LOAD-DEFLECTION BEHAVIORS AND PLASTIC FATIGUE OF WIDE-FLANGE BEAMS SUBJECTED TO ALTERNATING PLASTIC BENDING

PART III STEADY-STATE THEORY

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1. INTROUCTION

Previous theoretical predictions of load-deflection behaviors of wide-flange beams, columns and frames are mostly based upon bilinear or polylinear hysteretic stress-strain relations assumed without sufficient quantitative evidences and concerned primarily with initial transient behaviors [(I-3~10)] Nonlinear stress-strain relations have been applied only to virgin response [1], [2]. With the use of modern electronic computers and of the finite difference or finite element method, it does not appear to be very difficult to obtain numerical solutions to the load-deflection behaviors of, at least, members subjected to alternating plastic bending under the presence of an axial force [3], [4].

The purpose of the present paper is to derive an approximate but analytical solution to the steady-state load-deflection behaviors and energy absorption capacity based upon the concept of the steady-state alternating plastic bending and the concept of an approximately equivalent sandwich cross section. In view of the experimental evidence obtained in Part I, the flanges of the idealized sandwich section of a specimen may be assumed, to the first order approximation, to be subjected to alternating tension and compression of an equal strain amplitude in steady-state alternating bending. The skeleton and hysteretic stress-strain relations which have been derived in Part II are therefore applied. It will be shown that the numerical result obtained from the present theory predicts the experimental load-deflection curves, energy absorption per cycle considerably well. Since the maximum absolute strain amplitudes can also be estimated by the present theory, a prediction on the plastic bending fatigue ductility relation will also be made on the assumption that the tension-compression fatigue ductility relation derived in Part II may be applied to flanges of wide-flange specimens.

2. ASSUMPTIONS FOR APPROXIMATE ANALYSIS

The bending strain distributions shown in Figs. 5 and 6 of Part I have clearly indicated the existence of steady-state bending strain distributions. It should be noted, however, that the steady-state bending strain distributions in specimens subjected to larger deflection amplitudes are the results which include the effect of local and lateral deformations. It is observed that, the steady-state bending strain distribution in a specimen subjected to a smaller deflection amplitude is almost the same in upward and downward bending but that it has a larger amplitude in the direction of the virgin bending in a specimen subjected to a larger deflection amplitude. It is not clear from the experimental evidence so far obtained, whether this circumstance is solely due to the initial perfectly plastic behavior or to the lateral torsional deformation. Since, however, the plastic region in a specimen subjected to a smaller deflection amplitude has been indeed subjected to tension and compression of an almost equal strain amplitude, it is a reasonable first-order approximation to assume that a steady-state bending strain

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distribution has an equal amplitude in downward and upward bending. Deviation of the observed distribution from the assumed distribution may be regarded to be of a second order quantity. This assumption implies that, the steady-state stress distribution in a specimen at the prescribed maximum or minimum deflection configuration is governed by the skeleton stress-strain relations which have been derived in Part II. The stress distribution over a cross-section in a plastic region based upon Eq. (1) of Part II is as shown in Fig. 1 (b), if the elastic response is not to exhibit any softening behavior up to $\epsilon = \epsilon_y$ in the converging process to the skeleton curve given by Eq. (1), while the stress distribution based upon Eq. (5) of Part II is as shown in Fig. 1 (d). Although the sharp peak in Fig. 1 (b) is hypothetical, the form of Eq. (1) is advantageous in deriving a hysteretic load-deflection relation. On the other hand, Eq. (5) which appears to be more realistic, will be useful only for a skeleton load-deflection relation. The principal advantage of the above assumption is that an analytical steady-state solution may be derived in an explicit and closed form with the use of the "approximately equivalent sandwich section" defined below.

An idealized sandwich section consists of two flanges of concentrated areas carrying normal stresses only and a shear resistant core which does not carry normal stress. A sandwich section is said to be equivalent to the original wide-flange section if (1) the extreme fiber strains $\pm \epsilon_a$ and the curvature κ at the section are the same and (2) the corresponding extreme fiber stresses $\pm \sigma_a$ and the bending moments are the same. Due to the nonlinear stress-strain relation fitting considerably well to the experimental result, the equivalent section is dependent upon the nonlinear stress distribution. It is more convenient to define an approximately equivalent sandwich section which may be applied throughout a plastic region without regard to nonlinear stress distributions. The approximately equivalent sandwich section is defined as that section which has the original depth H and the equivalent concentrated flange area only at the particular section where $\sigma_a = \sigma_y$ and $\epsilon_a = \epsilon_y + \epsilon_{py}$. It is called "approximately equivalent" since the aforementioned conditions are satisfied only approximately at sections other than the particular section.

The equivalence conditions can be satisfied precisely throughout an elastic region. The concentrated area of a flange of the equivalent sandwich section in an elastic region is given by

$$A_e H = Z_e \dots \dots \dots (1)$$

where Z_e denotes the elastic section modulus. The concentrated area of the approximately equivalent sandwich section associated with the stress distribution shown in Fig. 1 (b), is given by

$$A_p = \left[\frac{\sigma_F}{\sigma_y} \right] (B - t_1) \left[1 - \frac{t_2}{H} \right] t_2 + \left[\frac{\sigma_R}{\sigma_y} \right] (4 - \pi) \left[\frac{1}{2} - \frac{t_2 + c}{H} \right] R^2 + \frac{1}{2} f_w t_1 H \dots \dots \dots (2a)$$

Fig.-1 Idealized Stress Distribution in a Steady-state

The symbols appearing in this equation are defined in Fig. 1. The average stresses σ_F in a flange and σ_R in a round corner may be evaluated by the standard Taylor expansion of $\epsilon(\sigma)$ about $\sigma = \sigma_y$ as follows :

$$\frac{\sigma_F}{\sigma_y} = 1 - \frac{(1+r)t_2}{(1+nr)H}, \quad \frac{\sigma_R}{\sigma_y} = 1 - \frac{2(1+r)(t_2+c)}{(1+nr)H} \dots \dots \dots (2b, c)$$

where $r = \epsilon_{py}/\epsilon_y$, as was defined in Part II.

