EARTHQUAKE RESPONSE PREDICTION OF REINFORCED CONCRETE BUILDINGS

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SYNOPSIS

This paper is an application of the random vibration theory in earthquake engineering, from which a general analytical method is developed for the response of a multi-story building with a linear stable system subjected to earthquake excitation.

This analytical method is applied to evaluate the response of a three story building subjected to an accelerometer recording of the Tokachi-Oki earthquake. Expected maximum story displacements and expected number of cycles with varying intensities of the repeated story shearing forces due to the strong ground motion are computed and compared with the corresponding values obtained by a deterministic analysis method.

Hence, a dynamic response prediction method for the expected intensity of future earthquakes is here by discussed.

1. INTRODUCTION

The Tokachi-Oki earthquake which occurred in the northern part of Japan on 16th, May, 1968 caused extensive damages to the columns of some reinforced concrete buildings.

Numerous investigations as well as inspections on this area have been carried out to determine the process to the damage in certain type of buildings.

This paper deals with an application of random vibration theory in earthquake engineering.

It is not always possible to fully define the excitation of a particular dynamic system, however, in such case it may be possible to characterize the excitation in a probabilistic manner which then makes it possible to predict with any degree of accuracy the future dynamic response of buildings withstanding strong motion earthquake excitation by probabilistic methods. This method differs from deterministic methods and the dynamic response predictions are made in a probabilistic sense only.

An idealized model based on investigations on damaged reinforced concrete buildings is shown in Fig. 1. The transmissible mechanism of general earthquake excitation considered is shown in Chart 1.

Fig. 1 Idealized Model of Building

Chart 1 Transmissible mechanism

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2. VIBRATION OF GROUND SURFACE

It has been suggested by Dr. Kanai and Dr. Tajimi that the autocorrelation and the power spectral density functions of the ground accelerations of many past earthquake motions are approximated by the following equations

$$R_{\ddot{x}_g}(\tau) = \pi \omega_g S_0 \left\{ \left(2\xi + \frac{1}{2\xi} \right) e^{-\xi \omega_g \tau} \cos \omega_g \tau + \frac{\xi}{\sqrt{1-\xi^2}} \left(\frac{1}{2\xi} - 2\xi \right) e^{-\xi \omega_g \tau} \sin \omega_g \tau \right\} \quad (1)$$

$$S_{\ddot{x}_g}(p) = \frac{1 + 4\xi^2 \left(\frac{p}{\omega_g} \right)^2}{\left[1 + (4\xi^2 - 2) \left(\frac{p}{\omega_g} \right)^2 + \left(\frac{p}{\omega_g} \right)^4 \right]} S_0 \quad (2)$$

where ξ and ω_g may be thought of as ground damping factor and predominant frequency, respectively, and where S_0 is a constant power spectral density function (see Fig. 2).

This idea has been presented as a result of analyses of many past earthquake motion considering that the spectrum observed at bedrock is characterized by a constant pattern, while the spectrum at the ground surface is multiplied by the vibration property of the ground layer.

The plot of Eq. (2) shows that the function has one high peak at $p = \omega_g$ as shown in Fig. 3(b).

However, the power spectral density functions observed at ground surface happen to show more than one distinguished peak.

In such case, the autocorrelation and the power spectral density functions may be expressed as the sum of each function.

$$R_{x_g}(\tau) = \sum_{j=1}^n \pi \omega_{gj} S_j \left\{ \left(2\xi_j + \frac{1}{2\xi_j} \right) e^{-\xi_j \omega_{gj} \tau} \cos \omega_{gj} \tau + \frac{\xi_j}{\sqrt{1-\xi_j^2}} \left(\frac{1}{2\xi_j} - 2\xi_j \right) e^{-\xi_j \omega_{gj} \tau} \sin \omega_{gj} \tau \right\} \quad (3)$$

$$S_{\ddot{x}_g}(p) = \sum_{j=1}^n \frac{1 + 4\xi_j^2 \left(\frac{p}{\omega_{gj}} \right)^2}{\left[1 + (4\xi_j^2 - 2) \left(\frac{p}{\omega_{gj}} \right)^2 + \left(\frac{p}{\omega_{gj}} \right)^4 \right]} S_j \quad (4)$$

Fig. 2 Auto-Correlation and Power Spectral Density Functions of White Noise

Fig. 3 Auto-Correlation and Power Spectral Density Function of Ground Acceleration

