INELASTIC BEHAVIOURS OF STRUCTURAL STEEL SECTIONS UNDER ALTERNATIVE LOADINGS

(2) FINAL STATE OF RESISTING MOMENT AND EXPERIMENTAL STUDY

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1. INTRODUCTION.

In the previous paper (Ref. 1), simple method of analysis was proposed in order to obtain the moment-curvature relationship of the cross sections of steel structural members subjected to constant axial load and alternately repeated bending, and using this method, moment-curvature curves for rectangular and H-sections of steel members under constant curvature amplitude were obtained assuming bi-linear stress-strain relation with a strain-hardening factor μ (Fig. 1). As a result of this analysis, it was pointed out that resisting moment of the cross section under such conditions increases gradually with the increase of the number of repetition of cycles, until it converges to a certain final value, and also average strain over the cross section increases gradually with the increase of the number of repetition of cycles, and especially in case that strain-hardening factor $\mu \neq 0$, it also converges to a certain constant value.

Fig.-1

In this paper, it is described about these final states of resisting moment and average strain of the cross section and also discussed about the experimental results on the rectangular steel section subjected to constant axial compression and alternative repeated bending under constant curvature amplitude.

2. FINAL STATE OF RESISTING MOMENT AND AVERAGE STRAIN.

Resisting moment of the cross section subjected to constant axial load and alternately repeated bending under constant curvature amplitude increases gradually with the increase of the number of repetition of cycles, until it converges to a certain final state (Ref. 1). In Figs. 2 and 3, absolute values of maximum resisting moment of the cross section are plotted for each cycle under various conditions.

These figures show the increasing process of maximum absolute value of resisting moment for rectangular and H-sections (H-100×100×6×8) under constant axial compression ratios $n=P/P_y=0.3, 0.6$, constant curvature amplitudes $\phi_{\max}=|\phi/\phi_y|_{\max}=5.0, 10.0$ and strain-hardening factors $\mu=0, 0.01, 0.02$. In the figures, dashed lines parallel to the abscissa represent the resisting moment in case that $n=0$ with same μ and $\phi_{\max}$. While Figs. 4 and 5 show the process of accumulation of average strain over the cross sections under the same conditions as Figs. 2 and 3, respectively. Analytical results for these inelastic cyclic behaviours were described in detail in the previous paper. As is seen from Figs. 2 and 3, resisting moments of the cross sections increases gradually with the increase of the number of cycles, until they converge to certain steady states, and especially in case that $\mu \neq 0$, these final values of resisting moments are seemed to coincide with that for $n=0$ (dashed lines in the Figs. 2 and 3) regardless of axial compression ratio, while in case that $\mu=0$ (elasto-plastic stress-strain relation), these

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converged values of resisting moments are dependent on axial compression ratio. This fact means that with the increase of the number of cycles, hysteresis loop of moment-curvature curve of the cross section under such conditions approaches gradually to its final steady stable loop.

These phenomena can be easily understood by considering stress and strain distributions over the cross section at the reversal point of the moment-curvature curves for each cycle. In Figs. 6 and 7, moment-curvature curves of the rectangular section in case that $\mu \neq 0$ and $\mu = 0$ are respectively shown

Fig.-6

Fig.-7

Rectangular-Section $n=0.6, \mu=0.01, \varphi_{max}=6.0$						
cycle	1	2	3	4	5	6
ϵ_1 / ϵ_s	2.60	-14.84	-7.24	-21.35	-10.05	-20.28
ϵ_5 / ϵ_s	-3.41	-8.62	-13.24	-17.34	-21.07	-24.24
ϵ_3 / ϵ_s	-9.41	-2.61	-10.24	-11.34	-27.09	-10.23
σ_1 / σ_s	1.02	-1.14	0.92	-1.22	0.84	-1.29
σ_5 / σ_s	-1.02	-1.07	-1.12	-1.18	-1.20	-1.23
σ_3 / σ_s	-1.08	0.96	-1.18	0.87	-1.06	0.81
m_{max}	1.0526	-1.1472	1.2146	-1.2688	1.3118	-1.3507
cycle	7	8	9	10	20	30
ϵ_1 / ϵ_s	-21.35	-35.99	-26.29	-40.43	-52.22	-55.94
ϵ_5 / ϵ_s	-27.35	-23.99	-32.52	-34.42	-46.22	-49.94
ϵ_3 / ϵ_s	-33.34	-23.99	-38.76	-28.42	-40.21	-43.94
σ_1 / σ_s	0.77	-1.34	0.73	-1.39	-1.51	-1.54
σ_5 / σ_s	-1.26	-1.28	-1.31	-1.33	-1.45	-1.49
σ_3 / σ_s	-1.32	0.75	-1.37	0.71	0.59	0.55
m_{max}	1.3730	-1.4058	1.4236	-1.4423	-1.5093	-1.5248

Fig.-8

under $n=0.6$ and $\varphi_{max}=5.0$, as examples, and under same conditions, stress and strain distribution over the cross section at the reversal points of these cyclic moment-curvature curves are shown in Figs. 8 and 9, respectively. As is seen from these figures, stress and strain distributions in case that $\mu \neq 0$ are quite different from those for that $\mu=0$ and the process to the final state are as follows;

