LIMIT ANALYSIS OF BEAM-COLUMN CONNECTIONS (IX)

— Experimental Verification —

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§1 Introduction

Theoretical researches on the yield strength of various types of beam-column connections have been presented in the preceding papers¹⁾, and it has been clarified how the yield strength of connections is affected by the aspect ratio of connections and the ratio of sectional area of members.

In this paper, the authors present an experimental research on the cruci-form connections subjected to anti-symmetrical bending, and verify the coincidence of theoretical results with experimental ones, and also evaluate the applicability of the theory to design practice.

§2 Connections of Wide-flange Beam and Column

2.1 Test specimens

The connections consisting of the same size members are tested and their types and dimensions are shown in Fig. 1, 2 and Table 1. As shown in Fig. 3, these six specimens correspond to Case A and D in the statical analysis and Case (a) and (c) in the kinematical analysis presented in Part I and II¹⁾, and in this figure parameters β and λ are defined as

$$\beta = \frac{A_f}{A_w}, \quad \lambda = \frac{t_p}{t} \quad \dots\dots\dots (2.1)$$

where A_f and A_w denote the sectional area of flange and web of the members, and t and t_p the thickness of member-web and

Fig. 1 Type of Connection

NAME OF SPECIMEN		H-1	H-2	H-3	H-4	H-5	H-6
DEPTH OF MEMBER $2h$ (cm)		19.69	19.70	19.49	19.28	19.60	19.61
WEB THICKNESS OF MEMBER t (cm)		0.60	0.59	0.59	0.60	0.60	0.59
WEB THICKNESS OF CONNECTION t_p (cm)		1.0	1.6	2.2	1.6	1.4	1.9
AREA OF FLANGE A_f (cm ²)		6.00	5.94	5.93	8.67	5.97	5.94
AREA OF WEB A_w (cm ²)		11.81	11.70	11.55	11.55	11.70	11.65
LENGTH OF MOMENT ARM l (cm)		90.3	90.3	90.3	90.45	90.3	90.3
YIELD POINT (t/cm ²)	CONNECTION-WEB $\bar{\sigma}_0$	2.47	2.51	2.47	2.51	2.55	2.46
	FLANGE OF MEMBER σ_0	2.82	2.82	2.82	2.72	2.82	2.82
	WEB OF MEMBER σ_0	2.82	2.82	2.82	2.82	2.82	2.82

Fig. 2 Test Apparatus

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Fig. 3 Classification of Analysis

Fig. 4

connection-web, respectively.

2.2 Test apparatus and procedure

The specimens are set up in a hydraulic universal testing machine as shown in Fig. 2, and alternately loaded through pin-roller bearings. After the repeated and completely reversed loads are applied by two cycles, the specimens are finally loaded until the connection-web deforms plastically and sufficiently. Increments of loading are 0.5 t in elastic range, 0.25 t in the plastic range, and those of unloading 0.5 or 1.0 t.

Deformations of connection-web are measured in the diagonal directions by dial gauges attached as shown in Fig. 4.

2.3 Experimental results

Load-deformation curve are shown in Fig. 5 to Fig. 10 where P is the applied total load shown in Fig. 2 and δ is the diagonal deformation of connection-web shown in Fig. 4.

2.4 Theoretical analysis

Yield moments M of connection are given from the preceding papers¹⁾ by the following equations.

For the Case A and (a)

$$M = \lambda \sigma_0 h^2 t \dots \dots \dots (2.2)$$

For the Case D and (c)

$$M = \left[\lambda - \frac{1}{4} (\lambda - 4\beta)^2 \right] \sigma_0 h^2 t \dots \dots \dots (2.3)$$

Fig. 5

Fig. 6

Fig. 7

Fig. 9

Fig. 8

Fig. 10

Since the test specimen is transversely loaded at the tips of members as shown in Fig. 2 and, on the other hand, the theoretical results are obtained for the case of anti-symmetrical pure bending, it is impossible to compare experimental results directly with theoretical ones. Hence, the moment M is approximately replaced by load P so as to work equivalently in the velocity fields shown in Fig. 11 (a) and (b), as follows :