Therefore, the variance of the ground acceleration $\ddot{x}_g$ with respect to time t is simply obtained according to the property of the autocorrelation function, i.e.,

$$\sigma_{\ddot{x}_g}^2 = R_{\ddot{x}_g}(0) = \pi \omega_g S_0 \left(2\xi + \frac{1}{2\xi} \right) \quad (5a)$$

or

$$R_{\ddot{x}_g}(0) = \sum_{j=1}^n \pi \omega_{gj} S_j \left(2\xi_j + \frac{1}{2\xi_j} \right) \quad (5b)$$

3. VIBRATION OF MULTI-DEGREE OF FREEDOM SYSTEMS

The foundations of buildings are supposed to be subjected to the above mentioned ground acceleration $\ddot{x}_g(t)$ in the lateral direction as shown in Fig. 1, where y_i ($i=1, 2, \dots, m$) denoted the absolute displacements of the floors.

The ground displacement is denoted by x_g .

Consequently, the floor displacements relative to the foundation are given by $(y_i - x_g)$ ($i=1, 2, \dots, m$).

$$[\backslash m_{\setminus}]\{\ddot{y}\} + [c]\{\dot{y} - \dot{x}_g\} + [k]\{y - x_g\} = 0 \quad \dots\dots\dots(6)$$

Introducing anew variable

$$z_i = y_i - x_g$$

the equations of motion become

$$[\backslash m_{\setminus}]\{\ddot{z}\} + [c]\{\dot{z}\} + [k]\{z\} = -[\backslash m_{\setminus}]\{\ddot{x}_g\} \quad \dots\dots\dots(7)$$

since $[\backslash m_{\setminus}]$ is a diagonal matrix and

$$\{\ddot{x}_g\} = \ddot{x}_g \begin{Bmatrix} 1 \\ \vdots \\ 1 \end{Bmatrix}$$

the right-hand side of Eq. (7) can be written as

$$-\ddot{x}\{m\}$$

where $\{m\}$ is a column matrix containing the masses $m_1, m_2, \dots, m_n$. Eq. (7) then becomes

$$[\backslash m_{\setminus}]\{\ddot{z}\} + [c]\{\dot{z}\} + [k]\{z\} = -\ddot{x}_g\{m\} \quad \dots\dots\dots(8)$$

The relative floor displacements z_i are expressed in terms of the normal modes ϕ and normal coordinates η_r ($r=1, 2, \dots, m$), where

$$z_i(t) = \sum \phi_{ir} \eta_r(t)$$

or

$$\{z\} = [\phi]\{\eta\}, \{\dot{z}\} = [\phi]\{\dot{\eta}\} \text{ and } \{\ddot{z}\} = [\phi]\{\ddot{\eta}\} \quad \dots\dots\dots(9)$$

Substituting Eq. (9) into Eq. (8) and premultiplying the resulting equation by $[\phi]^T$, a set of decoupled differential equations in the normal coordinates η_r is obtained as follows;

$$[\backslash M_r]\{\ddot{\eta}\} + 2\zeta[\backslash M_r]\{\dot{\eta}\} + [\omega_r^2][\backslash M_r]\{\eta\} = -\ddot{x}_g[\phi]^T\{m\} \quad \dots\dots\dots(10)$$

where

$$[\backslash M_r] = [\phi]^T[\backslash m_{\setminus}][\phi]$$

$$2\zeta[\backslash M_r] = [\phi]^T[c][\phi]$$

$$[\omega_r^2][\backslash M_r] = [\phi]^T[k][\phi]$$

The r -th row of Eq. (10) has the form

$$\ddot{\eta}_r + 2\zeta\dot{\eta}_r + \omega_r^2\eta_r = \frac{\Gamma_r}{M_r}\ddot{x}_g \quad \dots\dots\dots(11)$$

in which $\Gamma_r = \{\phi^{(r)}\}^T\{m\}$

The solution of Eq. (11) is given by

$$\eta_r = -\frac{\Gamma_r}{\omega_{dr}M_r} \int_{-\infty}^t \ddot{x}_g e^{-\zeta\omega_r(t-\tau)} \sin \omega_{dr}(t-\tau) d\tau \quad \dots\dots\dots(12)$$

where ζ , ω_r and ω_{dr} denote the structural damping ratio ($\zeta=0.05$), the undamped natural frequency and the damped frequency in the under critically damped case, respectively.