2.1 IN CASE THAT $\mu \neq 0$:

2.1.1 $n=P/P_y \neq 0$: As is seen from Fig. 8, with the increase of the number of repetition of cycles, average strain over the cross section converges gradually to a certain constant value and stress distribution becomes symmetric gradually about the central axis of the cross section and also the intersection of plastic and elastic range in compression side (point d in Figs. 10.a and 11.a) approaches to the central axis of the cross section. This inclination can be also observed for other conditions of

Rectangular-Section $n=0.6, \mu=0, \varphi_{\max}=6.0$						
cycle	1	2	3	4	5	6
ϵ_1/ϵ_y	2.40	-15.88	-10.00	-22.21	-22.35	-40.57
ϵ_2/ϵ_y	-3.63	-8.83	-14.03	-22.20	-22.30	-34.86
ϵ_3/ϵ_y	-2.66	-3.78	-22.05	-16.19	-24.41	-20.55
σ_1/σ_y	1.0	-1.0	1.0	-1.0	1.0	-1.0
σ_2/σ_y	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
σ_3/σ_y	-1.0	1.0	-1.0	1.0	-1.0	1.0
$M_{\max}$	0.9397	-0.9626	0.9630	-0.9623	0.9623	-0.9623
cycle	7	8	9	10	20	30
ϵ_1/ϵ_y	-34.71	-32.92	-47.08	-52.27	-127.02	-188.77
ϵ_2/ϵ_y	-40.73	-44.91	-33.09	-39.26	-121.01	-182.78
ϵ_3/ϵ_y	-44.76	-46.90	-32.11	-33.23	-115.00	-176.75
σ_1/σ_y	1.0	-1.0	1.0	-1.0	1.0	-1.0
σ_2/σ_y	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
σ_3/σ_y	-1.0	1.0	-1.0	1.0	-1.0	1.0
$M_{\max}$	0.9623	-0.9623	0.9623	-0.9623	-0.9623	-0.9623

Fig.-9

Fig.-10.a

Fig.-11.a

Fig.-10.b

Fig.-11.b

the axial compression ratio, strain-hardening factor and curvature amplitude. Therefore, final state of resisting moment is seemed to occur when average strain over the cross section converges to a certain constant value and the intersection of the plastic and elastic range in compression side coincides with the central axis of the cross section. These final states of strain and stress distribution for rectangular and H-sections are shown in Figs. 10.a and 11.a. For these distributions, consider the equilibrium equations to obtain the final value of the resisting moment;

Symbols used are shown in the figure and are arranged alphabetically in APPENDIX II.

(A) RECTANGULAR SECTION (Fig. 10.a) :

Average strain over the cross section and curvature amplitude are given by

$$\epsilon_0 = \frac{1}{2}(\epsilon_1 + \epsilon_2) \quad \dots\dots\dots (1)$$

$$\varphi_{\max} = |\phi/\phi_y|_{\max} = (\epsilon_1 - \epsilon_2)/2\epsilon_y \quad \dots\dots\dots (2)$$

in which $\phi_y = 2\epsilon_y/D$ and ϵ_y is the strain at yielding point. From the stress-strain relation,

$$\left. \begin{aligned} \epsilon_3/\epsilon_y &= \epsilon_0/\epsilon_y + 1, \\ \sigma_1/\sigma_y &= \mu(\epsilon_1/\epsilon_y - 1) + 1, \quad \sigma_3/\sigma_y = \mu(\epsilon_0/\epsilon_y) + 1, \\ \sigma_0/\sigma_y &= \mu(\epsilon_0/\epsilon_y + 1) - 1, \quad \sigma_2/\sigma_y = \mu(\epsilon_2/\epsilon_y + 1) - 1, \end{aligned} \right\} \quad \dots\dots\dots (3)$$

β and α in fig. 10.a are given by

$$\beta = (\epsilon_3 - \epsilon_0) / (\epsilon_1 - \epsilon_2) = 1/2 \varphi_{\max} \quad \dots\dots\dots (4)$$

$$\alpha = \beta \frac{\sigma_3}{\sigma_3 - \sigma_0} = \frac{\mu(\epsilon_1/\epsilon_y - \varphi_{\max}) + 1}{2 \varphi_{\max}(2 - \mu)} \quad \dots\dots\dots (5)$$

Constant axial compression ratio $n = P/P_y$ is given by

$$n = \left\{ \left(\frac{1}{2} - \beta \right) \frac{1}{2} (\sigma_1 + \sigma_3) + \frac{\alpha}{2} \sigma_3 + \frac{1}{2} (\beta - \alpha) \sigma_0 + \frac{1}{4} (\sigma_0 + \sigma_2) \right\} / \sigma_y \quad \dots\dots\dots (6)$$

substituting Eqs. (1)~(5) to Eq. (6),

$$n = \mu \epsilon_0 / \epsilon_y - (1 - \mu) / 2 \varphi_{\max} \quad \dots\dots\dots (7)$$

Rearranging Eq. (7) and representing by ϵ_0 ,

$$\epsilon_0 / \epsilon_y = n / \mu + (1 - \mu) / 2 \mu \varphi_{\max} \quad \dots\dots\dots (8)$$