For the Case (a)

$$\frac{P}{\sqrt{2}} (l-h) \dot{\theta} = M \dot{\theta}$$

or

$$P = \frac{\sqrt{2}}{l-h} M \quad \dots\dots\dots (2.4)$$

For the Case (c)

$$\frac{P}{\sqrt{2}} (l-\xi h) \dot{\theta} = M \dot{\theta}$$

or

$$P = \frac{\sqrt{2}}{l-\xi h} M \quad \dots\dots\dots (2.5)$$

where ξ is defined from eq. (5.8) in Part I¹⁾ as

(a) Case (a) (b) Case (c)

Fig. 11 Kinematical Velocity Fields

Table 3

NAME OF SPECIMEN		□-1	□-2	□-3	□-4	□-5	□-6	□-7	□-8	□-9
COLUMN DEPTH $2d$ (cm)		29.1	28.6	28.1	27.5	29.1	28.6	28.8	28.1	27.5
BEAM DEPTH $2\alpha d$ (cm)		28.1				43.1				
COLUMN PLATE THICKNESS t (cm)		1.2				1.9		1.2		
CONNECTION-WEB THICKNESS t_p (cm)		0.9	1.4	1.9	2.5	0.9	1.4	1.2	1.9	2.5
LENGTH OF MOMENT ARM (cm)	l_B	100.5	100.7	101.0	101.3	100.5	100.7	100.6	101.0	101.3
	l_C	101.0				93.5				
YIELD POINT (t/cm ²)	$\bar{\sigma}_0$	3.6		2.46	2.49	3.31	2.56	2.54	2.46	2.49
	σ_0	3.6		2.54		2.46		2.54		

where σ_0 is the yield point of column-web and $\bar{\sigma}_0$ that of connection web.

3.2 Test apparatus and procedure

The specimens are set up as shown in Fig. 13. Some of them are loaded by hydraulic testing machine and the others are set in a testing frame of steel and loaded by hydraulic jack.

Loads are applied in the similar way described in §2 except load increments, which of loading are about 5% of the yield load previously estimated and which of unloading about 10% during elastic behavior, but during plastic behavior loading is controlled with reasonable increments of deformation. Deformations of connection-web are measured in the diagonal directions by dial gauges attached as

Fig. 16

Fig. 18

Fig. 17

Fig. 19

Fig. 20

Fig. 23

Fig. 21

Fig. 24

Fig. 22

shown in Fig. 13.

3.3 Experimental results

The test results of $P-\delta$ curves are shown in Fig. 16 to Fig. 24 where P is the load applied by testing machine and δ the diagonal deformation of connection obtained by dial gauges as shown in Fig. 13.

3.4 Theoretical results

The yield moment M of each specimen can

Table 4

NAME OF SPECIMEN		□-1	□-2	□-3	□-4	□-5	□-6	□-7	□-8	□-9
LOWER BOUND OF M	CLASS. OF ANALYSIS	A ₃	B ₃	B ₂	C ₂	A ₂		B ₂	C ₂	C ₁
	NUMBER OF EQUATION	(3.3)	(3.6)	(3.5)	(3.8)	(3.2)		(3.5)	(3.8)	(3.7)
UPPER BOUND OF M	CLASS. OF ANALYSIS	b			c	b			c	
	NUMBER OF EQUATION	(4.24)			(4.25)	(4.24)			(4.25)	

be obtained from the equations listed in Table 4, where numbers of the equations refer to those in the previous paper Part V¹⁾.

And the loads P may be approximately expressed by term M , equating the works done by P and M in the velocity fields of the kinematical analysis shown in Fig. 25, and so we obtain

$$\frac{P}{\sqrt{2}} [l_B - \alpha d + (1 - \xi)d] \dot{\phi} = M \dot{\phi} \quad \dots\dots\dots (3.2)$$

then the load P is given as;

$$P = \frac{\sqrt{2}}{l_B - \alpha d + (1 - \xi)d} M \quad \dots\dots\dots (3.3)$$

where the ξ is given by

$$\xi = \begin{cases} 1.0 & \text{for the case (b)} \\ 2.0 - \frac{\lambda \alpha}{\sqrt{3}} & \text{for the case (c)} \end{cases} \quad \dots\dots\dots (3.4)$$

The calculated yield load P of each specimen is shown in Table 5 and also plotted in Fig. 16 to Fig. 24.

