CONVERGENCE OF FOURIER SERIES SOLUTIONS IN THE ELASTIC ANALYSES OF THE SHEAR WALLS SUBJECTED TO THE VARIOUS EXTERNAL PLANE FORCES

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Introduction

A series of the elastic analysis of the shear walls by Airy's stress function have been reported^{1)~3)} by Dr. M. Tomii and the author. These were analysed by using the stress function which consists of the series and the polynomials. The stresses, displacements, etc. were expanded into the Fourier series, and then these values are approximated. In this paper, before the applications of these analyses, the accuracy of the calculated values of the stresses, displacements, etc. is examined.

The quarter part ($x \geq 0, y \geq 0$) of each shear wall subjected to the fundamental plane forces and moments :

I. which are antimetric with regard to the longitudinal and transversal center lines of the shear walls,

II. which are symmetric with regard to the longitudinal and transversal center lines of the shear walls,

III. which are antimetric with regard to the longitudinal center line and symmetric to the transversal center line of the shear wall, were analysed.

These shear walls were called shear wall I, II, and III.

The author shows the continuity conditions between the wall and frame as follows. There are four elements which have satisfied their continuity conditions on each boundary between the wall and beam, and the wall and column. These are the shearing force, the normal force, the unit elongation parallel to the boundary, and the normal displacement. Two (shearing force, normal displacement) of the four elements on the boundaries of the wall were given to the frame, and the remaining two elements (normal force, unit elongation parallel to the boundary) on the boundaries of the frame were derived, and the two equations for the continuity conditions of the remaining two elements were made on the boundaries. These equations were derived in such a way as to develop each element on the wall and frame into the Fourier series. As these two equations are taken into account by the limited number (r_m, r_n) of terms of the Fourier series, the continuity conditions are not precisely satisfied.

In this paper, the author shows the relations between the convergence and the number of terms of the Fourier series in the analyses of the shear wall I, and II.

The definitions of the shear wall (see Fig. 5.1) in this paper are the same as the shear walls which were used in the numerical examples in the previous papers^{1)~3)}. The stress distributions in the wall and frame, and the displacements of the shear wall have been reported in the previous papers^{1)~3)}.

§ 5.1 The convergence and the number of the terms of the Fourier series in the analysis of the shear wall (see Fig. 5.2) loaded antimetrically with regard to x -axis and y -axis respectively

The continuity conditions on the boundaries between the wall and frame

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5.2 b) are shown in Figs. 5.3 a, 5.4 a, 5.5 a for the three numbers ($r_m=r_n=5, 20, 45$) of the terms of the Fourier series, and those of the unit elongations (each left side of eqs. 5.4 a, 5.4 b, 5.5 a, 5.5 b) are shown in Figs. 5.3 b, 5.4 b, 5.5 b for the three numbers ($r_m=r_n=5, 20, 45$) of the Fourier series.

The errors by the partial sum in the Fourier expansion are derived from the Bessel's inequality as follows :

Fig. 5.3 a Continuity conditions of the normal stresses (per unit length) on the boundaries between the wall and frame ($r_m, r_n=5$)

Fig. 5.3 b Continuity conditions of the unit elongations parallel to and on the boundaries between the wall and frame ($r_m, r_n=5$)

Fig. 5.4 a Continuity conditions of the normal stresses (per unit length) on the boundaries between the wall and frame ($r_m, r_n=20$)

Fig. 5.4 b Continuity conditions of the unit elongations parallel to and on the boundaries between the wall and frame ($r_m, r_n=20$)

Fig. 5.5 a Continuity conditions of the normal stresses (per unit length) on the boundaries between the wall and frame ($r_m, r_n=45$)

Fig. 5.5 b Continuity conditions of the unit elongations parallel to and on the boundaries between the wall and frame ($r_m, r_n=45$)

$$\Phi = \frac{\sum_{m=1}^{2r_m-1} (t \sigma_{ym})^2}{\frac{2}{a} \int_0^a (t \sigma_y|_{y=b})^2 dx} < 1 \quad (\text{for eq. 5.1 a}) \dots\dots\dots (5.7 a)$$

$$\Phi = \frac{\sum_{m=1}^{2r_m-1} (w_{a1m})^2}{\frac{2}{a} \int_0^a (w_{a1})^2 dx} < 1 \quad (\text{for eq. 5.2 a}) \dots\dots\dots (5.8 a)$$

$$\Phi = \frac{\sum_{n=1}^{2r_n-1} (t \sigma_{xn})^2}{\frac{2}{b} \int_0^b (t \sigma_x|_{x=a})^2 dy} < 1 \quad (\text{for eq. 5.1 b}) \dots\dots\dots (5.7 b)$$