While resisting moment of the cross section is given by

$$\begin{aligned} \frac{1}{6} m = \frac{1}{6} \frac{M}{M_y} = & \left[\left(\frac{1}{2} - \beta \right) \left\{ \frac{1}{2} - \frac{1}{2} \left(\frac{1}{2} - \beta \right) \right\} \sigma_3 + \left(\frac{1}{2} - \beta \right) \left\{ \frac{1}{2} - \frac{1}{3} \left(\frac{1}{2} - \beta \right) \right\} \frac{1}{2} (\sigma_1 - \sigma_3) \right. \\ & \left. + \alpha \left(\beta - \frac{\alpha}{3} \right) \frac{1}{2} \sigma_3 + (\beta - \alpha) \frac{1}{3} (\beta - \alpha) \frac{1}{2} \sigma_0 - \frac{1}{2} \cdot \frac{1}{4} \sigma_0 - \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} (\sigma_2 - \sigma_0) \right] / \sigma_y \quad \dots\dots\dots (9) \end{aligned}$$

and also substituting Eqs. (1)~(5) to Eq. (9)

$$m = \mu \varphi_{\max} + \frac{3}{2} (1 - \mu) - \frac{1 - \mu}{2 \varphi_{\max}^2} \quad \dots\dots\dots (10)$$

Eqs. (8) and (10) show the final values of average strain and resisting moment of the rectangular section which is subjected to a constant axial compression and alternately repeated bending under constant curvature amplitude $\varphi_{\max}$, with strain-hardening factor μ .

(B) H-SECTION (Fig. 11.a) :

For H-section, average strain and resisting moment of the cross section can be obtained by the same procedure as the rectangular section. Average strain over the cross section and curvature amplitude are given by the same equations as Eqs. (1) and (2), respectively.

From the hysteretic relation of stress and strain at the final state, (Fig. 11.a, $\beta D < D/2 - T_f$ i.e., $\varphi_{\max} > 1/(1 - 2 T_f/D)$).

$$\left. \begin{aligned} \epsilon_3 / \epsilon_y &= \epsilon_0 / \epsilon_y + 1, & \epsilon_4 / \epsilon_y &= \epsilon_1 / \epsilon_y - 2 \varphi_{\max} T_f / D, \\ \epsilon_5 / \epsilon_y &= \epsilon_1 / \epsilon_y - 2 \varphi_{\max} + 2 \varphi_{\max} \cdot T_f / D, & & \\ \sigma_1 / \sigma_y &= \mu (\epsilon_1 / \epsilon_y - 1) + 1, & \sigma_4 / \sigma_y &= \mu (\epsilon_1 / \epsilon_y - 2 \varphi_{\max} T_f / D - 1) + 1, \\ \sigma_3 / \sigma_y &= \mu (\epsilon_1 / \epsilon_y - \varphi_{\max}) + 1, & \sigma_0 / \sigma_y &= \mu (\epsilon_1 / \epsilon_y - \varphi_{\max} + 1) - 1, \\ \sigma_5 / \sigma_y &= \mu (\epsilon_1 / \epsilon_y - 2 \varphi_{\max} + 2 \varphi_{\max} \cdot T_f / D + 1) - 1, & \sigma_2 / \sigma_y &= \mu (\epsilon_1 / \epsilon_y - 2 \varphi_{\max} + 1) - 1 \end{aligned} \right\} \quad \dots\dots\dots (11)$$

α and β in fig. 11. a are respectively given by the same equations as Eqs. (4) and (5), and the constant axial compression ratio is given by

$$\begin{aligned} nS = & \left\{ T_f \frac{1}{2} (\sigma_1 + \sigma_4) B + \left(\frac{D}{2} - \beta D - T_f \right) \frac{1}{2} (\sigma_3 + \sigma_4) T_W + \frac{\alpha D}{2} \sigma_3 T_W \right. \\ & \left. + \frac{1}{2} (\beta - \alpha) D \sigma_0 T_W + \left(\frac{D}{2} - T_f \right) \frac{1}{2} (\sigma_5 + \sigma_0) T_W + T_f \frac{1}{2} (\sigma_5 + \sigma_2) B \right\} / \sigma_y \quad \dots\dots\dots (12) \end{aligned}$$

in which S is the area of H-section ($S = 2 T_f B + D T_W - 2 T_f T_W$).

Substituting Eqs. (1), (2), (4), (5) and (11) to Eq. (12), axial compression ratio is

$$n = \frac{\mu \epsilon_0}{\epsilon_y} + \frac{(\mu - 1) T_W D}{2 \varphi_{\max} \cdot S} \quad \left(\text{with } \varphi_{\max} > \frac{1}{1 - 2 \cdot T_f / D} \right) \quad \dots\dots\dots (13)$$

Therefore, average strain ϵ_0 over the cross section can be obtained from Eq. (13),

$$\epsilon_0 / \epsilon_y = \frac{n}{\mu} + \frac{1 - \mu}{2 \mu \varphi_{\max}} \cdot \frac{T_W D}{S} \quad \dots\dots\dots (14)$$