The autocorrelations of η_r and $\dot{\eta}_r$ can be expressed as follows;

$$\begin{aligned} R_{\eta_r}(\tau) &= E[\eta_r(t)\eta_r(t+\tau)] \\ &= \frac{\Gamma_r^2}{\omega_r^4 M_r^2} \int_{-\infty}^{\infty} \frac{S_{\ddot{x}_g}(p) \cdot e^{ip\tau}}{\left[1 + 2i\zeta\left(\frac{p}{\omega_r}\right) - \left(\frac{p}{\omega_r}\right)^2\right] \left[1 - 2i\zeta\left(\frac{p}{\omega_r}\right) - \left(\frac{p}{\omega_r}\right)^2\right]} dp \quad \dots\dots\dots(13) \end{aligned}$$

$$\begin{aligned} R_{\dot{\eta}_r}(\tau) &= E[\dot{\eta}_r(t)\dot{\eta}_r(t+\tau)] = -R''_{\eta_r}(\tau) \\ &= \frac{\Gamma_r^2}{\omega_r^4 M_r^2} \int_{-\infty}^{\infty} \frac{S_{\ddot{x}_g}(p) p^2 e^{ip\tau}}{\left[1 + 2i\zeta\left(\frac{p}{\omega_r}\right) - \left(\frac{p}{\omega_r}\right)^2\right] \left[1 - 2i\zeta\left(\frac{p}{\omega_r}\right) - \left(\frac{p}{\omega_r}\right)^2\right]} dp \quad \dots\dots\dots(14) \end{aligned}$$

Therefore, the mean squares of the responses

$\overline{z_i^2(x, t)}$ and $\overline{\dot{z}_i^2(x, t)}$ become

$$\overline{z_i^2(x, t)} = R_{z_i}(0) = \sum_{r=1}^m \Phi_{ir}^2(x) \frac{\Gamma_r^2}{\omega_r^4 M_r^2} \int_{-\infty}^{\infty} |H(\omega_r p)|^2 S_{\ddot{x}_g}(p) dp \quad (15)$$

$$\overline{\dot{z}_i^2(x, t)} = R_{\dot{z}_i}(0) = \sum_{r=1}^m \Phi_{ir}^2(x) \frac{\Gamma_r^2}{\omega_r^4 M_r^2} \int_{-\infty}^{\infty} |H(\omega_r p)|^2 p^2 S_{\ddot{x}_g}(p) dp \quad (16)$$

where $i=1, 2, \dots, m$ represents the levels of the building in Fig. 1 and

$$|H(\omega_r p)|^2 = \frac{1}{\left[1 + 2i\zeta\left(\frac{p}{\omega_r}\right) - \left(\frac{p}{\omega_r}\right)^2\right] \left[1 - 2i\zeta\left(\frac{p}{\omega_r}\right) - \left(\frac{p}{\omega_r}\right)^2\right]} \quad (17)$$

$$S_{\ddot{x}_g}(p) = \sum_{j=1}^n \frac{\left[1 + 2i\xi_j\left(\frac{p}{\omega_{gj}}\right)\right] \left[1 - 2i\xi_j\left(\frac{p}{\omega_{gj}}\right)\right]}{\left[1 + 2i\xi_j\left(\frac{p}{\omega_{gj}}\right) - \left(\frac{p}{\omega_{gj}}\right)^2\right] \left[1 - 2i\xi_j\left(\frac{p}{\omega_{gj}}\right) - \left(\frac{p}{\omega_{gj}}\right)^2\right]} S_j \quad (18)$$

Substituting the above equations into Eqs. (15) and (16) the mean square values are written in the following equivalent forms, i.e.,

$$\overline{z_i^2(x, t)} = \sum_{j=1}^n \sum_{r=1}^m \Phi_{ir}^2(x) \frac{\Gamma_r^2 S_j}{\omega_r^4 M_r^2} \int_{-\infty}^{\infty} \left| \frac{iBp + 1}{1 + iA_1 p - A_2 p^2 - iA_3 p^3 + A_4 p^4} \right|^2 dp \quad (19)$$

$$\overline{\dot{z}_i^2(x, t)} = \sum_{j=1}^n \sum_{r=1}^m \Phi_{ir}^2(x) \frac{\Gamma_r^2 S_j}{\omega_r^4 M_r^2} \int_{-\infty}^{\infty} \left| \frac{-Bp^2 + ip}{1 + iA_1 p - A_2 p^2 - iA_3 p^3 + A_4 p^4} \right|^2 dp \quad (20)$$