The third term representing the contribution of the web stresses to the resultant moment may be

evaluated by an integration similar to that employed in Ref. [1, 2] and f_w is given by

$$f_w = \frac{1}{3(1+r)^2} \left\{ 2 + \frac{3(1+n)r}{n+2} + \frac{3nr^2}{2n+1} \right\} + \frac{1}{2(1+r)^2} \left[\frac{\sigma_m}{\sigma_y} \right] \left\{ \frac{1}{3} \left[\frac{\sigma_m}{\sigma_y} \right] + \frac{2r}{n+2} \left[\frac{\sigma_m}{\sigma_y} \right]^{n+1} + \frac{r^2}{2n+1} \left[\frac{\sigma_m}{\sigma_y} \right]^{2n} - 1 \right\} \dots\dots\dots (2d)$$

where

$$\left[\frac{\sigma_m}{\sigma_y} \right] + r \left[\frac{\sigma_m}{\sigma_y} \right]^n = 1 \quad (\sigma_m > 0)$$

The resultant moment corresponding to this stress distribution is denoted by M_y , while that of elastic stress distribution with the extreme fibre stress σ_y by M_0 . The ratio M_y/M_0 is denoted by μ_0 . It is noted that $A_p/A_e = \mu_0$.

The concentrated area of the approximately equivalent sandwich section associated with the stress distribution shown in Fig. 1(d) may also be represented by Eq. (2a) if n is replaced by $\bar{n}$ in Eq. (2b and c) and if f_w is estimated by

$$f_w = \frac{1}{3(1+r)^2} + \frac{\bar{n}}{2\bar{n}+1} \left[1 - \frac{r^3}{(1+r)^2} \right] \dots\dots\dots (2e)$$

The resultant moment corresponding to this stress distribution is denoted by M_s and the ratio M_s/M_0 , by μ_s .

3. STEADY-STATE SOLUTION

3.1 SKELETON LOAD-DEFLECTION CURVE-1 In accordance with the foregoing assumptions and idealization, a beam has the flange area A_e in an elastic region and A_p in a plastic region, and is subjected to alternating bending as a cantilever shown in Fig. 2(c). The coordinate system and symbols to be used are shown in Fig. 2. It is convenient to introduce the following dimensionless quantities :

$$\xi = \frac{x}{L}, \quad \eta = \frac{y}{L}, \quad m = \frac{M}{M_y}, \quad q = \frac{QL}{M_y} \dots\dots\dots (3)$$

The lower flange stresses in the elastic and plastic regions may then be written as

$$\frac{\sigma}{\sigma_y} = \frac{A_p}{A_e} \frac{\sigma A_e H}{\sigma_y A_p H} = \mu_0 m = \mu_0 \frac{Qx}{M_y} = \mu_0 q \xi \dots\dots\dots (4)$$

$$\frac{\sigma}{\sigma_y} = \frac{\sigma A_p H}{\sigma_y A_p H} = m = q \xi \dots\dots\dots (5)$$

respectively. The curvature-displacement relation may be reduced to

$$\kappa L = L \frac{d^2 y}{dx^2} = \frac{d^2 \eta}{d\xi^2} = \eta'' \dots\dots\dots (6)$$

where a prime denotes differentiation with respect to ξ .

After a steady-state has been attained in a few cycles of alternating plastic bending, the state of stress in a flange at a maximum deflection may be considered to be governed by the skeleton stress-strain relation. The variation of the state of stress as the load and deflection are varied, must be considered to be governed by the hysteretic stress-strain relation.

If it may be assumed that no softening occurs in the elastic region and that the skeleton stress-strain relation

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} \left[1 + r \left| \frac{\sigma}{\sigma_y} \right|^{n-1} \right] \dots\dots\dots (7)$$

Fig.-2

which has been derived in Part II, can be applied only to the plastic region in the steady state, then the corresponding skeleton load-deflection relation may be obtained as follows.

The elastic region is defined by

$$0 \leq |M| \leq M_0 \quad \text{or} \quad 0 \leq \xi \leq 1/|q|\mu_0 \quad (8)$$

The differential equation governing the deflection η_e in the elastic region may be written, in view of Eq. (4), as

$$-\frac{\eta_e''}{\kappa_y L} = \frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} = \mu_0 q \xi \quad (9)$$

Substitution of Eq. (5) into Eq. (7) and use of Eq. (6) provide the differential equation governing the deflection η_p in the plastic region.

$$-\frac{\eta_p''}{\kappa_y L} = \frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} \left\{ 1 + r \left| \frac{\sigma}{\sigma_y} \right|^{n-1} \right\} = q \xi \{ 1 + r |q \xi|^{n-1} \} \quad (10)$$