Resisting moment of the cross section is given by

$$M = T_f \left(\frac{D}{2} - \frac{T_f}{2} \right) B \sigma_4 + T_f \left(\frac{D}{2} - \frac{T_f}{3} \right) \frac{1}{2} (\sigma_1 - \sigma_4) B + \left(\frac{D}{2} - T_f - \beta D \right)$$

$$\begin{aligned}
& \times \left(\frac{D}{2} - T_f - \frac{D}{4} + \frac{T_f}{2} + \frac{\beta D}{2} \right) T_W \sigma_3 + \left(\frac{D}{2} - T_f - \beta D \right) \left(\frac{D}{2} - T_f - \frac{D}{6} + \frac{T_f}{3} + \frac{\beta D}{3} \right) T_W \\
& \times \frac{1}{2} (\sigma_4 - \sigma_3) + \alpha \left(\beta - \frac{\alpha}{3} \right) D^2 T_W \frac{1}{2} \sigma_3 + \frac{1}{3} (\beta - \alpha)^2 D^2 T_W \frac{1}{2} \sigma_0 - \frac{1}{2} \left(\frac{D}{2} - T_f \right)^2 T_W \sigma_0 \\
& - \frac{2}{3} \left(\frac{D}{2} - T_f \right)^2 T_W \frac{1}{2} (\sigma_5 - \sigma_0) - T_f \left(\frac{D}{2} - \frac{T_f}{2} \right) B \sigma_5 - T_f \left(\frac{D}{2} - \frac{T_f}{3} \right) \frac{1}{2} (\sigma_2 - \sigma_5) B \dots \dots \dots (15)
\end{aligned}$$

and also substituting Eqs. (1), (2), (4), (5) and (11) to Eq. (15),

$$\begin{aligned}
m = \frac{M}{M_y} = \mu \cdot \varphi_{\max} + \frac{3(1-\mu)D}{2(D-2T_f)} - \frac{(1-\mu)D^3}{2\{BD^3 - (B-T_W)(D-2T_f)^3\}} \left(\frac{T_W}{\varphi_{\max}^2} + \frac{6BT_f}{D-2T_f} \right) \\
\text{(with } \varphi_{\max} > 1/(1-2T_f/D)) \dots \dots \dots (16)
\end{aligned}$$

2.1.2 $n=P/P_y=0$: On the other hand, when there is no axial compression load over the cross section, neutral axis always coincides with the central axis of the cross section and resisting moment of the cross section can be easily obtained as follows;

(A) RECTANGULAR SECTION (Fig. 10.b):

Curvature amplitude is defined by

$$\varphi_{\max} = \epsilon_1/\epsilon_y \dots \dots \dots (17)$$

From the stress and strain relation,

$$\sigma_1/\sigma_y = \mu(\epsilon_1/\epsilon_y - 1) + 1 = \mu(\varphi_{\max} - 1) + 1 \dots \dots \dots (18)$$

Since strains vary linearly from the neutral axis (assumption 1 in Ref. 1),

$$r = \frac{1}{2} \left(1 - \frac{1}{\epsilon_1/\epsilon_y} \right) = \frac{1}{2} \left(1 - \frac{1}{\varphi_{\max}} \right) \dots \dots \dots (19)$$

Resisting moment of the cross section is given by

$$\frac{1}{6} \cdot \frac{1}{2} \cdot m = r \left(\frac{1}{2} - r \right) + \frac{1}{2} \left(\frac{\sigma_1}{\sigma_y} - 1 \right) r \left(\frac{1}{2} - r \right) + \left(\frac{1}{2} - r \right) \frac{2}{3} \left(\frac{1}{2} - r \right) \frac{1}{2} \dots \dots \dots (20)$$

Substituting Eqs. (17)~(19) to Eq. (20),

$$m = \mu \cdot \varphi_{\max} + \frac{3}{2} (1-\mu) - \frac{1-\mu}{2 \varphi_{\max}^2} \dots \dots \dots (21)$$

(B) H-SECTION (Fig. 11.b):

Stress and strain relation over the cross section are given by

$$\left. \begin{aligned}
\epsilon_1/\epsilon_y &= \varphi_{\max}, & \epsilon_4/\epsilon_y &= \epsilon_1/\epsilon_y - 2\varphi_{\max} T_f/D, \\
\sigma_1/\sigma_y &= \mu(\epsilon_1/\epsilon_y - 1) + 1, & \sigma_4/\sigma_y &= \mu(\epsilon_4/\epsilon_y - 1) + 1,
\end{aligned} \right\} \dots \dots \dots (22)$$

Resisting moment is then

$$\begin{aligned}
\frac{1}{2} M &= T_f \left(\frac{D}{2} - \frac{T_f}{2} \right) B \sigma_4 + T_f \left(\frac{D}{2} - \frac{T_f}{3} \right) \frac{1}{2} (\sigma_1 - \sigma_4) B \\
&+ (rD - T_f) \left\{ \frac{D}{2} - T_f - \frac{1}{2} (rD - T_f) \right\} T_W \sigma_y + (rD - T_f) \left\{ \frac{D}{2} - T_f - \frac{1}{3} (rD - T_f) \right\} T_W \\
&\times \frac{1}{2} (\sigma_4 - \sigma_y) + \left(\frac{1}{2} - r \right) D \frac{2}{3} \left(\frac{1}{2} - r \right) D \frac{1}{2} T_W \sigma_y \dots \dots \dots (23)
\end{aligned}$$

Substituting Eqs. (19) and (22) to Eq. (23),

$$m = \mu \cdot \varphi_{\max} + \frac{3(1-\mu)D}{2(D-2T_f)} - \frac{(1-\mu)D^3}{2\{BD^3 - (B-T_W)(D-2T_f)^3\}} \left(\frac{T_W}{\varphi_{\max}^2} + \frac{6BT_f}{D-2T_f} \right) \dots \dots \dots (24)$$

As is seen from Eqs. (10), (21) for rectangular section and Eqs. (16), (24) for H-section, final value of resisting moment of the cross section subjected to cyclic bending and constant axial compression load under constant curvature amplitude is independent on a axial compression ratio n except for strain-hardening factor μ and curvature amplitude $\varphi_{\max}$, and this value coincides with those for $n=0$.