in which

$$A_1 = 2\left(\frac{\xi_j}{\omega_{gj}} + \frac{\zeta}{\omega_r}\right)$$

$$A_2 = \frac{1}{\omega_{gi}^2} + 4\xi_j \zeta \frac{1}{\omega_{gj} \omega_r} + \frac{1}{\omega_r^2}$$

$$A_3 = 2\left(\frac{\xi_j}{\omega_{gj} \omega_r^2} + \frac{\zeta}{\omega_{gj}^2 \omega_r}\right)$$

$$A_4 = \frac{1}{\omega_{gj}^2 \omega_r^2}$$

and

$$B = \frac{2\xi_j}{\omega_{gj}}$$

These integrations are carried out using the complex plane and contour integration such that

$$\overline{z_i^2(x, t)} = \sum_{j=1}^n \sum_{r=1}^m \Phi_{ir}^2(x) \frac{\Gamma_r^2 S_j}{\omega_r^4 M_r^2} \pi \frac{A_2 A_3 - A_1 A_4 + A_3 B^2}{A_1(A_2 A_3 - A_1 A_4) - A_3^2} \quad (21)$$

$$\overline{\dot{z}_i^2(x, t)} = \sum_{j=1}^n \sum_{r=1}^m \Phi_{ir}^2(x) \frac{\Gamma_r^2 S_j}{\omega_r^4 M_r^2} \pi \frac{A_3 + A_1 B^2}{A_1(A_2 A_3 - A_1 A_4) - A_3^2} \quad (22)$$

The most interesting property of dynamic response is the mean square values of relative displacements at each floor level denoted by $\{z_i(x, t) - z_{i+1}(x, t)\}^2$.

Using the relations

$$\overline{\{z_i(x, t) - z_{i+1}(x, t)\}^2} = \overline{z_i^2(x, t)} - 2\overline{z_i(x, t)z_{i+1}(x, t)} + \overline{z_{i+1}^2(x, t)} \quad (23)$$

$$\overline{\{\dot{z}_i(x, t) - \dot{z}_{i+1}(x, t)\}^2} = \overline{\dot{z}_i^2(x, t)} - 2\overline{\dot{z}_i(x, t)\dot{z}_{i+1}(x, t)} + \overline{\dot{z}_{i+1}^2(x, t)} \quad (24)$$

the mean square values of the relative displacements and their differential forms with respect to time t are obtained as follows;

$$\overline{\{z_i(x, t) - z_{i+1}(x, t)\}^2} = \sum_{j=1}^n \sum_{r=1}^m \{\Phi_{ir}^2(x) - 2\Phi_{ir}(x)\Phi_{i+1,r}(x) + \Phi_{i+1,r}^2(x)\} \cdot \frac{S_j \Gamma_r^2}{\omega_r^4 M_r^2} \pi \frac{A_2 A_3 - A_1 A_4 + A_3 B^2}{A_1(A_2 A_3 - A_1 A_4) - A_3^2} \quad (25)$$

$$\overline{\{\dot{z}_i(x, t) - \dot{z}_{i+1}(x, t)\}^2} = \sum_{j=1}^n \sum_{r=1}^m \{\Phi_{ir}^2(x) - 2\Phi_{ir}(x)\Phi_{i+1,r}(x) + \Phi_{i+1,r}^2(x)\} \cdot \frac{S_j \Gamma_r^2}{\omega_r^4 M_r^2} \pi \frac{A_3 + A_1 B^2}{A_1(A_2 A_3 - A_1 A_4) - A_3^2} \quad (26)$$

4. NUMBER OF CYCLES IN RESPONSE DURING STRONG MOTION EXCITATION

The probability density function for crossings of response $\alpha(t)$ at level V with positive velocity is given by Roce's relation if the process is normal or Gaussian with zero mean value, i.e.,

$$Q(V) = \frac{1}{2\pi} \cdot \frac{\sigma_{\dot{\alpha}}}{\sigma_{\alpha}} \exp\left\{-\frac{1}{2} \frac{V^2}{\sigma_{\alpha}^2}\right\} \dots\dots\dots (27)$$

σ_{α} being the root mean square level of α .

Fig. 4 Response of a narrow band System

The expected number of crossings of the level V with positive slope during the time T is, then, estimated as follows;

$$N_{V^+} = \frac{T}{2\pi} \frac{\sigma_{\dot{\alpha}}}{\sigma_{\alpha}} \exp\left(-\frac{V^2}{2\sigma_{\alpha}^2}\right) \dots\dots\dots (28)$$

where the time T represents the duration of the strong earthquake excitation.