The general solutions to Eq. (9) and (10) may readily be obtained. Integration constants can be determined by the boundary conditions $\eta_e(0)=0$ and $\eta_p'(1)=0$ and the continuity conditions $\eta_e(1/|q|\mu_0) = \eta_p(1/|q|\mu_0)$ and $\eta_e'(1/|q|\mu_0) = \eta_p'(1/|q|\mu_0)$. The solution $\eta_p(\xi)$ may be written as

$$\eta_p = -\kappa_y L q \left\{ \left(\frac{\xi^3}{6} - \frac{\xi}{2} \right) + \frac{r|q|^{n-1}}{n+1} \left[\frac{\xi^{n+2}}{n+2} - \xi \right] + \frac{1-\mu_0}{3|q|^3\mu_0^3} + \frac{r}{n+2} \frac{1}{|q|^3\mu_0^{n+2}} \right\} \quad (11)$$

The deflection amplitude $\eta_p(1) = \delta_a$ for a load amplitude q_a is given by

$$\delta_a = \kappa_y L q_a \left\{ \frac{1}{3} \left[1 + \frac{\mu_0 - 1}{|q_a|^3 \mu_0^3} \right] + \frac{r|q_a|^{n-1}}{n+2} \left[1 - \frac{1}{|q_a|^{n+2} \mu_0^{n+2}} \right] \right\} \quad (12)$$

($|q_a| \geq 1/\mu_0$)

If the beam is entirely elastic, then $A = A_e$ throughout and the deflection is given by

$$\delta_{ae} = \frac{1}{3} \kappa_y L q_a \mu_0 \quad (13)$$

The plastic component δ_{ap} of the deflection has been called the plastic deflection and is defined as the difference $\delta_a - \delta_{ae}$. The expression of δ_{ap} may be reduced to

$$\delta_{ap} = \kappa_y L q_a \left\{ -\frac{1}{3} (\mu_0 - 1) \left[1 - \frac{1}{|q_a|^3 \mu_0^3} \right] + \frac{r|q_a|^{n-1}}{n+2} \left[1 - \frac{1}{|q_a|^{n+2} \mu_0^{n+2}} \right] \right\} \quad (14)$$

The curve described by Eq. (12) is the skeleton load-deflection curve. It should be noted that Eq. (12) does not represent the load-deflection curve in the usual sense and only gives the load amplitude for a prescribed deflection amplitude. Eq. (11) represents the deflected shape at the state of load and deflection expressed by a tip of a hysteresis curve.

3.2 HYSTERETIC LOAD-DEFLECTION CURVE If the load-deflection curve of a beam starting from a state of deflection ($\delta_a > 0$, $q_a > 0$) on the skeleton load-deflection curve passes through its symmetric point ($-\delta_a$, $-q_a$) on the skeleton curve when subjected to reversed bending and if the state of deformation at ($-\delta_a$, $-q_a$) is the precise reverse of that at (δ_a , q_a), then there exists indeed a closed hysteretic load-deflection curve in the steady-state. The lower flange stress and strain at (δ , q) denoted by $\sigma(\xi)$ and $\varepsilon(\xi)$, respectively are governed by the hysteretic stress-strain relation;

$$\frac{\varepsilon(\xi) - \varepsilon_a(\xi)}{2 \varepsilon_y} = \frac{\sigma(\xi) - \sigma_a(\xi)}{2 \sigma_y} \left\{ 1 + r \left| \frac{\sigma_a(\xi)}{\sigma_y} \right|^{n-n'} \left| \frac{\sigma(\xi) - \sigma_a(\xi)}{2 \sigma_y} \right|^{n'-1} \right\}$$

which has been derived in Part II. If the curvature and bending moment distributions at (δ_a , q_a) and at a subsequent state (δ , q) are denoted by $\kappa_a(\xi)$, $\kappa(\xi)$ and $m_a(\xi)$ and $m(\xi)$, respectively, then the curvature-moment relation may be reduced to

$$\frac{\kappa(\xi) - \kappa_a(\xi)}{2 \kappa_y} = \frac{m(\xi) - m_a(\xi)}{2} \left\{ 1 + r |m_a(\xi)|^{n-n'} \left| \frac{m(\xi) - m_a(\xi)}{2} \right|^{n'-1} \right\} \quad (16)$$

The differential equation governing the deflection $\eta_p(\xi)$ in the plastic region may be written as

$$\eta_p'' - \eta_a'' = -2 \kappa_y L \left[\frac{q - q_a}{2} \right] \left\{ \xi + r q_a^{n-n'} \left| \frac{q - q_a}{2} \right|^{n'-1} \xi^n \right\} \quad (17)$$

where η_a denotes the deflection function at (δ_a , q_a). It should be noted that the power of ξ in the nonlinear term is again n . The general solutions to Eq. (17) and for the elastic region may readily

be obtained. Use of the boundary and continuity conditions in the general solutions furnishes the deflection at a subsequent state (δ, q) and the hysteretic load-deflection relation as follows :

$$\eta_p - \eta_a = -2 \kappa_y L \left[\frac{q - q_a}{2} \right] \left\{ \left[\frac{\xi^3 - 3\xi}{6} - \frac{\mu_0 - 1}{3 q_a^3 \mu_0^3} \right] + r q_a^{n-n'} \left| \frac{q - q_a}{2} \right|^{n'-1} \left[\frac{1}{(n+1)} \left(\frac{\xi^{n+2}}{n+2} - \xi \right) + \frac{1}{(n+2) q_a^{n+2} \mu_0^{n+2}} \right] \right\} \dots\dots\dots (18)$$

$$\delta - \delta_a = \kappa_y L (q - q_a) \left\{ \frac{1}{3} \left[1 + \frac{\mu_0 - 1}{q_a^3 \mu_0^3} \right] + \frac{r q_a^{n-n'}}{n+2} \left| \frac{q - q_a}{2} \right|^{n'-1} \left[1 - \frac{1}{q_a^{n+2} \mu_0^{n+2}} \right] \right\} \dots\dots\dots (19)$$