2.2 IN CASE THAT $\mu=0$:

In this case (Figs. 7,9), distribution of the stress and strain is different from the case that $\mu \neq 0$, that is, at the final state, intersection of the plastic and elastic range (Fig. 10.a and 11.a, point d) does not converge to the central axis of the cross section and also average strain over the cross section increases infinitely being proportional to the number of repetition of cycles (Figs. 4,5), and never

converges to a certain limited value. As is seen from Figs. 7 and 9, increase of resisting moment due to the repetition of cycles is quite small in comparison with the case that $\mu \neq 0$. In this case, resisting moment for final state is easily obtained from the stress and strain distribution at the final state, i.e., for rectangular section,

$$m = \frac{3}{2}(1-n^2) - \frac{1}{8\varphi_{\max}^2} \dots\dots\dots(25)$$

and for the H-section, various cases are considered, for instance, in case that

$$-\frac{T_W \cdot D}{S} \frac{3}{2\varphi_{\max}} < n < -\frac{T_W \cdot D}{S} \left(1 - \frac{2T_f}{D} - \frac{1}{2\varphi_{\max}}\right)$$

in which

$$S = 2T_f \cdot B + D \cdot T_W - 2T_f \cdot T_W,$$

resisting moment for final state is

$$m = \frac{3}{2} \frac{D}{D-2T_f} - \frac{D}{\{BD^3 - (B-T_W)(D-2T_f)^3\}} \left\{ \frac{3BT_f D^2}{D-2T_f} + \frac{3}{2} \frac{S^2}{T_W} n^2 + \frac{T_W \cdot D^2}{8\varphi_{\max}^2} \right\} \dots\dots(26)$$

As is seen from Eqs. (25) and (26), final resisting moment of the cross section in case that $\mu=0$ is dependent on $\varphi_{\max}$ and $n=P/P_y$, and this fact is different from the case that $\mu \neq 0$.

3. EXPERIMENTAL RESEARCH.

Numerous experiments have been performed on the steel structures under alternately repeated loadings. Experimental studies on the steel members with rectangular section subjected to alternative bending under constant axial tension were carried out by H. Hagura (Ref. 2). According to his studies, it is pointed out that the assumption used for the analysis, (that is, the stress-strain relation is elasto-plastic one), is adequate until strain is less than strain hardening region, however, after strain hardening, discrepancy between his theory and experimental results becomes considerably pronounce.

Herein, in order to make clear the validity of the method proposed by us (Ref. 1), experimental study on moment-curvature relation of rectangular section subjected to constant axial compression and alternately repeated bending under constant curvature amplitude is carried out for three cases of axial compression ratio.

4. DESCRIPTION OF TEST.

Cross sections of the specimens are rectangular and its mechanical properties are shown in Table. 1 and pertinent dimensions of the specimens are as follows, that is, cross section is 40 mm $\times$ 20 mm, and 260 mm length which is cut out from 20 mm thickness mild steel plate. Vertical compression load was applied with the oil pressure testing machine with 50 ton capacity and alternative bending was applied with two bolts which were attached through the pin support in perpendicular direction to the arm plates attached rigidly to the upper and lower end of the specimen. Bending was applied to the specimen about the weak axis. Load applied to the bolt was measured by wire strain gauges pasted to the bolt. Vertical load applied to the specimen was always kept constant throughout experiment. Bending moment M at the mid span of the member was calculated by the following equation; $M = P \cdot \Delta + A(l + \Delta)$, in which P =vertical axial compression load, Δ =horizontal deflection at the mid span of the member, A =axial force of the bolt and l =arm length of the arm plate, while curvature of the cross section was measured by plastic wire strain gauges located at the mid span of the member. Throughout this experiment, member was scarcely subjected to bending about the strong axis.

5. EXPERIMENTAL RESULTS AND CONSIDERATION.

Figs. 12, 13 and 14 show the moment-curvature curves obtained from the experiment. Axial compression ratio are $n=0, 0.3$, and 0.6 , and curvature amplitude are $\varphi_{\max}=3.0, 5.0$ and 5.0 for each

Table-1

Material	σ_y kg/cm ²	σ_u kg/cm ²	ϵ_y	E kg/cm ²	E_{st} kg/cm ²
SS 41	2,310	4,503	1127×10^{-6}	2.05×10^6	4.8×10^4

Fig.-12

Fig.-13

axial load ratio, respectively. In the figures, solid lines show moment-curvature curves obtained from the method in Ref. 1 in which stress-strain relation is of bi-linear with a strain-hardening factor $\mu = 0.01$. While geometrical configurations of virgin curve and hysteresis curves are shown in Figs. 15~17. In the figures, each of hysteresis curves is halved and their ordinates of reversal points are replaced to origin at (0, 0). Maximum strains in the cross section which occurred throughout the experiment are about 5000×10^{-6} , 15000×10^{-6} , and 20000×10^{-6} for $n=0$, 0.3 and 0.6, respectively.

