \right\} \quad (4.12)$$

where

$$\begin{aligned} \sigma_x^{(0)-X} &= \sigma_x^{(0)-Y} = \sigma_x^{(0)}, \\ \sigma_y^{(0)-X} &= \sigma_y^{(0)-Y} = \sigma_y^{(0)}, \\ \tau_{xy}^{(0)-X} &= \tau_{xy}^{(0)-Y} = \tau_{xy}^{(0)}. \end{aligned}$$

In this expression the 1st term only is concerned with the external forces on each side, but the others have no concern with them and changes the stress distribution, and hence have influence on the strain potential energy. Therefore the moduli are determined to minimize the strain potential energy, so that

$$\frac{\partial U}{\partial \lambda_1} = \dots = \frac{\partial U}{\partial \lambda_n} = \frac{\partial U}{\partial \mu_1} = \dots = \frac{\partial U}{\partial \mu_n} = 0 \quad (4.13)$$

If the connection is square and the external forces on each side are equal, it follows that

$$\lambda_n = \mu_n \dots\dots\dots (4.14)$$

Namely, directly, substituting (4.6) and (4.9) into (4.5), $c_0, d_0, \dots, c_n$ and d_n are determined. These stresses are represented by

$$\sigma_x^{(n)XY}, \sigma_y^{(n)XY}, \tau_{xy}^{(n)XY},$$

and we consider the following stresses :

$$\sigma_x^{(n)XY} - \sigma_x^{(n-1)XY}, \sigma_y^{(n)XY} - \sigma_y^{(n-1)XY}, \tau_{xy}^{(n)XY} - \tau_{xy}^{(n-1)XY}, \dots\dots\dots (4.15)$$

which have self-equilibrium of force, namely the external forces on each side are zero, the distribution of the normal stress takes $(2n-2)$ -th and $2n$ -th power of y on the side AB and x on BC . Finally

$$\left. \begin{aligned} \sigma_x &= \sigma^{(0)} + \sum_{n=1}^{\infty} \nu_n (\sigma_x^{(n)XY} - \sigma_x^{(n-1)XY}), \\ \sigma_y &= \sigma_y^{(0)} + \sum_{n=1}^{\infty} \nu_n (\sigma_y^{(n)XY} - \sigma_y^{(n-1)XY}), \\ \tau_{xy} &= \tau_{xy}^{(0)} + \sum_{n=1}^{\infty} \nu_n (\tau_{xy}^{(n)XY} - \tau_{xy}^{(n-1)XY}), \end{aligned} \right\} \dots\dots\dots (4.16)$$

where

$$\sigma_x^{(0)XY} = \sigma_x^{(0)}, \sigma_y^{(0)XY} = \sigma_y^{(0)}, \tau_{xy}^{(0)XY} = \tau_{xy}^{(0)}.$$

And similarly

$$\frac{\partial U}{\partial \nu_1} = \dots = \frac{\partial U}{\partial \nu_n} = 0. \dots\dots\dots (4.17)$$

In this way, the true solution of the rectangular beam-column connection as a rectangular plate under the axial load in fig. 3 is obtained to take m in (4.1) to infinite, applying the principle of least energy.

Abridging this regular method, DIRECTLY, we can obtain the solution, if we do not mind the troublesome, Namely, in eqs (4.5) there are $2n+2$ constants which are determined by the external forces and the principle of least energy; from the former, 2 constants are determined :

$$\left. \begin{aligned} c_0 &= f_1(c_1, d_1, \dots, c_n, d_n), \\ d_0 &= f_2(c_1, d_1, \dots, c_n, d_n), \end{aligned} \right\} \dots\dots\dots (4.18)$$

then the strain potential energy is

$$U = U(c_1, d_1, \dots, c_n, d_n), \dots\dots\dots (4.19)$$

from the the later the remaining constants are determined :

$$\frac{\partial U}{\partial c_1} = \frac{\partial U}{\partial d_1} = \dots = \frac{\partial U}{\partial c_n} = \frac{\partial U}{\partial d_n} = 0 \dots\dots\dots (4.20)$$

§5 The symmetrical load with respect to the y -axis

If the external forces acts on each side as in fig. 4, the next analytic functions are considered;

$$\left. \begin{aligned} g(z) &= i \sum c_m z^{2m-1}, \\ h(z) &= i \sum d_m z^{2m-1}, \end{aligned} \right\} (m=1, 2, 3, \dots) \dots\dots\dots (5.1)$$

where c_m and d_m are unknown real constants.