$(q_a \geq 1/\mu_0)$

It may readily be confirmed that the curve defined by Eq. (19) passes through the point $(-\delta_a, -q_a)$ and that $\eta_p(\xi) = -\eta_a(\xi)$ at $(-\delta_a, -q_a)$. Similarly the load-deflection curve starting from $(-\delta, -q_a)$ may be shown to pass through (δ_a, q_a) . Eq. (19) gives therefore a half of the steady-state hysteresis loop and the other half may be described by the equation similar to Eq. (19) in which $\delta - \delta_a$ and $q - q_a$ are replaced by $\delta + \delta_a$ and $q + q_a$, respectively. It may therefore be concluded that, once the state of deformation represented by $\pm \eta_a(\xi)$ is attained at $(\pm \delta_a, \pm q_a)$ through the converging process described in the preceding section, the subsequent load-deflection curve is a steady-state hysteresis loop consisting of the curve given by Eq. (19) and the other half. This hysteresis loop is the asymptote to which an actual load-deflection curve will approach after a number of cycles. It has been shown in Part I, that a beam apparently attains a steady state quite rapidly in a few cycles.

3.3 PLASTIC ENERGY DISSIPATION The plastic energy dissipated in a cantilever beam during a steady-state hysteretic cycle can be evaluated with the use of Eqs. (12) and (19). If the new coordinates $\bar{q} = q - q_a$ and $\bar{\delta} = \delta - \delta_a$ with the origin at the tip (δ_a, q_a) of a hysteresis loop are used, then the area d_p enclosed by the hysteresis loop may be represented by

$$d_p = M_y \left[4 q_a \delta_a - 2 \int_0^{-2q_a} \bar{\delta} d\bar{q} \right] \dots\dots\dots (20)$$

Substitution of Eq. (12) and Eq. (19) into Eq. (20) provides

$$d_p = \kappa_y L M_y \left[\frac{4r}{n+2} \right] \left[\frac{n'-1}{n'+1} \right] \left[q_a^n - \frac{1}{q_a^2 \mu_0^{n+2}} \right] q_a \dots\dots\dots (21)$$

3.4 $\delta_{ap} \sim N_c$ RELATION For a prescribed δ_a , Eq. (12) determines q_a and hence the strain at any point may be calculated by Eq. (5) and Eq. (7) or by Eq. (10). The largest extreme fiber strain for a prescribed δ_a occurs at $\xi=1$ and the total and plastic strain amplitudes are given by

$$\epsilon_a = \epsilon_y (q_a + r q_a^n) \dots\dots\dots (22a)$$

$$\epsilon_{ap} = \epsilon_{py} q_a^n \dots\dots\dots (22b)$$

respectively, in tension and compression. If these estimates of the largest strains are accurate and if the $\epsilon_{ap} \sim N_c$ relation, Eq. (7) of Part II may be applied also to ϵ_{ap} here, an idealized but approximate $\delta_{ap} \sim N_c$ relation can be derived therefrom.

3.5 SKELETON LOAD-DEFLECTION CURVE-2 If the skeleton stress-strain relation given by Eq. (5) of Part II :

$$\frac{\epsilon}{\epsilon_y} = (1+r) \left[\frac{\sigma}{\sigma_y} \right] \left[\frac{\sigma}{\sigma_y} \right]^{\bar{n}-1}$$

is applied to the present problem, the corresponding stress distribution at a section in a plastic region is as shown in Fig. 1 (d). Through a manipulation similar to that of Section 3.1, the skeleton load-deflection relation may be obtained and reduced to

$$\delta_a = \kappa_y L q_a \left\{ \left[\frac{1+r}{\bar{n}+2} \right] q_a^{\bar{n}-1} + \left[\frac{1}{3 \mu_s^2} - \left(\frac{1+r}{\bar{n}+2} \right) \frac{1}{\mu_s^{\bar{n}+2}} \right] / q_a^3 \right\} \dots\dots\dots (23)$$

where $q_a = Q_a L / M_s > 0$. The largest absolute extreme fibre strain must now be estimated by Eq. (5) of Part II rewritten above.

$$\epsilon_a = \epsilon_y (1+r) q_a^{\bar{n}}, \quad \epsilon_{ap} = \epsilon_y (1+r) q_a^{\bar{n}} - \epsilon_y q_a \dots\dots\dots (24)$$

4. NUMERICAL RESULT AND DISCUSSION

Table 1 summarises the cross-sectional properties, material properties and some nondimensionalized