Summarizing the experimental results; (1) For the virgin curve, the larger is the axial compression load, the smaller is the resisting moment of the section, and their geometrical configurations are almost elasto-plastic types. (For every case, strains in the cross sections were less than strain hardening region). (2) Geometrical configuration of the virgin curve is considerably different from the hysteresis curves, while each of hysteresis curves has similar configurations. (3) With the increase of the number of repetition of cycles, resisting moment of the cross section becomes gradually larger and its tendency becomes remarkable as axial compression ratio becomes larger, however, in case that $n=0$, increase of the resisting moment is scarcely observed. (4) Elastic modulus of the moment-curvature curve does not vary. (5) With the increase of the number of repetition of cycles, hysteresis curves are inclined to converge to a steady loop.

On the other hand, comparing the experimental results with theoretical ones, (1) Elastic modulus of the experimental moment-curvature curves coincides with that of theoretical ones. (2) For the virgin curve, slope of plastic range of the experimental curves is less than that of the theoretical one. As the reason of this, since strain over the cross section is less than strain hardening region, it is considered that the elasto-plastic stress-strain relation is more reasonable rather than bi-linear one assumed for the analysis, however, even moment-curvature curve obtained from elasto-plastic stress-strain relation (dashed and dotted lines in Figs. 15~17) cannot explain this experimental result completely. (3) At the transient portion from elastic region to the plastic one of moment-curvature curve, theoretical resisting moment is greater than experimental one. As the reason of this, creep of the material and the fact that the actual stress-strain relation of the material is more gentle than that used for analysis

Fig.-14

Fig.-15

Fig.-16

Fig.-17

are considered. (4) Maximum resisting moment obtained from the experiment is approximately same as the theoretical one.

While, Fig. 18 shows a process of accumulation of average strain over the cross section for the increase of the number of repetition of cycles. In the figure, solid line represents the experimental

result and dashed line is the theoretical one. From this figure and the theoretical result shown in Fig. 13 of Ref. 1, it can be understood that for initial few cycles, strain-hardening factor μ used for analysis may be greater, while after few cycles, may be smaller than actual value. Consequently, strain-hardening factor μ used for analysis is considered to vary with the number of cycles. Strains over the cross section varied approximately linearly throughout this experiment.

6. CONCLUSION.

As is pointed out in the previous paper (Ref. 1), resisiting moment of the cross section subjected to constant axial compression load and alternately repeated bending under constant curvature amplitude increases gradually with the increase of the number of repetition of cycles, until it converges to a certain limited value, and also average strain over the cross section increases gradually, and especially in case that $\mu \neq 0$, it converges to a final value. In this paper, these final values of resisting moment and average strain over the cross section were obtained from the stress and strain distribution for these final states and also experiments on the rectangular steel section were performed to make clear the validity of this method. According to the experimental results, cyclic behaviour of the rectangular cross section subjected to constant axial compression can be approximately explained by the method proposed by writers.

Calculations in this analysis were done on the digital computer FACOM 230-60, at Computer Center, Kyushu University.

APPENDIX I : REFERENCE;

- (1) S. Igarashi, C. Matsui, K. Yoshimura and K. Matsumura; "Inelastic Behaviours of Structural Steel Sections under Alternative Loadings", (Part 1) Trans. A.I.J., No. 169, Mar., 1970.
- (2) H. Hagura; "Research on the Elasto-Plastic Analysis of Steel Section subjected to Alternative Load", (Part 4), Trans. A.I.J., No. 125, Jul., 1966. in Japanese.

APPENDIX II : NOTATION;

- A =axial force of the bolt.
 B, D =width and depth of the cross section.
 l =arm length of the plate.
 m =dimensionless bending moment= M/M_y .
 M, M_y =bending moment and yield moment= $\sigma_y \cdot Z$, respectively.
 n =axial load ratio= P/P_y
 P, P_y =total and yielding axial load over the cross section.
 S =area of H-section= $2T_f B + DT_w - 2T_f T_w$.
 T_f, T_w =thickness of flange and web of the H-section.
 Z =elastic section modulus.
 μ =strain-hardening factor.
 σ_y, ϵ_y =stress and strain at initial yielding point.
 σ_0, ϵ_0 =average stress and strain over the cross section.
 $\sigma_1, \epsilon_1, \sigma_2, \epsilon_2$ =stress and strain at the extreme fiber, respectively.
 ϕ_{max} =curvature amplitude ($=|\phi/\phi_y|_{max}$).
 ϕ, ϕ_y =curvature and initial yielding curvature= $2 \cdot \epsilon_y/D$, respectively.