For $m=1$ the stresses of (5.1) yielded from (2.1) that

$$\left. \begin{aligned} \sigma_x^{(0)} &= -(2c_1 + 0.5d_1)y, \\ \sigma_y^{(0)} &= 0.5d_1y, \\ \tau_{xy}^{(0)} &= -0.5d_1x, \end{aligned} \right\} \dots\dots\dots (5.2)$$

where two unknown constants are determined from only the external forces, namely, if the bending moment M_1 acts on the side AB , then

$$M_1 = \int_{-b}^b \sigma_x^{(0)}|_{x=a} \cdot y dy,$$

so that

$$\sigma_x^{(0)}|_{x=a} = \frac{M_1}{2} \frac{3}{b^3} y, \dots\dots\dots (5.3)$$

Fig. 4

and if the axial force N_2 acts on BC , then

$$N_2 = \int_{-a}^a \sigma_y^{(0)}|_{y=b} dx,$$

so that

$$\sigma_y^{(0)}|_{y=b} = \frac{N_2}{2} \frac{1}{a} \dots\dots\dots (5.4)$$

Substituting (5.3) and (5.4) into (5.2), c_1 and d_1 are determined

Then n -th approximate solution is obtained in the similar way as the previous sections.

For $m=n+1$ in (5.1), it follows from (2.1) that

$$\left. \begin{aligned} \sigma_x^{(n)} &= \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^{r+1} \left\{ {}_{2m-1}C_{2r+1} + (2m-1) \times {}_{2m-2}C_{2r} \right\} c_m + \frac{{}_{2m-1}C_{2r+1}}{2} d_m \Big\} x^{2m-2r-2} y^{2r+1}, \\ \sigma_y^{(n)} &= \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^{r+1} \left\{ {}_{2m-1}C_{2r+1} - (2m-1) \times {}_{2m-2}C_{2r} \right\} c_m - \frac{{}_{2m-1}C_{2r+1}}{2} d_m \Big\} x^{2m-2r-2} y^{2r+1}, \\ \tau_{xy}^{(n)} &= \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^r \left\{ (2m-1) \times {}_{2m-2}C_{2r+1} c_m \times \frac{y^2}{x^2} - \frac{{}_{2m-1}C_{2r}}{2} d_m \right\} x^{2m-2r-1} y^{2r} \end{aligned} \right\} \dots (5.5)$$

From which two solutions are obtained depending on the distribution of the normal stress on each side.

(1) For M_1 on the side AB , the distribution of the normal stress on AB is selected to take $(2n+1)$ -th power ;

$$\sigma_x^{(n)}|_{x=a} = \frac{M_1}{2} \frac{2n+3}{b^{2n+3}} y^{2n+1} \dots\dots\dots (5.6)$$

and for N_2 on the side BC , it is linear

$$\sigma_y^{(n)}|_{y=b} = \frac{N_2}{2} \frac{1}{a} \dots\dots\dots (5.7)$$

Substituting (5.6) and (5.7) into (5.5), then $c_1, d_1, \dots, c_{n+1}$ and d_{n+1} are determined.

(2) For M_1 on the side BC , it is linear :

$$\sigma_x^{(n)}|_{x=a} = \frac{M_1}{2} \frac{3}{b^3} y, \dots\dots\dots (5.8)$$

and for N_2 on the side BC , it is $2n$ -th power :

$$\sigma_y^{(n)}|_{y=b} = \frac{N_1}{2} \frac{2n+1}{a^{2n+1}} x^{2n} \dots\dots\dots (5.9)$$

Substituting (5.8) and (5.9) into (5.5), then $c_1, d_1, \dots, c_{n+1}$ and d_{n+1} are determined.

For the case (1) the stresses are represented by

$$\sigma_x^{(n)-X}, \sigma_y^{(n)-X}, \tau_{xy}^{(n)-X},$$

and for the case (2)

$$\sigma_x^{(n)-Y}, \sigma_y^{(n)-Y}, \tau_{xy}^{(n)-Y}$$

And finally we consider the equations as eqs (4.12) and (4.13). Then the last eqs (4.12) are the n -th approximate solution.