$$\Phi = \frac{\sum_{n=1}^{2r_n-1} (w_{b1n})^2}{\frac{2}{b} \int_0^b (w_{b1})^2 dy} < 1 \quad (\text{for eq. 5.2 b}) \dots\dots\dots (5.8 b)$$

$$\Phi = \frac{\sum_{m=1}^{2r_m-1} (\varepsilon_{xm})^2}{\frac{2}{a} \int_0^a (\varepsilon_x|_{y=b})^2 dx} < 1 \quad (\text{for eq. 5.4 a}) \dots\dots\dots (5.9 a)$$

$$\Phi = \frac{\sum_{m=1}^{2r_m-1} (\varepsilon_{a1m})^2}{\frac{2}{a} \int_0^a (\varepsilon_{a1})^2 dx} < 1 \quad (\text{for eq. 5.5 a}) \dots\dots\dots (5.10 a)$$

$$\Phi = \frac{\sum_{n=1}^{2r_n-1} (\varepsilon_{yn})^2}{\frac{2}{b} \int_0^b (\varepsilon_y|_{x=a})^2 dy} < 1 \quad (\text{for eq. 5.4 b}) \dots\dots\dots (5.9 b)$$

$$\Phi = \frac{\sum_{n=1}^{2r_n-1} (\varepsilon_{b1n})^2}{\frac{2}{b} \int_0^b (\varepsilon_{b1})^2 dy} < 1 \quad (\text{for eq. 5.5 b}) \dots\dots\dots (5.10 b)$$

Fig. 5.6 a Convergence of the means of the squares of the errors by expanding the normal stresses(per unit length) on the boundaries between the wall and frame into the Fourier series

Fig. 5.6 b Convergence of the means of the squares of the errors by expanding the unit elongations parallel to and on the boundaries between the wall and frame into the Fourier series

Fig. 5.7 a Convergence and number of the terms of the Fourier series with respect to the bending moment $M_a|_{x=a}$, the shearing force $Q_a|_{x=a}$, the axial tensile force $N_a|_{x=a}$ in the beam

The ratios ϕ in eqs. 5.7 a~5.10 b are shown in Figs. 5.6 a, 5.6 b.

The conditions of the convergence with regard to the bending moment M_a , shearing force Q_a , and axial force N_a of the beam are shown in Fig. 5.7 a, and those with regard to the bending moment M_b , shearing force Q_b , and the axial force N_b , of the column are shown in Fig. 5.7 b.

In Fig. 5.8, the conditions of the convergence with regard to the displacements at the corner of the frame are shown.

§ 5.2 The convergence and the number of the terms of the Fourier series in the analysis of the shear wall (see

Fig. 5.9) loaded symmetrically with regard to the x -axis and y -axis respectively

The continuity conditions on the boundaries between the wall and frame

A. By expanding the normal force $t\sigma_y|_{y=b}$ of the wall, and that w_{a1} of the beam in the Fourier series with regard to $\cos \xi_m (\xi_m = m\pi/2a \cdot x)$:

$$t\sigma_y|_{y=b} \doteq t \sum_{m=2,4,6,\dots} \sigma_{ym} \cos \xi_m + t\sigma_{y0} \dots \dots \dots (5.11 a)$$

$$w_{a1} \doteq \sum_{m=2,4,6,\dots} w_{a1m} \cos \xi_m + w_{a10} \dots \dots \dots (5.12 a)$$

and making the right side of eq. 5.11 a equal to that of eq. 5.12 a, r_m (number of $m=2, 4, 6, \dots$) + 1 equations for the continuity of the normal stresses (per unit length) on the boundary between the wall beam are obtained as follows:

$$t\sigma_{ym} = w_{a1m} \text{ (for each } m=2, 4, 6, \dots) \text{ and } t\sigma_{y0} = w_{a10} \dots \dots \dots (5.13 a)$$

B. By expanding the normal force $t\sigma_x|_{x=a}$ of the wall, and that w_{b1} of the column in the Fourier series with regard to $\cos \eta_n (\eta_n = n\pi/2b \cdot y)$:

$$t\sigma_x|_{x=a} \doteq t \sum_{n=2,4,6,\dots} \sigma_{xn} \cos \eta_n + t\sigma_{x0} \dots \dots \dots (5.11 b)$$

Fig. 5.7 b Convergence and number of the terms of the Fourier series with respect to the bending moment $M_b|_{y=b}$, the shearing force $Q_b|_{y=b}$, the axial tensile force $N_b|_{y=b}$, in the column