Fig.—18

繰返し組合せ応力を受ける鋼構造部材断面の弾塑性性状(梗概)

(その 2. 抵抗モーメントの終局状態に関する理論的実験的研究)

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1. 序

前報 (Ref. 1) では定軸圧下繰返し曲げを受ける鋼構造部材断面のモーメント (M)—曲率 (ϕ) 関係を求める簡単な解析法を述べ、その方法を用いて矩形および H 型断面についてその応力度 (σ)—歪度 (ϵ) 関係を歪硬化係数 (μ) を有する Bi-linear 型に仮定した時の (図 1) 繰返し M - ϕ 曲線を種々の条件下で求めた。その結果、断面の軸圧比 $n(=P/P_y)=0$ の下で定曲率振幅の繰返し曲げを受ける際の M - ϕ 曲線の処女載荷曲線と履歴分枝曲線の幾何学的形状は前者が後者の $1/2$ のスケールで表わせるが、断面が定軸圧を受ける場合 ($n \neq 0$) には繰返し数の増加とともに抵抗モーメントは徐々に増加し、終局的にはある一定値に収斂する事、同時に断面内の平均歪度も蓄積され、特に歪硬化係数 $\mu \neq 0$ の場合、この蓄積歪度もある値に収斂する事がわかった。この場合、抵抗モーメントは $\mu \neq 0$ ならば軸圧比 n の値に関係なく、歪硬化係数 μ 、定曲率振幅 $\phi_{\max}$ が与えられれば常に一定値に収斂し、その値は同じ μ 、 $\phi_{\max}$ を有する軸圧比 $n=0$ の場合の抵抗モーメントに一致する事、 $\mu=0$ の場合はこの収斂値は軸圧比 n によって異なる事を指摘した。本論では抵抗モーメントおよび蓄積歪度の終局状態について詳しく論じ、あわせて矩形断面について行なった定軸圧定曲率下繰返し曲げ実験を報告する。

2. 抵抗モーメントおよび平均歪度の終局状態

図 2, 3 は矩形および H 型断面 ($H-100 \times 100 \times 6 \times 8$) が定曲率振幅の繰返し曲げを受ける際の、曲げモーメント曲率曲線の各サイクルの除荷点における抵抗モーメントの絶対値を軸圧比 n 、歪硬化係数 μ および定曲率振幅 $\phi_{\max}$ に対して図示したものである。図中の横軸に平行な破線は、同じ μ 、 $\phi_{\max}$ で $n=0$ の場合の抵抗モーメントを示す。一方、図 4, 5 は図 2, 3 と同じ条件下における断面内の平均歪度の蓄積過程を各サイクルに対して示したものである。これらの非弾性繰返し挙動に関する

解析的な考察は前報 (Ref. 1) で述べた。

一般に構造部材断面の有する M - ϕ 関係において、モーメント (M) は応力度 (σ) 分布、曲率 (ϕ) は歪度 (ϵ) 分布にて与えられるが、 σ - ϵ 関係にある履歴法則が与えられると σ -分布は ϵ -分布と一意的であるので抵抗モーメントの増大は平均歪度の増大と密接に関係がある。この現象を理解するには、解析で得られた M - ϕ 曲線の除荷点における断面内の応力度分布、歪度分布を各サイクルに対して考えればよい。図 6, 7 に $n=0.6$ 、 $\phi_{\max}=5.0$ の場合の M - ϕ 曲線を $\mu \neq 0$ と $\mu=0$ の場合についてそれぞれ図示し、これらの M - ϕ 曲線の各除荷点における歪度分布と応力度分布を図 8, 9 に示す。抵抗モーメントが徐々に増大し、終局状態に至る過程は次の通りである。

2.1 $\mu \neq 0$ の場合

$n=P/P_y \neq 0$ の場合は図 8 より、サイクル数の増大につれて断面の平均歪度は徐々にある値に収斂し、また応力度分布は前後のサイクルに対して中心軸について対称分布に近づき、また同時に圧縮側の弾性域と塑性域の交点 (図 10.a, 11.a の d 点) は徐々に中心軸に近づく。この傾向は他の種々の条件下でも $\mu \neq 0$ 、 $n \neq 0$ の場合には必ずみられる。それ故、抵抗モーメントが収斂した終局状態では、平均歪度もある一定値に収斂し、それは圧縮側の応力度分布の弾性域と塑性域の交点が中心軸に一致したときに相当する。この事は応力度と歪度の履歴を考えれば容易に理解される。図 10.a と 11.a にこの終局状態における歪度と応力度の分布を矩形と H 型断面についてそれぞれ示すが、これらの分布に対して応力度歪度関係を考慮して断面の平均歪度および抵抗モーメントを求めると、矩形 H 型断面に対してそれぞれ式 (8), (10) および式 (14), (16) が得られる。一方 $n=0$ の場合の抵抗モーメントを求めると矩形、H 型断面に対してそれぞれ式 (21), (24) になる。式 (10), (21) および式 (16), (24) より、定軸圧定曲率振幅下繰返し曲げを受ける抵抗モーメントの終局値は軸圧比 n に関係なく、 μ 、 $\phi_{\max}$ が同じで $n=0$ の場合の抵抗モーメント

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に一致する事がわかる。

2.2 $\mu=0$ の場合

断面内の応力度および歪度分布は $\mu \neq 0$ の場合とかなり異なる(図9)。即ち、終局状態で断面の平均歪度は一定値に収斂せず、また応力度分布の圧縮側の弾性域と塑性域の交点は断面の中心軸に収斂しない。また平均歪度はサイクル数に比例して無限に上昇する。この場合抵抗モーメントの終局値について式(25)、(26)に矩形およびH型断面の場合を示す。