Also, directly, we can obtain the solution from (5.5) : by the external forces, two constants are determined :

$$\left. \begin{aligned} c_1 &= f_1(c_2, d_2, \dots, c_{n+1}, d_{n+1}), \\ d_1 &= f_2(c_2, d_2, \dots, c_{n+1}, d_{n+1}), \end{aligned} \right\} \dots\dots\dots (5.10)$$

then the strain potential energy is

$$U = U(c_2, d_2, \dots, c_{n+1}, d_{n+1}), \dots\dots\dots (5.11)$$

from the principle of least energy the remaining constants are determined :

$$\frac{\partial U}{\partial c_2} = \frac{\partial U}{\partial d_2} = \dots = \frac{\partial U}{\partial c_{n+1}} = \frac{\partial U}{\partial d_{n+1}} = 0 \dots\dots\dots (5.12)$$

§ 6 The symmetrical load with respect to the X-axis

This type is the same one as § 5, but x and y are interchanged with each other (see fig. 4 and 5).

Then

$$\begin{aligned} g(z) &= \sum c_m z^{2m-1}, \\ h(z) &= \sum d_m z^{2m-1}, \end{aligned} \quad (m=1, 2, 3, \dots) \quad (6.1)$$

where c_m and d_m are real numbers.

The fundamental solution, or the o -th approximate solution is obtained for $m=1$ in (6.1);

$$\left. \begin{aligned} \sigma_x^{(0)} &= (c_1 + 0.5 d_1)x, \\ \sigma_y^{(0)} &= (c_1 - 0.5 d_1)x, \\ \tau_{xy}^{(0)} &= -(c_1 + 0.5 d_1)y, \end{aligned} \right\} \quad (6.2)$$

where c_1 and c_1 are determined from

$$\sigma_x^{(0)}|_{x=a} = \frac{N_1}{2} \frac{1}{b} \quad (6.3)$$

and

$$\sigma_y^{(0)}|_{y=b} = -\frac{M_2}{2} \frac{3}{a^3} x. \quad (6.4)$$

For $m=n+1$ in (6.1)

$$\left. \begin{aligned} \sigma_x^{(n)} &= \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^r \left\{ \left({}_{2m-1}C_{2r} - (2m-1) \times {}_{2m-2}C_{2r+1} \times \frac{y^2}{x^2} \right) c_m + \frac{{}_{2m-1}C_{2r}}{2} d_m \right\} x^{2m-2r-1} y^{2r}, \\ \sigma_y^{(n)} &= \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^r \left\{ \left({}_{2m-1}C_{2r} + (2m-1) \times {}_{2m-2}C_{2r+1} \times \frac{y^2}{x^2} \right) c_m \times -\frac{{}_{2m-1}C_{2r}}{2} d_m \right\} x^{2m-2r-1} y^{2r}, \\ \tau_{xy}^{(n)} &= \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^{r+1} \left\{ (2m-1) \times {}_{2m-2}C_{2r} c_m + \frac{{}_{2m-1}C_{2r+1}}{2} d_m \right\} x^{2m-2r-2} y^{2r+1}, \end{aligned} \right\} \quad (6.5)$$

where the unknown constants are determined from

$$(1) \quad \sigma_x^{(n)}|_{x=a} = \frac{N_1}{2} \frac{2n+1}{b^{2n+1}} y^{2n}, \quad (6.6)$$

$$\sigma_y^{(n)}|_{y=b} = \frac{M_2}{2} \frac{3}{a^3} x \quad (6.7)$$

or from

$$(2) \quad \sigma_x^{(n)}|_{x=a} = \frac{N_1}{2} \frac{1}{b} \quad (6.8)$$

$$\sigma_y^{(n)}|_{y=b} = \frac{M_2}{2} \frac{2n+3}{a^{2n+3}} x^{2n+1} \quad (6.9)$$

After this we omit, because of the same as § 5.

§ 7 Displacement

For the Yokota formula the displacement u in x direction is represented by

$$Eu = -(1+\nu)y I_m[g(z)] + (1-\nu) \operatorname{Re} \left[\int g(z) dz \right] + \frac{1+\nu}{2} \operatorname{Re} \left[\int h(z) dz \right], \quad (7.1)$$

and displacement v in y direction is represented by

$$Ev = -(1+\nu)y \operatorname{Re}[g(z)] + 2 I_m \left[\int g(z) dz \right] + \frac{1+\nu}{2} I_m \left[\int h(z) dz \right]. \quad (7.2)$$