Table 5

NAME OF SPECIMEN	□-1	□-2	□-3	□-4	□-5	□-6	□-7	□-8	□-9
α	0.966	0.983	1.0	1.022	1.482	1.507	1.497	1.534	1.567
λ	0.750	1.167	1.583	2.042	0.637	0.767	1.0	1.533	2.042
ξ	1.0			0.795	1.0			0.642	0.152
LOWER BOUND OF P (t)	24.52	38.70	36.43	46.87	35.72	43.51	38.68	54.11	57.02
UPPER BOUND OF P (t)	24.52	38.70	36.43	47.02	36.05	43.99	39.08	54.79	58.25

§4 Conclusions

Since it is difficult to define a yield point on the $P-\delta$ curve obtained from experimental data²⁾, the comparison of experimental and theoretical results are discussed on the experimental $P-\delta$ curves as they are.

In Figs. 5 to 10 obtained from wide-flange specimens, the calculated value seems to be located a little above the proportional limit and near the point where excessive deformation initiates.

In Figs. 16 to 24, for the box-type specimens, it is also observed that the theoretical analysis gives the reasonable results except for the specimen □-9, for which the calculated yield load is somewhat underestimated.

Accordingly, it may be concluded that analytical results predict satisfactorily the "yield" loads where connections initiate the excessive deformation.

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Fig. 25 Kinematically Admissible Velocity Field

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柱はり接合部の極限解析 (IX) 梗概

— 実 験 的 検 証 —

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§1 は し が き

本論文 (I)~(VIII)¹⁾において、種々の形式の柱はり接合部の降伏強度について理論解析を行ない、柱、はり材および接合部の形状比の接合部降伏強度に対する影響を検討してきた。

ここでは逆対称曲げを受ける十字形接合部の実験を行ない、理論解析の妥当性と設計への適用性を検討する。

§2 H形柱、はりの接合部

2.1 実 験 計 画

試験体は同一 H 形断面材で構成される十字形接合部で、形状を Fig. 1, 2 に示し、各部寸法を Table 1 に示す。各試験体は理論解析上 Fig. 3 に示すような領域に属しており、同図でパラメーター β , λ は式 (2.1) で与える。

2.2 実験方法・装置

試験装置は油圧形試験機を用い、試験体を Fig. 2 のように設置した。各加力点はピンローラー支持として、材に軸力が生じないようにした。載荷は正負交番で2回行なったあと、最終回は接合部が十分塑性変形を起こすまで加力した。接合部の変形測定は Fig. 4 のように、対角線方向の変形をダイヤルゲージによって行なった。

2.3 実 験 結 果

実験で得られた荷重変形関係は各試験体について、Fig. 5 から Fig. 10 に示す通りである。

2.4 理 論 値

理論解析による接合部の降伏モーメントは式 (2.2) または (2.3) で求められるが、実験では柱、はりにせん断力を伴っているため、これを Fig. 11 に示す機構を用いて修正し、降伏荷重を式 (2.4) または式 (2.5) で求める。ただしパラメーター β と λ は、式 (2.7) を用いる。

§3 H形はりとはり柱よりなる接合部

3.1 実 験 計 画

試験体形状を Fig. 12, 13 に示し、各部の寸法を Table 3 に示す。Fig. 14, 15 は解析 (その V¹⁾) で得られた解の分類であり、各試験体の位置を示してある。図中 α ははり柱せい比で、 λ は式 (3.1) で与えられるパラメーターである。

3.2 実験方法・装置

実験は Fig. 13 のような状態で行なわれた。実験方法は §2 の場合とほぼ同様である。

3.3 実 験 結 果

実験による $P-\delta$ 図を Fig. 16~24 に示した。

3.4 理 論 値

各試験体の理論降伏モーメント M は Table 4 に一覧した式により求められるが、荷重 P との関係は、§2 と同様の方法により、式 (3.3) で与えられる。

§4 む す び

実験による降伏荷重を $P-\delta$ 図から定義するのは容易でない²⁾、解析結果を直接実験結果の図上に記入して比較する。

Fig. 5~10 は H 形材相互の接合部についての結果で、計算値は実験の弾性限荷重をやや上回り、急激な塑性流れが始まる付近に位置している。

箱形と H 形材の接合部の場合も Fig. 16~24 に示す通り、図-9 の予想荷重がやや低く過ぎる以外は極めて妥当な結果を得ている。

従って理論解析の結果は予想「降伏荷重」として適当であると言える。

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