両式よりわかる様に、 $\mu=0$ の場合の抵抗モーメントの終局値は軸力比 $n(=P/P_y)$ に支配される。

3. 実験

すでに羽倉(Ref. 2)は定軸圧下繰返し曲げを受ける際の $M-\phi$ 関係を実験的に求め、断面の一部が歪硬化域に入ると elasto-plastic 型の $\sigma-\epsilon$ 関係のモデルを基礎にした理論解析では相当の誤差を生じる事、および平面保持の仮定は断面の一部が歪硬化域に入るまでの間はある程度成立する事等を指摘している。本論で述べる実験は前報に述べた理論の妥当性を確かめるためのもので、手始めに矩形断面について定軸圧定曲率振幅下繰返し曲げ実験を行ない、前報理論の導く所と比較検討した。

4. 実験概要

使用鋼材はSS41でその機械的性質を表1に示す。供試体は断面が40mm×20mm長さ260mmの矩形断面で20mm厚の板より削り取ったものである。軸圧比 $n=0, 0.3, 0.6$ の3種につき、曲率振幅を $\phi_{\max}=5.0, 5.0, 3.0$ として実験した。実験は50tonのアムスラー型試験機で定軸圧を加え、繰返し曲げ応力は供試体の上下端にそれぞれ剛に取り付けた板を通してこれにピン接合されたボルトにナットを摩擦のない様にスラストベアリングをはさんで加えた。ボルトに加えた荷重はボルト中央部に貼付けた4枚のW.S.G.の平均歪度より測定した。ボルトに加える荷重の大小により軸圧も変動するが、これは常に試験機の荷重が定荷重に保たれる様にした。曲げは供試体の弱軸方向に加えたが、始めに定軸圧を加える際に強軸方向に曲げが加わらぬ様強軸方向にもピンを挿入している。本実験中、強軸方向に曲げはほとんど加わらなかった。

5. 実験結果および考察

図12, 13, 14に軸圧比 $n=0, 0.3, 0.6$ を受ける場合の $M-\phi$ 曲線を示す。図中、実線は実験曲線、破線は $\mu=0.01$ を有するBi-linear型の $\sigma-\epsilon$ 関係より求めた理論曲線である。また図15, 16, 17には履歴曲線の除荷点を原点にスケールを1/2倍して処女載荷曲線と同じ座標系に図12, 13, 14をそれぞれ書き直したものを示す。

断面内の最大歪度は軸圧比 $n=0, 0.3, 0.6$ に対して約0.5, 1.5, 20(%)である。

本実験の結論とする事は、(1) 処女載荷曲線は軸圧が大きい程抵抗モーメントは小さく、ほぼ完全弾塑性型である。(いずれの場合も断面内の歪度は処女載荷曲線では歪硬化域に入っていない)。(2) 処女載荷曲線と履歴分枝曲線の幾何学的形状は異なる。(3) 軸圧が大きい程繰返しによる抵抗モーメントの上昇割合は大きい。(4) 軸圧が零の場合、繰返しによる抵抗モーメントはほとんど上昇しない。(5) 繰返し回数が増えるるとループは安定し、定常ループに収斂する。(6) 処女載荷曲線の塑性域勾配は理論曲線よりも小さいが、これは $\sigma-\epsilon$ の単調載荷曲線が歪硬化域以前ではほぼ $\mu=0$ の傾向がある事にもよるが、それだけでは充分説明できない。(7) 弾性域から塑性域にはいる部分で理論曲線が実験曲線より外側に張り出しているが、これは材料の $\sigma-\epsilon$ 関係が解析に用いたBi-linear型より滑らかな連続曲線である事や材料のクリープ現象などが原因と思われる。(8) 抵抗モーメントの最大値はそれぞれのサイクルに対して理論値とほぼ一致している。一方、断面の平均歪度(ϵ_0)の蓄積状態を理論値と比較した図18と理論値(Ref. 1の図13)とをあわせて考えると繰返しの初期の数サイクルでは材料の歪硬化係数 μ が解析に用いた $\mu(=0.01)$ より小さく、繰返し数が増大するにつれ、 μ の値が大きくなる傾向を持つものと思われる。なお平面保持の仮定は本実験に関してはほぼ成立していた。

6. 結語

前報(Ref. 1)で指摘したごとく、定軸圧定曲率下繰返し曲げを受ける鋼構造部材断面の抵抗モーメントは繰返し回数の増加に伴い徐々に増加し、終局的にある一定値に収斂する。また断面内の平均歪度も繰返す毎に蓄積される。 $\mu \neq 0$ の場合、平均歪度の蓄積はある値に収斂し、抵抗モーメントの収斂値は軸圧比 n の値に関係なく、与えられた μ および $\phi_{\max}$ に対しては常に一定値を示し、しかもその値は同じ $\mu, \phi_{\max}$ を有する軸圧比 $n=0$ の場合の抵抗モーメントに一致するが、 $\mu=0$ の場合には、この収斂値は軸圧比 n により異なる値を示す。本論ではこの終局状態に関する考察を行ない、あわせて矩形断面について行なった実験について述べた。

即ち、前報に述べた傾向を実証し、定軸圧定曲率下における繰返し曲げを受ける矩形断面の非弾性挙動を説明するのに本論はほぼ有効である事を示した。なお本実験は予備実験であり、今後、更に種々の断面、鋼種について同種の実験を行ない、また測定方法についても検討を加える予定である。