Table 1

Specimen No.	Series A						Series B				
	11	12	13	14	15	16	17	18	19	20	21
M_p/σ_y (cm ³)	91.4	92.6	91.5	92.0	90.4	84.8	89.4	88.3	88.6	89.3	88.9
M_p (tcm)	264	258	265	258	268	238	256	249	254	251	251
σ_y (t/cm ²)	2.89	2.79	2.90	2.81	2.97	2.80	2.87	2.82	2.87	2.81	2.83
ϵ_y (x10 ⁻³)	1.38	1.33	1.38	1.34	1.41	1.33	1.37	1.34	1.37	1.34	1.35
ϵ_{yp} (x10 ⁻³)	2.87	2.36	2.90	2.47	3.23	2.38	3.35	3.20	3.35	3.13	3.25
r	2.08	1.77	2.10	1.84	2.29	1.79	2.52	2.38	2.52	2.34	2.41
n	5.5						4.3				
M_y/σ_y (cm ³)	88.1	89.2	88.1	88.7	87.1	81.9	86.1	85.1	85.4	85.9	85.6
M_y (tcm)	255	249	256	246	259	229	247	240	245	241	242
M_o/σ_y (cm ³)	79.9	80.7	79.7	80.3	78.7	74.5	78.0	77.1	77.2	77.8	77.5
μ_o	1.103	1.105	1.106	1.104	1.107	1.100	1.103	1.103	1.106	1.103	1.105
Q_a (ton)	1.45	1.49	1.58	1.57	1.66	1.54	1.56	1.63	1.65	1.50	1.44
$q_a=QL/M_y$	1.023	1.079	1.108	1.149	1.151	1.211	1.135	1.222	1.210	1.116	1.072
Δ_a (mm)	50	60	75	80	90	100	75	90	100	60	50
δ_a	0.0278	0.0333	0.0416	0.0445	0.0500	0.0555	0.0416	0.0500	0.0555	0.0333	0.0278
δ_{ap}	0.0089	0.0148	0.0217	0.0248	0.0289	0.0345	0.0213	0.0286	0.0338	0.0138	0.0087
$\bar{n}$	4.5						3.8				
r_s	0.723	0.746	0.719	0.746	0.701	0.745	0.640	0.650	0.640	0.661	0.647
M_s/σ_y (cm ³)	88.5	89.5	88.4	89.1	87.5	82.1	86.0	85.0	85.2	85.9	85.5
M_s (tcm)	256	250	256	249	260	230	247	240	244	241	242

data from the test result of all the H-specimens. The elastic and plastic section modulae have been computed from the cross-sectional dimensions which were measured by vernier calipers and micrometers. The yield stress may vary from specimen to specimen even within the same series. The tension-compression tests were conducted only on two series of plate specimens cut out of two different H-specimens. Since the virgin load-deflection curve of every H-specimen exhibited a perfectly-plastic behavior, however, the yield stresses were computed from the fully-plastic moments which were considered to have been attained. The yield stress of B-17 so computed is 2.87 t/cm², while the average lower yield stress obtained from the tension-compression tests on B-17 plate specimens was 2.94 t/cm². Since the material skeleton curves have been obtained only on one H-specimen in a Series in Part II, it is assumed here that all the H-specimens in a Series have the same material skeleton curve. From Fig. 5 of Part II, ϵ_{py} may then be read for each yield stress and $r=\epsilon_{py}/\epsilon_y$ may be computed. The values of M_y/σ_y and M_s/σ_y were computed with the use of Eqs. (2) and measured cross-sectional dimensions. The averages of a pair of measured load amplitudes and plastic deflection amplitudes, respectively are also included in Table 1 as Q_a and Δ_{ap} together with their dimensionless values.

The theoretical skeleton load-deflection curves for each H-specimen may be drawn with the use of Eqs. (14) and (23). Figs. 3 (a) and (b) show the log-log plots of the theoretical $q_a \sim \delta_{ap}$ relations and the corresponding experimental points for Series A and B, respectively. Since M_s is very close to M_y for all the H-specimens as shown in Table 1, an experimental load amplitude normalized with respect to M_s/L has been regarded, in the scale of Fig. 3, to be the same as the amplitude normalized with respect to M_y/L . It is apparent that both of the theoretical curves predicted by Eq. (14) and (23) which are different from specimen to specimen fit the experimental points very well mostly within a few percent deviations and that the predictions due to Eq. (23) are slightly closer to experimental points.

Fig. 4 shows three theoretical hysteretic load-deflection curves given by Eq. (19) and the corresponding experimental curves to be compared with. Figs. 5 (a) and (b) show some theoretical $[q-$

Fig.-3 a, b

Fig.-4

q_a]- $[\delta-\delta_{ap}]$ curves in log-log scale to compare with nondimensionalized experimental points. Apparently, the theoretical curves predict the experimental result considerably well except near the shoulders which are sharper than those of experimental curves due to the sandwich approximation.

The plastic energy dissipated in a hysteretic cycle has been computed by means of Eq. (21) in the dimensionless form and plotted in Fig. 6. Since the theoretical hysteresis curves have sharper shoulders than the experimental curves as has been pointed out above, Eq. (21) overestimates the plastic energy. It is observed that the theoretical predictions are considerably close to the straight lines fitting the experimental points and that the deviations are mostly about 10% of the experimental values. Although no simple relation between d_p/M_y and δ_{ap} can be derived from Eq. (14) and Eq. (21), it has been observed in the process of computation that the theoretical result is fairly close to the evaluation by means of the second term of Eq. (14), which is given by

Fig.-5 a, b

Fig.-6

Fig.-7

$$\frac{d_p}{M_y} = 4 \left[\frac{n'-1}{n'+1} \right] \delta_{ap} q_a \dots \dots \dots (25)$$

Strains were measured at the sections located at 6 cm from the respective midspans. The theoretical estimates of strains must therefore be made for $\xi=0.967$. The solid and dot-dash lines in Fig. 7 are the theoretical estimates due to Eq. (10) and Eq. (24), respectively. Since the experimental points for specimens B-18 and B-19 show considerable scatter, it is difficult to estimate the essential deviation of the theoretical values from the experimental values. It may, however, be observed that the deviations from the average values of the experimental points are roughly within 20% of the average values.