Now we show the displacement of n -th solution, namely eqs (3.35), (4.5), (5.5) and (6.5); for the inverse-symmetrical load, it follows from (2.3) that

$$Eu^{(n)} = \sum_{m=0}^{n+1} \sum_{r=0}^m (-1)^{r+1} \left[\left\{ (1+\nu) {}_{2m}C_{2r} + (1-\nu) \frac{{}_{2m+1}C_{2r+1}}{2m+1} \right\} c_m + \frac{1+\nu}{2} \frac{{}_{2m+1}C_{2r+1}}{2m+1} d_m \right] x^{2m-2r} y^{2r+1} \quad (7.3)$$

$$Ev^{(n)} = \sum_{m=0}^{n+1} \sum_{r=0}^m (-1)^r \left[\left\{ (1+\nu) {}_{2m}C_{2r+1} \times \frac{y^2}{x^2} + 2 \frac{{}_{2m+1}C_{2r}}{2m+1} \right\} c_m - \frac{1+\nu}{2} \frac{{}_{2m+1}C_{2r}}{2m+1} d_m \right] x^{2m-2r+1} y^{2r} \quad (7.4)$$

for the axial load, it follows from (2.4) that

$$Eu^{(n)} = \sum_{m=0}^n \sum_{r=0}^m (-1)^r \left[\left\{ -(1+\nu) {}_{2m}C_{2r+1} \times \frac{y^2}{x^2} + (1-\nu) \frac{{}_{2m+1}C_{2r}}{2m+1} \right\} c_m + \frac{1+\nu}{2} \frac{{}_{2m+1}C_{2r}}{2m+1} d_m \right] x^{2m-2r+1} y^{2r}$$

Fig. 5

$$\dots\dots\dots(7.5)$$

$$Ev^{(n)} = \sum_{m=0}^n \sum_{r=0}^m (-1)^{r+1} \left[\left\{ (1+\nu) {}_{2m}C_{2r} - 2 \frac{{}_{2m+1}C_{2r+1}}{2m+1} \right\} c_m + \frac{1+\nu}{2} \frac{{}_{2m+1}C_{2r+1}}{2m+1} d_m \right] x^{2m-2r} y^{2r+1} \dots\dots(7.6)$$

for the symmetrical load with respect to the y -axis

$$Eu^{(n)} = \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^{r+1} \left[\left\{ (1+\nu) {}_{2m-1}C_{2r} + (1-\nu) \frac{{}_{2m}C_{2r+1}}{2m} \right\} c_m + \frac{1+\nu}{2} \frac{{}_{2m}C_{2r+1}}{2m} d_m \right] x^{2m-2r-1} y^{2r+1} \dots\dots(7.7)$$

$$Ev^{(n)} = \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^r \left[\left\{ (1+\nu) {}_{2m-1}C_{2r+1} \times \frac{y^2}{x^2} + 2 \frac{{}_{2m}C_{2r}}{2m} \right\} c_m - \frac{1+\nu}{2} \frac{{}_{2m}C_{2r}}{2m} d_m \right] x^{2m-2r} y^{2r} \dots\dots(7.8)$$

for the symmetrical load with respect to the x -axis

$$Eu^{(n)} = \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^r \left[\left\{ -(1+\nu) {}_{2m-1}C_{2r+1} \times \frac{y^2}{x^2} + (1-\nu) \frac{{}_{2m}C_{2r}}{2m} \right\} c_m + \frac{1+\nu}{2} \frac{{}_{2m}C_{2r}}{2m} d_m \right] x^{2m-2r} y^{2r} \dots\dots(7.9)$$

$$Ev^{(n)} = \sum_{m=1}^{n+1} \sum_{r=0}^m (-1)^{r+1} \left[\left\{ (1+\nu) {}_{2m-1}C_{2r} - 2 \frac{{}_{2m}C_{2r+1}}{2m} \right\} c_m + \frac{1+\nu}{2} \frac{{}_{2m}C_{2r+1}}{2m} d_m \right] x^{2m-2r-1} y^{2r+1} \dots(7.10)$$

Then the origin does not displace :

$$\left. \begin{aligned} u(x=0, y=0) &= 0, \\ v(x=0, y=0) &= 0 \end{aligned} \right\} \dots\dots\dots(7.11)$$

§ 8 Example

We solve the most important type for us, namely the inverse-symmetrical load, because there is little space left. The problem is as follows (see fig. 2) ;

$$a=b=1 \text{ cm} \dots\dots\dots(8.1)$$

$$\left. \begin{aligned} Q_1 &= Q_2 = 5.556 \text{ kg} \\ M_1 &= M_2 = -25.31 \text{ kg} \cdot \text{cm} \end{aligned} \right\} \dots\dots\dots(8.2)$$

From eqs (3.3) and (3.4) and (3.5), eq (3.2) yields that

$$\left. \begin{aligned} \sigma_x^{(0)} &= -37.96 xy, \\ \sigma_y^{(0)} &= -37.96 xy, \\ \tau_{xy}^{(0)} &= -22.53 + 18.98 x^2 + 18.98 y^2, \end{aligned} \right\} \dots\dots\dots(8.3)$$

which is the fundamental solution, or the o -th approximate solution.