The most expected maximum response amplitude during the time T is simply approximated by setting the number of crossings of the level $V_{\max}^+$ equal to unity, i.e.,

$$V_{\max}^+ = \sqrt{2\sigma_{\alpha}^2 \log \frac{T}{2\pi} \frac{\sigma_{\dot{\alpha}}}{\sigma_{\alpha}}} \dots\dots\dots (29)$$

5. ACCELEROMETER RECORDINGS OF THE TOKACHI-OKI EARTHQUAKE

Many strong motion accelerometer recordings were obtained during the Tokachi-Oki earthquake, of which two are shown in Fig. 5.

One was recorded on a SMAC-B2, the station is situated at the port of Hachinohe and operated by the Port and Harbor Research Institute. The instrument is placed on hollow concrete blocks 1.4 m by 1.4 m with a height of 0.5 m. The blocks are on four 200 mm diameter piles. The soil is sand to a depth of about 10 m, below this a clay layer extends for about 20 m.

Fig. 5 Accelerometer recording Tokachi-Oki Earthquake

The recording at Hiroo was recorded on a SMAC-B, this station is operated by the Building Research Institute and located on a diluvium terrace which is covered by a layer of volcanic clay. The foundation for the instrument is a 5.6 m by 5.6 m concrete block with a height of about 0.2 m and rests directly on the ground floor slab of a one-story wooden building. This concrete slab can be assumed to be structurally independent of the wooden building.

The autocorrelation and the power spectral density functions of the accelerometer recordings for

the duration of the strong motion were computed, one of them is shown in Fig. 6, where a different unit of spectral density is used. The differences arise due to the use of cycles per unit time instead of radians per unit and also taking into account of only the positive frequencies. The spectral density will be denoted by $W(f)$ where f is frequency in cycles per unit time. The relation between $S(p)$ and $W(f)$ is simply

$$W(f) = 4\pi S(p) \dots \dots \dots (30)$$

Regarding that each power spectral density function can be the response of a single degree of freedom system represented by Eq. (2) when subjected to a white noise excitation $W_g(f) = W_0$, the function will be modified by applying the method of least squares as indicated by a broken line shown in Fig. 6.

Fig. 6 Auto-Correlation and Power spectral Density functions (Hiroo EW)

The constant power spectral density function S_0 , the ground damping factor ξ and the predominant ground frequency ω_g will be estimated from each modified function obtained by the method of least squares as shown in the following table.

Table 1

		$\frac{\text{cm}^2}{\text{sec}^4} / \frac{\text{rad}}{\text{sec}}$	ξ_j	$\omega_{gj} \frac{\text{rad}}{\text{sec}}$
HACHINOHE EW		$S_0 = 11.04608$	$\xi = 0.11714$	$\omega_g = 5.6862$
"	NS	$= 13.404$	$= 0.17114$	$= 2.5234$
HIROO	EW	$= 10.44$	$= 0.1309$	$= 27.61$
"	NS	$= 15.78$	$= 0.2559$	$= 25.06$

6. RESPONSE OF A REINFORCED CONCRETE BUILDING EXCITED BY THE TOKACHI-OKI EARTHQUAKE

To clarify sections 3 and 4 the response of a three story building subjected to the Tokachi-Oki earthquake in the lateral direction as shown in Fig. 7 is presented as an example using the data of the Hiroo EW recording given in Table 1.

The mass and stiffness in the y coordinates are

$$\begin{aligned} m_1 &= 0.153 \text{ ton} \cdot \text{sec}^2 / \text{cm} & k_1 &= 326.80 \text{ ton} / \text{cm} \\ m_2 &= 0.252 \text{ " } & k_2 &= 325.80 \text{ " } \\ m_3 &= 0.255 \text{ " } & k_3 &= 386.70 \text{ " } \end{aligned}$$

Fig. 7 Model

The normal modes and natural frequencies of the building are given by

$$\{\phi^{(1)}\} = \begin{Bmatrix} 1.0000 \\ 0.8323 \\ 0.4367 \end{Bmatrix} \quad \omega_1 = 18.87 \text{ rad/sec}$$

$$\{\phi^{(2)}\} = \begin{Bmatrix} 1.0000 \\ -0.1936 \\ -1.0090 \end{Bmatrix} \quad \omega_2 = 50.49 \text{ rad/sec}$$

$$\{\phi^{(3)}\} = \begin{Bmatrix} 1.0000 \\ -1.1610 \\ 0.8157 \end{Bmatrix} \quad \omega_3 = 67.94 \text{ rad/sec}$$

The property of the ground layer under the building and the constant power spectral obtained from the Hiroo EW accelerometer recording are repeated here convenience.

$$S_0 = 10.44 \frac{\text{cm}^1}{\text{sec}^4} \frac{\text{rad}}{\text{sec}}$$

$$\omega_g = 27.61 \text{ rad/sec}$$

$$\xi = 0.1309$$

Using Eqs. (21), (22), (25) and (26), the mean square values of z_i and $\dot{z}_i$ and the mean square values of $z_i - z_{i+1}$ at the floor levels in the building are computed.