Fig. 5.8 Convergence and number of the terms of the Fourier series with respect to the terms of the Fourier series with respect to the displacements u_{ac} , v_{bc} at the corner of the frame

$$w_{b1} \doteq \sum_{n=2,4,6,\dots} w_{b1n} \cos \eta_n + w_{b10} \dots \dots \dots (5.12 b)$$

and making the right side of eq. 5.11 b equal to that of eq. 5.12 b, r_n (number of $n=2, 4, 6, \dots$) + 1 equations for the continuity of the normal stresses (per unit length) on the boundary between the wall and column are obtained as follows :

$$t \sigma_{xn} = w_{b1n} \text{ (for each } n=2, 4, 6, \dots) \text{ and } t \sigma_{x0} = w_{b10} \dots \dots \dots (5.13 b)$$

C. By expanding the unit elongations $\varepsilon_x|_{y=b}$ of the wall, and that ε_{a1} of the beam in the Fourier series with regard to $\cos \xi_m$:

$$\varepsilon_x|_{y=b} \doteq \sum_{m=2,4,6,\dots} \varepsilon_{xm} \cos \xi_m + \varepsilon_{x0} \dots \dots \dots (5.14 a)$$

$$\varepsilon_{a1} \doteq \sum_{m=2,4,6,\dots} \varepsilon_{a1m} \cos \xi_m + \varepsilon_{a10} \dots \dots \dots (5.15 a)$$

and making the right side of eq. 5.13 a equal to that of eq. 5.14 a, r_m equations for the continuity of the unit elongations are obtained as follows.

$$\varepsilon_{xm} = \varepsilon_{a1m} \text{ (for each } m=2, 4, 6, \dots) \text{ and } \varepsilon_{x0} = \varepsilon_{a10} \dots \dots \dots (5.16 a)$$

D. By expanding the unit elongation $\varepsilon_y|_{x=a}$ of the wall, and that ε_{b1} of the column in the Fourier series with regard to $\cos \eta_n$:

$$\varepsilon_y|_{x=a} \doteq \sum_{n=2,4,6,\dots} \varepsilon_{yn} \cos \eta_n + \varepsilon_{y0} \dots \dots \dots (5.14 b)$$

$$\varepsilon_{b1} \doteq \sum_{n=2,4,6,\dots} \varepsilon_{b1n} \cos \eta_n + \varepsilon_{b10} \dots \dots \dots (5.15 b)$$

and making the right side of eq. 5.14 b equal to that of eq. 5.15 b, r_n equations for the continuity of the unit elongations parallel to and on the boundary between the wall and column are obtained as follows.

$$\varepsilon_{yn} = \varepsilon_{b1n} \text{ (for each } n=2, 4, 6, \dots) \text{ and } \varepsilon_{y0} = \varepsilon_{b10} \dots \dots \dots (5.16 b)$$

The continuity condition diagrams of the normal stresses (each left side of eqs. 5.11 a, 5.11 b, 5.12 a, 5.12 b) are shown in Figs. 5.10 a, 5.11 a, 5.12 a for the three numbers ($r_m=r_n=5, 20, 45$) of the terms of the Fourier series, and those of the unit elongations (each left side of eqs. 5.14 a, 5.14 b, 5.15 a, 5.15 b) are shown in Figs. 5.10 b, 5.11 b, 5.12 b for the three numbers ($r_m=r_n=5, 20, 45$) of the Fourier series.

The errors by the partial sum in the Fourier expansion are derived from the Bessel's inequality as follows :

$$\phi = \frac{2(t \sigma_{y0})^2 + \sum_m^{2r_m} (t \sigma_{ym})^2}{\frac{2}{a} \int_0^a (t \sigma_y|_{y=b})^2 dx} < 1 \text{ (for eq. 5.11 a)} \dots \dots \dots (5.17 a)$$

Fig. 5.9 Shear wall subjected to the forces which are symmetric with regard to the x -axis and the y -axis respectively

Fig. 5.10 a Continuity conditions of the normal stresses (per unit length) on the boundaries between the wall and frame ($r_m, r_n=5$)