From eqs (3.7) and (3.11) and (3.9), eq (3.6) yields that

$$\left. \begin{aligned} \sigma_x^{(1)-XY} &= -63.27 xy + 63.27 x^3 y - 63.27 xy^3, \\ \sigma_y^{(1)-XY} &= -63.27 xy - 63.27 x^3 y + 63.27 xy^3, \\ \tau_{xy}^{(1)-XY} &= -26.75 + 31.64 x^2 + 31.64 y^2 + 15.82 x^4 - 94.91 x^2 y^2 + 15.82 y^4 \end{aligned} \right\} \dots\dots\dots(8.4)$$

Then eq (3.18) yields that

$$\left. \begin{aligned} \sigma_x^{(1)-XY} - \sigma_x^{(0)} &= -25.31 xy + 63.27 x^3 y - 63.27 xy^3, \\ \sigma_y^{(1)-XY} - \sigma_y^{(0)} &= -25.31 xy - 63.27 x^3 y + 63.27 xy^3 \\ \tau_{xy}^{(1)-XY} - \tau_{xy}^{(0)} &= -4.22 + 12.66 x^2 + 12.66 y^2 + 15.82 x^4 - 94.91 x^2 y^2 + 15.82 y^4 \end{aligned} \right\} \dots\dots\dots(8.5)$$

From eqs (8.3) and (8.5), the 1st approximate solution (3.19) is obtained ;

$$\left. \begin{aligned} \sigma_x &= -37.96 xy + \nu_1 (-25.21 xy + 63.27 x^3 y - 63.27 xy^3), \\ \sigma_y &= -37.96 xy + \nu_1 (-25.31 xy - 63.27 x^3 y + 63.27 xy^3), \\ \tau_{xy} &= -22.53 + 18.98 x^2 + 18.98 y^2 + \nu_1 (-4.22 + 12.66 x^2 + 12.66 y^2 + 15.82 x^4 - 94.91 x^2 y^2 + 15.82 y^4), \end{aligned} \right\} \dots\dots\dots(8.6)$$

where ν_1 is obtained from (3.20) ;

$$\nu_1 = - \frac{3.16 - 3.25 \nu}{26.46 - 20.97 \nu} \dots\dots\dots(8.7)$$

so that

$$\left. \begin{aligned} \nu_1 &= -0.099 \quad (\nu=0.1), \\ \nu_1 &= -0.067 \quad (\nu=0.3). \end{aligned} \right\} \dots\dots\dots(8.9)$$

For $\nu=0.1$, the 1st approximate solution yield that

$$\left. \begin{aligned} \sigma_x &= -35.43 xy - 6.33 x^3 y + 6.33 xy^3, \\ \sigma_y &= -35.43 xy + 6.33 x^3 y - 6.33 xy^3, \\ \tau_{xy} &= -22.11 + 17.71 x^2 + 17.71 y^2 - 1.58 x^4 + 9.49 x^2 y^2 - 1.58 y^4 \end{aligned} \right\} \dots\dots\dots(8.10)$$

See fig. 6.

Fig. 6

It is known from this example that : the stresses near the center are not so different from the fundamental solution, but near the corner, the more the stresses of self-equilibrium are taken, the more they change. See STRESS ANALYSIS OF +TYPE BEAM-COLUMN CONNECTIONS (III).

§ 9 Conclusion

In this method, we divide the solution into a particular solution which satisfies the external forces and serial general solutions which satisfy the non-existence of the external forces; namely a particular solution of an inhomogeneous equation and serial solutions of its homogeneous equation in Galerkin method.