Regarding that the duration of the strong earthquake excitation was $t=28.0$ sec., the expected maximum displacements relative to the base and the expected maximum story displacements at the floor levels are obtained from Eq. (29) and the above mean square values.

These results are compared with the corresponding values computed by a deterministic analysis method as shown in Table 2.

Table 2

		Non-deterministic	Deterministic
Max. displacement relative to base	D_1	1.541 cm	1.2202 cm
"	D_2	1.283 cm	1.0188 cm
"	D_3	0.674 cm	0.5429 cm
Maximum story displacement	d_1	0.264 cm	0.2456 cm
"	d_2	0.612 cm	0.5225 cm
"	d_3	0.674 cm	0.5429 cm

The expected number of cycles with the varying intensities of the repeated story shearing forces at the floor levels can be estimated for the duration of the strong motion from Eq. (23) and the mean square values as shown in Table 3.

Table 3

	3 rd story		2 nd story		1 st story	
Max. story shear P max.	86.1 t	cycles 1.0	199.4 t	cycles 1.0	260.6 t	cycles 1.0
0.9P	77.5	2.36	179.5	2.35	234.5	2.35
0.8P	68.9	5.11	159.5	5.03	208.5	5.03
0.7P	60.3	10.1	139.6	9.86	182.4	9.86
0.6P	51.7	18.2	114.6	17.7	156.4	17.7
0.5P	43.1	29.9	99.7	28.9	130.3	29.0
0.4P	34.4	44.9	79.8	43.4	104.2	43.4
0.3P	25.8	61.7	59.8	59.3	78.2	59.4
0.2P	17.2	77.4	39.9	74.3	52.1	74.3
0.1P	8.6	88.7	19.9	85.0	26.1	85.0
0.0P	0.0	92.8	0.0	88.9	0.0	88.9

7. DISCUSSION

It is clear from the above analysis that the response of a building due to a future strong earthquake excitation is predictable if the bedrock power spectral density S_0 and the property of the layer ω_g and ξ can be evaluated at the building site.

Moreover, to design a building with safety for life and property in mind, the expected maximum power spectral density S_0 max. should be estimated, which seems to be closely related to the property

of the layer under the building.

For this reason, the autocorrelations $R_{\ddot{x}_g}(0)$ of ground accelerations $\ddot{x}_g$ are calculated from many past earthquake recordings and the mean value and the variance are

$$\overline{R_{\ddot{x}_g}(0)} = 3,470.93 \text{ gal}^2$$

$$\sigma R_{\ddot{x}_g}^2(0) = 1.4293 \times 10^6 \text{ gal}^4$$

The possible maximum $R_{\ddot{x}_g}(0)$ may be taken as the design level of $3 \sigma R_{\ddot{x}_g}^2(0)$ from an engineering point of view, although a precise conclusion cannot be drawn due to insufficient number of data of Table 4.

$$R_{\ddot{x}_g}(0) \text{ max} = \overline{R_{\ddot{x}_g}(0)} + 3 \sigma R_{\ddot{x}_g}(0) = 7,057.49 \text{ gal}^2 \quad \dots\dots\dots(31)$$

Therefore, the possible maximum power spectral density $S_0 \text{ max}$, is derived from Eq. (5) as follows;

$$S_0 \text{ max} = \frac{R_{\ddot{x}_g}(0) \text{ max}}{\pi \omega_g \left(2\xi + \frac{1}{2\xi} \right)} \div \frac{7,000.0}{\pi \omega_g \left(2\xi + \frac{1}{2\xi} \right)} \quad \dots\dots\dots(32)$$