If the largest extreme fibre strain may be assumed to obey the plastic fatigue ductility relation, Eq. (7) in Part II, then the theoretical $\delta_{ap} \sim N_c$ relations may be obtained for all the H-specimens. The solid and dot-dash lines in Fig. 8 show the $\delta_{ap} \sim N_c$ curves computed from Eqs. (14) and (22)

Fig.-8

and from Eqs. (23) and (24), respectively. Apparently the difference between the two estimates is only slight. These idealized $\delta_{ap} \sim N_c$ relations are based upon, among others, the assumptions that no local or lateral torsional deformation which would increase local strains occurs and that the effect of stress and strain gradients is of a second-order quantity. When the idealized $\delta_{ap} \sim N_c$ curves are compared with the experimental $\delta_{ap} \sim N_{HF}$ relations, it must be cautioned that the definition of N_{HF} is different from that of N_c . It has already been pointed out in Part I that a number of hair cracks had been observed long before a deep crack penetrated through a flange of a H-specimen. The detection of haircracks was made only with naked eyes and was not of a uniform accuracy. In spite of these reservations, the idealized $\delta_{ap} \sim N_c$ curves in Fig. 8 have revealed some features of the fatigue ductility of wide-flange members and explained partly the tendency that a pair of H-specimens with different skeleton stress-strain relations exhibit different fatigue ductility relations even when subjected to the same deflection amplitude. Because of the difference of definitions of N_c and N_{HF} , it may be of interest to draw $\delta_{ap} \sim N_c$ curves for the longitudinal strains at the inner sides of flanges. If these strains may be estimated from the linear distribution over a cross-section, the curves may be obtained by a simple translation of the previous $\delta_{ap} \sim N_c$ curves as shown by the dashed lines in Fig. 8.

5. CONCLUSION

The following conclusions may be drawn from the result :

(1) The steady-state theory proposed here as a first-order approximation is able to predict with a considerable accuracy the steady-state load-deflection behaviors of rolled wide-flange members subjected to alternating forced displacements. For specimens subjected to larger deflection amplitudes, the theory is able to predict the initial steady-state behaviors more accurately than the later deteriorated behaviors due to the development of local and lateral torsional deformations. The theoretical skeleton curves based upon the two different skeleton stress-strain relations fit the experimental points very well mostly within a few percent deviations. The theoretical hysteresis curves also predict the experimental curves considerably well except near the sharper shoulders.

(2) The present theory is able to estimate the plastic energy dissipated during a steady-state hysteretic cycle also with a considerable accuracy. The order of magnitude of the overestimation due to the sharper shoulders mentioned above is about 10% of respective experimental values.

(3) Because of the scatter in measured strain values, it was difficult to assess the deviations of the theoretical predictions from the experimental values. It may, however, be stated that the devia-

tions are about 20% from the respective averages of the measured strains.

(4) The idealized $\delta_{ap} \sim N_c$ relations derived from the present theory can not be directly compared with the experimental $\delta_{ap} \sim N_{HF}$ relations because of the difference of definitions of N_c and N_{HF} and because of the variations of strain amplitudes due to local buckling and lateral torsional deformations which were developed in specimens subjected to larger deflection amplitudes. Nevertheless the idealized $\delta_{ap} \sim N_c$ curves appear to have accounted partly for the fact that a pair of H-specimens of different steels with different skeleton stress-strain relations exhibit different fatigue ductility even when subjected to the same forced deflection amplitude.

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交番塑性曲げをうける H 型鋼梁の荷重変位挙動と塑性疲労 (梗概)

— 第Ⅲ部 定常状態理論 —

正 会 員	棚	橋	諒 ¹⁾
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1. 序

本論文の目的は、定常状態交番塑性曲げの概念と、近似等価サンドウィッチ断面の概念に基いて、定常状態荷重変化挙動とエネルギー吸収能力に対する近似解析解を導く事にある。第Ⅰ部で得られた実験結果から見て、近似等価サンドウィッチ断面のフランジは、定常状態交番塑性曲げにおいて、第1近似としては完全両振定歪振巾引張圧縮をうけるものと仮定してよい。従って第Ⅱ部で導かれた骨格及び履歴応力歪関係式が適用される。本理論から得られる数値計算結果が、第Ⅰ部の実験で得られた荷重変位挙動と1サイクル当り消費エネルギーをかなりよく予測し得ている事が示される。絶対最大歪振巾も本理論で予測され得るので、第Ⅱ部で導いた引張圧縮塑性疲労延性関係式がH型鋼供試体のラフンジにも適用され得るものと仮定すれば、塑性曲げ疲労延性関係式の予測も可能である。

2. 近似解析の仮定

第Ⅰ部 Fig. 5, 6 に示される曲げ歪分布は定常状態曲げ歪分布の存在を示している。第Ⅰ部において指摘したように、第1近似としては、定常状態曲げ歪分布が上方及び下方曲げのいずれにも等しい振巾を有するものと仮定する事は妥当である。この仮定は、最大あるいは最小変位状態における定常状態応力分布が、第Ⅱ部で導いた骨格応力歪曲線で支配される事を意味する。第Ⅱ部(1)式で与えられる骨格応力歪曲線への収束過程において、降伏歪迄は弾性応答が軟化現象を示さないと仮定した場合の塑性域内の断面の応力分布は Fig. 1 (b) に、第Ⅱ部(5)式に基づく応力分布は Fig. 1 (d) に示す。

理想化サンドウィッチ断面は、垂直応力のみを支持する集中断面積の両フランジと、垂直応力を伝えずせん断にのみ抵抗するコアとから成る。サンドウッチ断面は、もし(1)縁歪 $\pm \epsilon_a$ と曲率 κ が同一で、(2)対応する縁応力 $\pm \sigma_a$ と曲げモーメントが同一であるならば、元のH型鋼断面と等価であると称される。この等価断面は一般にその非線型応力分布に依存する。故に非線型応力分布の如何に拘らず塑性域全体に適用され得る近似等価