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十型梁・柱接合部の応力解析 (II) (梗概)

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§4 軸 荷 重

外力が図-3のように作用するとき、解析関数を式(4.1)のように取る。 $m=1$ に対して、応力は式(4.2)になり、二つの未知定数が、外力のみによって満足されるので、基本解または0次近似解となる。前節と同様に、自己釣合の応力系を低次から順次選んで高次の近似解を得る。正解はそれらを無限に選ぶことによって得られる。また、これらの解は式(4.5)からわかるように、直接近似解を得ることができる。すなわち、 $m=n$ に対して、未知定数は $n+2$ 個ある。外力を満足するように二つの未知定数は n 個の未知定数によって表わされる。このとき、この歪ポテンシャル・エネルギーは $2n$ 個の未知定数によって表わされ、これらの未知定数は、エネルギーの最小原理により決定される：式(4.20)。

接合部が正方形で、外力が等しければ、式(4.15)を使って得られる。

§5 y 軸対称荷重§6 x 軸対称荷重

同様にして、基本解に順次自己釣合の応力系に係数をかけ合わせたものを加え、エネルギーの最小原理にしたがって、それらの係数を決定することによって得られる。また、直接法によっても得られる。

§7 変 位

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(昭和44年7月15日日本橋受理・討論期限昭和45年6月末日)

標田方程式に対して、 x 方向の変位には式(7.1)で、また、 y 方向の変位 v では式(7.2)で示される。ここでは、各型で、 n 次近似解を得るために、基本式に m を代入した応力に対応する変位を式(7.3)から式(7.10)で与えた。これらの変位式で、原点が不動点になる。

§8 例 題

紙面の都合で、逆対称荷重についてのみ扱い、光弾性の実験による値と比較した。問題は正方形接合部で、各辺で同じ大きさの外力を与えたものとした：式(8.1)と式(8.2)。基本解は(8.3)で表わされる。式(8.4)は各辺で直応力は3次関数のみで、かつ、外力を満足している解になる。式(8.4)から、式(8.3)を引くと、式(8.5)の最も低次の自己釣合の応力系が得られる。これに係数 ν_1 をかけて、基本解に加えたものが1次近似解になり、係数 ν_1 は歪ポテンシャル・エネルギーを最小になるように決定される、すなわち、式(8.7)。図6参照。

この例題でわかるように、中心部の応力は基本解と比較して大差ないが、隅の応力は、自己釣合の応力系を多く取るほど変化してゆく。

§9 ま と め

この方法では、解を外力を満足する一つの特殊解と、外力不存在条件(力の自己釣合)を満足する級数の型の一般解とに分けた。すなわち、これはガラーキン法における、非同次方程式の一つの特殊解とその同次式の級数の型の一般解との和による解と類似のものである。

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**VIBRATION AND STATIC LOADING TEST OF
HACHINOHE TECHNICAL COLLEGE**

NO. 2—Static Loading Tests—

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by Dr. HIROYUKI AOYAMA, Assoc. of Prof. of Tokyo Univ.,
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**STRESS ANALYSIS OF + TYPE BEAM — COLUMN
CONNECTION (II)**

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by Dr. GENGU MATSUI, Prof. of Waseda Univ. and TOSHIO
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**INELASTIC BEHAVIORS OF STRUCTURAL STEEL SECTIONS
UNDER ALTERNATIVE LOADING**

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(1) Method of Analysis and Examples

by Dr. SADAYOSHI IGRASHI, Prof. Osaka Univ., CHIAKI
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A NOTE ON ESTIMATION OF PROBABLE ACCELERATIONS IN BEDROCK

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by Dr. HIROSHI TAGIME. Prof. of Nihon Univ. Member of A.I.J.

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DAYLIGHTING CALCULATION BY LUMINOUS FLUX TRANSFER METHOD

(Interreflection Method)

by KOZO ISHINO, Assistant of Nagoya Univ. and MASAYUKI
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Many studies of interreflection based upon the mathematical analysis and experimental data have been made in the idealized rooms. Luminous Analogue Computer, which was originated with O'Brien in the Department of Engineering, University of California or digital computer may be employed for the solution of simultaneous equations which describe the interreflection systems. These computers are applied for the prediction of interior illumination or daylighting calculation and the analysis of heat transfer problem. As already published by O'Brien, the percent error of the analogue solution relative to the digital solution results from inaccurate settings of the resistances and errors in the potentiometer or galvanometer readings, but its difference is very few. The predetermination of daylighting design is based upon the many independent element or variables. (ex. room geometry; room properties and characteristics which involve reflectance; location and size of lighting window, limitation of depth, effect of inter-reflection; construction and material of window; block planning etc.)

In this paper, daylighting design problem of an idealized room with a unilateral lighting window is solved by computer, in regard to the fundamental analytical data such as luminous emittance ratio, direct and total illumination. Moreover, using the point-by-point method, direct and total illumination distribution of the floor are calculated and compared.