Table 4

	$R_{\ddot{x}_g}(0)$ and T			
Vernon, Calif. Oct. 2, 1933	NO 8 E	4,660.5 gal ²	0-9 sec	S 82 E 4,102.8 gal ² 0-9 sec
Los Angeles Subway Terminal Oct. 2, 1933	N 39 E	3,416.8 "	0-9 "	N 51 W 3,088.4 " 0-9 "
El Centro, Calif. May 18, 1940	NS	4,213.7 "	0-29 "	EW 6,023.9 " 0-29 "
Santa Barbara, Calif. June 30, 1941	N 45 E	—		S 45 E 2,907.3 " 0-10 "
Hiroo May 16, 1968	NS	2,626.4 "	0-28 "	EW 3,551.2 " 0-28 "
Hachinohe May, 16, 1968	NS	1,703.8 "	0-34 "	EW 1,875.4 " 0-34 "

The predominant frequency ω_g and the damping factor ξ should also be measured with accuracy before the construction commences.

It has been suggested that these values can be approximated from microtremor measurements or artificial earthquake methods.

However, the considerable techniques and modification will be required to evaluate these values with accuracy because of the extremely different characteristics of these excitations.

Furthermore, from the experimental data in the past it is observed that the accelerometer recordings measured at the base of a building during an excitation are more or less different from those recorded on the surface of the ground depending on the ground condition at the building site.

These involve many problems to be solved and are the subjects for future researches.

8. ACKNOWLEDGEMENT

The authors wish to thank Dr. Joseph Penzien, Professor of the University of California, Berkeley for suggesting this problem and for stimulating interest in it.

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RC 構造物の地震応答予測¹⁾(梗概)

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1. 緒 言

本研究の目的は、構造物の地震動に対する動的応答を予測する場合の（即ち、動的設計に通ずる）確率論的解析方法を確立することである。

従来の地震動に対する動的応答計算は、実際に得られた地震記録を入力とし、step-by-stepで行う。この方法で構造物の動的設計を行う場合に問題となるのは、構造物のモデル化にあるのは勿論であるが、入力地震波の選択にある。入力地震波としてどの波を採用するか、あるいは最大加速度をいくりにするかで応答値はいくらでも変わってしまう。将来の設計をするに当り、過去の特定の地震波を採用するのは如何にも不合理である。この点から、実際問題として数種の地震波を用いて、平均的なあるいは包絡線的な考え方で設計をしているわけであるが、それでもなお採用した地震波群が、将来起り得る地震波のパターンを網羅しているかどうかという点で不安が残るわけである。構造物が建てられる地盤上で、将来どのような地震動が記録されるかは不明である（波形に関して）。従来行われている step-by-step の応答計算は、動的解析には欠かせぬものであるが、構造物の動的設計には上記の理由から不適当と考えられる。

本研究では構造物の動的設計におけるこのような欠点を除いて、まず地震波は波形を問題とせず、その特性のみを用いて応答予測を行なおうとするものである。ある地域で記録される地震波の特性は、地震自体の特性、その地域の地盤の特性、震央距離等によって統計的、確率的に予測しうるものと考えられる。本研究は、これらの特性が何らかの方法によって定められた時に、これらを入力として与える事によって、ある地域に建てられる構造物の予想応答値を求めようとするものである。これから得られる応答値は、統計的、確率的に得られるものであるから、従来の応答計算法による動的設計時に考えられる危険性は除かれるわけである。

本論文は、最初に理論を述べた後、'68年十勝沖地震時に得られた北海道広尾 SMAC 記録から、この地震波の特性を抽出し、これを入力として、同地震時に被害を受けた RC 構造物の応答を予測し、従来法による応答値との比較を行なって、本理論の例題としてある。

2. 解析理論

地震力が構造物に入り、破壊を生ぜしめるまでの伝播経路を Chart 1 に示す。今、ある地震に対して地震基盤においてパワースペクトル密度が、一定値 S_0 であるとすると。この場合の地盤面の自己相関関数とパワースペクトル密度関数は、金井、多治見両博士により、式 (1)、式 (2) で与えられる。式 (2) は $P=\omega_g$ で一つのピークを有する関数であるが、今いくつかのピークを有する場合に適用するために式 (3)、式 (4) を考えることとする。

つぎにこれらの特性を有する地盤面上の構造物を考える。構造物の塑性域を考えない場合には、式 (6) 以下式 (11) までのように各次のモードに分解して計算することができる。その際の解は式 (12) で与えられる。この解およびこの解の1階微分の自己相関をとると、それぞれ式 (13)、式 (14) となる。両式中で $\tau \rightarrow 0$ とすれば応答の二乗平均値を与えることになる。これが式 (15)、式 (16) である。これらを整理したのが式 (21) および式 (22) である。これが構造物基礎に対する絶対変位および速度の二乗平均値である。層間変位および速度の二乗平均値は式 (25)、式 (24) を用いて、式 (25)、式 (26) で示される。