サンドウィッチ断面を定義すると好都合である。この近似等価サンドウィッチ断面とは、塑性域内で $\sigma_a = \sigma_y$ で且 $\epsilon_a = \epsilon_y + \epsilon_{py}$ である特定の断面においてのみ等価条件をすべて満足しているように等価集中断面積が決定された断面と定義される。この特定断面以外は正確には等価条件が満足されないという意味で近似等価と称する。

等価条件は弾性域では正確に満足され、フランジ集中断面積は(1)式で与えられる。塑性域では前述の定義から、近似等価集中断面積は(2a~d)式で与えられる。この時の応力分布に対応するモーメントを M_y 、弾性限モーメントを M_0 で記す。また、 M_y/M_0 の比を μ_0 と記す。Fig. 1 (d) に示す応力分布に基づく近似等価サンドウィッチ断面のフランジ集中断面積は(2b, c)式の n を $\bar{n}$ でおきかえて f_w を(2e)式で評価すれば、やはり(2a)式で与えられる。この時の応力分布に対応するモーメントを M_s 、 M_s/M_0 の比を μ_s で表わす。

3. 定常状態解

3.1 骨格荷重変位曲線-1 前記の仮定と理想化によって、H型鋼梁は弾性域ではフランジ面積 A_e 、塑性域では A_p を有するサンドウィッチ梁として扱われる。Fig. 2 のような座標系と記号を用い、(3)式の無次元量を用いる。Fig. 2 (a) の片持梁の弾性域、塑性域の下フランジ応力は夫々(4),(5)式で与えられる。定常状態に達して後の最大変位位置におけるフランジ応力状態は骨格応力歪曲線で支配されると考えられる。弾性域では繰返軟化現象を示さないと仮定すると、第Ⅱ部で導かれた骨格応力歪関係式(7)は塑性域にのみ適用される。(4),(5),(6)式から、弾性域と塑性域に対するたわみ曲線の微分方程式は夫々(9),(10)式で与えられる。境界条件と連続条件を満足する解は(11)式で、変位振巾と荷重振巾の関係は(12)式で与えられる。(13)を(12)式から引けば(14)式の塑性変位振巾の表現を得る。

3.2 履歴荷重変位曲線 上記の骨格荷重変位曲線上の一点($\delta_a > 0$, $q_a > 0$)で表わされる変形状態から発する荷重変位曲線がその骨格曲線上の対称点($-\delta_a$, $-q_a$)を通過するならば、そして($-\delta_a$, $-q_a$)における変形状態が丁度(δ_a , q_a)の変形状態を表わす関数に負号を付した関数で表わされるならば、確かに閉じた定

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常状態履歴ループが存在する。この際の塑性域内の断面のフランジの応力、至は第Ⅱ部で導いた履歴応力歪関係式で支配され、対応する曲率モーメント関係式は(16)式で与えられる。塑性域のたわみ曲線の微分方程式は(17)式で、境界条件と連続条件を満足する解は(18)式で夫々与えられる。履歴荷重変位曲線は(19)式で表現され、確かに骨格曲線上の対称点を通る事、且その変位位置で確かに負号を付した関数で表わされる変形状態になっている事が確められる。従って前節に述べた収束過程を経て骨格曲線上の一点で表わされる変形状態に達したならば、以後の荷重の荷重変位曲線は(19)式で表わされる曲線と他半分から成る閉じた履歴ループである。この履歴ループは、現実の荷重変位曲線が何サイクルかの後に近づく漸近曲線である。

3.3 塑性エネルギー消費 (12)式と(19)式に基いて定常状態1履歴サイクル中に片持梁の中で消費される塑性エネルギーは(20)式で評価され得る。

3.4 $\delta_{ap} \sim N_c$ 関係式 指定された変位振巾に対する荷重振巾が(12)式で求められると、(5),(7)式から至も評価できる。最大縁歪とその塑性成分とは(22a, b)式で与えられる。もしこの歪評価が正確で、且ここで求められた縁歪の塑性成分に対して第Ⅱ部の(7)式の塑性歪振巾一亀裂発生回数関係式が適用されてよいならば、塑性変位振巾一亀裂発生関係式の近似式が得られる。

3.5 骨格荷重変位曲線-2 第Ⅱ部の(5)式の骨格応力歪関係式がこの問題に適用されるならば、骨格荷重変位曲線は(23)式の形に帰せられ、最大絶対縁歪とその塑性成分は(24)式で与えられる。

4. 数値計算結果とその検討

表1は、すべてのH型鋼供試体の断面性状、材料性質及び実験結果の無次元化データを示す。降伏応力は供試体毎に異なるが、引張圧縮試験は2体のH型鋼供試体から採取した板状試験片について行ったのみである。他方処女荷重変位曲線が完全塑性挙動を示したので全塑性モーメント値に達したものとして降伏応力を逆算した。材料の骨格応力歪曲線はシリーズ毎に同じと仮定した。

理論的な骨格荷重変位曲線は、各H供試体について、(14),(23)式を用いて描く事ができる。Fig. 3(a), (b)は夫々、Series A, B に対する理論的骨格荷重変位曲線と実験点の両対数プロットを示す。表1に示すように、すべてのH供試体の M_s の値は夫々、対応する M_y の値に非常に近いので、Fig. 3のスケールでは M_s/L を M_y/L と同じと見なしてよい。(14),(23)式で予測された供試体毎に異なる理論的骨格荷重変位曲線は、殆んどの実験値に対して数%の差を認めるに過ぎない。