つぎに応答波形の一定レベルに対する繰り返し回数を計算してみる。応答波形を $\alpha(t)$ として $\alpha(t)$ がレベル V を正の傾きで切る確率密度関数は、 $\alpha(t)$ の平均値が零で、かつ正規分布しているならば式 (28) で与えられる。また、この式を用いて $\alpha(t)$ の最大値レベルを計算するには、 $N_V^+=1$ とおけばよい。従って式 (29) が得られるわけである。本文第3項までで、 σ_a , $\sigma_{\dot{a}}$ に対するものが計算されているから、式 (29) を用いることによって応答の最大値を予測することができる。

1) 昭和44年8月北海道大会で発表したものである。

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(昭和44年9月30日本稿受理・討論期限昭和45年8月末日)

3. 計 算 例

本理論の計算例として、1968年の十勝沖地震をとりあげてみる。入力としては北海道広尾のSMAC記録を利用して、通常の応答計算値と比較してみる。構造物は同地震時に被害を受けた3層RC造を用いる。

Fig. 5の広尾EWの記録の自己相関関数、パワースペクトル密度関数を計算するとFig. 6のようになる。Fig. 6右図中の点線のカーブは実線が式(2)の関係にあるものとして、最小二乗法により求めたものである。Table 1にはこうして得られた地震波の特性を示すファクター、 S_0 , ξ , ω_g の値を示してある。広尾EWに対する計算結果をTable 2に示す。左欄が本法による結果、右欄が通常の応答計算結果である。Table 3には構造物各層における剪断力の繰り返し回数を計算してある。

4. 検 討

以上の解析から明らかなように、構造物の所在する場所での基盤のパワースペクトル密度 S_0 と、地盤の特性

ξ , ω_g が知られれば、将来起り得る強震に対する構造物の応答を予測できる。したがって S_0 , ξ , ω_g を定めることが重要となるわけであるが、このうち ξ , ω_g は常時微動、人工地震等によって、ある程度近似値が得られる。今 Table 4 に示される地震の自己相関関数 ($\tau=0$) の値の平均値 $\overline{R_{\ddot{x}_g}(0)}$, および分散 $\sigma_{R_{\ddot{x}_g}(0)}$ を用いると $R_{\ddot{x}_g}(0)$ の最大値は式(31)で表わされよう。さらに式(5a)を用いると式(32)の関係が導かれる。したがって ξ , ω_g が何等かの方法で決まれば S_0 の予想最大値 $S_{0\max}$ を計算することができるわけである。

式(31)の数値はあくまで Table 4 の地震からのみ計算したものである。実際にはより広範なデータを用いての分類的处理も必要になろう。

最後に、この問題を提起し、かつそれに対する興味を喚起して下さったカリフォルニア大学バークレイ教授、ジョセフ・ペンゼン博士に深く感謝の意を表したい。

TRANSACTIONS OF THE ARCHITECTURAL INSTITUTE OF JAPAN

May 1970

Number 171

SYNOPSIS

U.D.C. 691.87 : 691.32

REINFORCED CONCRETE WITH BUNDLED BAR

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U.D.C. 621.791.75 : 691.87

ARC WELDED CONNECTIONS FOR HIGH STRENGTH DEFORMED BARS

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The arc welded joints of high strength deformed bars have been studied. Butt, lap, and splice joints were used. The ultimate strength of high strength deformed bars varied from 60,000 psi to 80,000 psi. Ordinary manual arc welding and enclosed arc welding were used. The results are obtained as follows: (a) the bend test about the short beaded is the most convenient method to determine the weldability of the bars; (b) suitable preheating is needed to obtain good welded joints.

U.D.C. 550 : 34 : 624.012.4

EARTHQUAKE RESPONSE PREDICTION OF REINFORCED CONCRETE BUILDING

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U.D.C. 624.072.2 : 624.041.62

EXPERIMENTAL STUDY OF THE INELASTIC STEEL BEAM (2)

(Beam under Moment Gradient)

By Dr. **TOSHIRO SUZUKI**, Assist. Prof. Nagoya Institute of Technology and **TETSURO ONO**, Graduate Student Nagoya Institute of Technology. Members of A.I.J.

In the previous paper, writers have reported about the post buckling behavior of the inelastic steel beams