Fig. 4は(19)式で与えられる3個の履歴荷重変位曲線と対応する実験履歴ループを示す。Fig. 5(a), (b)

は、無次元化された実験点と比較すべき理論的履歴荷重一塑性変位曲線の両対数紙上表示である。明らかに、理論曲線はやや鋭い肩部分を除いて、実験結果をかなりよく予測し得ている。

1履歴サイクル当り消費される塑性エネルギーは(21)式で計算される。Fig. 6に図示した理論的評価は実験点を結ぶ直線とかなり近く、その過大評価の程度は約10%である。1サイクル当り消費エネルギーと塑性変位振巾との間の関係は簡単な表現に帰せられないが、(25)式の評価にかなり近い。

スパン中央から6cm離れた断面において計測された至と(10)式(24)式による理論的予測値の比較をFig. 7に示す。計測値が等しい歪振巾でないために、理論値と実験値の差を表現する事は困難であるが、計測歪の平均値からの理論値の偏差は約20%程度と云える。

最大縁歪が第Ⅱ部(7)式の塑性疲労延性関係式に従うと仮定するとすべてのH型鋼供試体の曲げ塑性疲労延性関係式($\delta_{ap} \sim N_c$)が得られる。(14),(22)式と(23),(24)式から計算された $\delta_{ap} \sim N_c$ 曲線はFig. 8で実線と鎖線で示されている。これらの理想化された $\delta_{ap} \sim N_c$ 関係式が、実験で得られた $\delta_{ap} \sim N_{HF}$ 関係式と比較される際に注意すべきは、 N_c と N_{HF} の定義の差、大変位振巾時の局部座屈、横振れ変形に伴う歪の局部的増大による疲労寿命の低下等である。これらの条件にも拘らず、 $\delta_{ap} \sim N_c$ 関係曲線は実験結果の傾向を或る程度解明し、骨格応力歪曲線の差が、曲げ塑性疲労延性の差を説明し得る一因子である事を示した。

5. 結 論

(1) 第1近似としてここに提案された定常状態理論は、交番強制定変位を課せられるH型鋼部材の定常状態荷重変位挙動をかなりの精度で予測し得る。2個の異なる骨格応力歪関係式に基づく骨格荷重変位曲線は、実験点に対して多くは、数%程度の偏差で非常によく合う。理論的履歴荷重変位曲線は、やや鋭い肩部以外は実験曲線をかなりよく予測し得ている。

(2) 本理論はまた定常状態の一履歴サイクル当り消費される塑性エネルギーをかなりの精度で評価し得る。過大評価の程度は約10%である。

(3) 実験で計測された至の平均値に比較して、同位置の歪の理論的予測値の偏差は20%程度である。

(4) 本理論から導かれた $\delta_{ap} \sim N_c$ 関係は前節で述べた理由により実験で得られた $\delta_{ap} \sim N_{HF}$ 関係と直接比較され得ないが、理想化された $\delta_{ap} \sim N_c$ 関係は、異なる骨格応力歪曲線を有する異なるSS41鋼材の一对のH型供試体が同一の変位振巾の試験でも異なる塑性曲げ疲労延性を示した事実の理由を部分的に明らかにしたと思われる。

$2m$: The Crystal Size
 λ : The Wave Length

U.D.C. 69.022.327 : 539.4

THEORY OF THE CHAIN CONSTRUCTION WITH ELASTIC JOINTS (6)

Limit Analysis of the Chain Plate (1)

by HOKATO EGAMI, Assis. Prof. of Fukui Univ. Member of A.I.J.

In the previous paper, the author reported the theory of the Chain Construction with elastic joints from the results of the experimental study on the Sandwich Curtain Wall. It was shown that the theory could be applied to the wall with close approximation in its vertical direction. From the results of the other experimental study, it was shown that the theory could be applied to the each and every construction with elastic joints (Chain Plate).

The present paper shows that this theory could be applied to the new Building Element consisted of slab unit and ceiling unit, by the experimental study on this Building Element. And approachability of limit analysis on the Building Element are found to be possible as the result of an experiment on the element with special joints.

U.D.C. 624.042.7 : 624.043

STUDY ON ASEISMATIC ELASTIC DESIGN BY STATISTICAL METHOD

by YOSHIKAKI NAKAO, Mizubushi Heavy Industries Ltd., Member of A.I.J.

Such steel structures as steel stacks, boiler frames, etc. are usually designed by the elastic theory. The author tried to establish the method of the aseismatic elastic design by statistical method.

First he derived the formulae to estimate the response of general structures to the ground motion of two perpendicular directions, assuming that the seismic phenomena is a steady, Gaussian, and ergodic process. These formulae are proved to have sufficient accuracy by the random shaking table tests for the models of several framed structures. Second he applied the theory to the actual seismic waves, introducing the concept of the average earthquake continuation time.

Finally a practical response spectrum is proposed, which covers the whole response range to the seismic wave recorded in the past.

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LOAD-DEFLECTION BEHAVIORS AND PLASTIC FATIGUE OF WIDE-FLANGE BEAMS SUBJECTED TO ALTERNATING PLASTIC BENDING

—Part III, Steady-State Theory—

(See Page 35)

by Dr. RYO TANABASHI, Prof. of Kyoto Univ., Dr. YOSHITSURA YOKOO, Prof. of Kyoto Univ. and TSUNEYOSHI NAKAMURA, Assistant Prof. of Kyoto, Members of A.I.J.