Fig. 5.10 b Continuity conditions of the unit elongations parallel to and on the boundaries between the wall and frame ($r_m, r_n=5$)

$$\Phi = \frac{2(w_{a10})^2 + \sum_m^{2r_m} (w_{a1m})^2}{\frac{2}{a} \int_0^a (w_{a1})^2 dx} < 1 \quad (\text{for eq. 5.12 a}) \dots\dots\dots (5.18 a)$$

$$\Phi = \frac{2(t\sigma_{x0})^2 + \sum_n^{2r_n} (t\sigma_{xn})^2}{\frac{2}{b} \int_0^b (t\sigma_x|_{x=a})^2 dy} < 1 \quad (\text{for eq. 5.11 b}) \dots\dots\dots (5.17 b)$$

$$\Phi = \frac{2(w_{b10})^2 + \sum_n^{2r_n} (w_{b1n})^2}{\frac{2}{b} \int_0^b (w_{b1})^2 dy} < 1 \quad (\text{for eq. 5.12 b}) \dots\dots\dots (5.18 b)$$

$$\Phi = \frac{2(\varepsilon_{x0})^2 + \sum_m^{2r_m} (\varepsilon_{xm})^2}{\frac{2}{a} \int_0^a (\varepsilon_x|_{y=b})^2 dx} < 1 \quad (\text{for eq. 5.14 a}) \dots\dots\dots (5.19 a)$$

$$\Phi = \frac{2(\varepsilon_{a10})^2 + \sum_m^{2r_m} (\varepsilon_{a1m})^2}{\frac{2}{a} \int_0^a (\varepsilon_{a1})^2 dx} < 1 \quad (\text{for eq. 5.15 a}) \dots\dots\dots (5.20 a)$$

$$\Phi = \frac{2(\varepsilon_{y0})^2 + \sum_n^{2r_n} (\varepsilon_{yn})^2}{\frac{2}{b} \int_0^b (\varepsilon_y|_{x=a})^2 dy} < 1 \quad (\text{for eq. 5.14 b}) \dots\dots\dots (5.19 b)$$

Fig. 5.11 a Continuity conditions of the normal stresses (per unit length) on the boundaries between the wall and frame ($r_m, r_n=20$)

Fig. 5.11 b Continuity conditions of the unit elongations parallel to and on the boundaries between the wall and frame ($r_m, r_n=20$)

Fig. 5.12 a Continuity conditions of the normal stresses (per unit length) on the boundaries between the wall and frame ($r_m, r_n=45$)

Fig. 5.12 b Continuity conditions of the unit elongations parallel to and on the boundaries between the wall and frame ($r_m, r_n=45$)

$$\Phi = \frac{2(\varepsilon_{b10})^2 + \sum_n^{2r_n} (\varepsilon_{b1n})^2}{\frac{2}{b} \int_0^b (\varepsilon_{b1})^2 dy} < 1 \quad (\text{for eq. 5.15 b}) \dots\dots\dots (5.20 b)$$

The ratios Φ in eqs. 5.17 a 5.20 b are shown in Figs. 5.13 a, 5.13 b.

The conditions of the convergence with regard to the bending moment M_a , the shearing force Q_a , and the axial force N_a of the beam are shown in Fig. 5.14 a, and those with regard to the bending moment M_b , the shearing force Q_b , and axial force N_b of the column are shown in Fig. 5.14 b.

Fig. 5.13 a Convergence of the means of the squares of the errors by expanding the normal stresses (per unit length) the between the wall and frame into on the boundaries Fourier series

Fig. 5.13 b Convergence of the means of the squares of the errors by expanding the unit elongations parallel to and on the boundaries between the wall and frame into the Fourier series

Fig. 5.14 a Convergence and number of the terms of the Fourier series with respect to the bending moment $M_a|x=a$, the shearing force $Q_a|x=a$, the axial tensile force $N_a|x=a$, the shearing force $Q_a|x=a$, the axial tensile force $N_a|x=a$, in the beam

Fig. 5.14 b Convergence and number of the terms of the Fourier series with respect to the bending moment $M_b|y=b$, the shearing force $Q_b|y=b$, the axial tensile force $N_b|y=b$, the bending moment $M_b|y=b$, the shearing force $Q_b|y=b$, the axial tensile force $N_b|y=b$, in the column

In Figs. 5.15, the conditions of the convergence with regard to the displacements at the corner of the frame are shown.

Conclusions

The elements (stresses, displacements, etc.) which are derived by integrating the elements (unit stresses, unit elongations) which are used for the continuity conditions on the boundaries between the wall and frame, are obtained with the accurate results when the calculating are performed with the ten terms of the Fourier series. The values of the stresses and displacements for five and for ten terms differ slightly (two per cents).

The mean squares of the errors by the partial sums in the Fourier expansion are less than two per cents with respect to the elements (unit stresses, unit elongations) which are calculated with the twenty terms of the Fourier series.

The author showed the relations of the convergence and the terms of the numbers of the Fourier series with respect to the analyses of the shear wall I and II. It seems unnecessary to investigate the relations of the convergence and terms of the number of the Fourier series with respect to the shear wall III, because the shear wall III is the combination of the shear wall I and II.

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Fig. 5.15 Convergence and number of the terms of the Fourier series with respect to the displacements U_{ac} , V_{bc} , at the corner of the frame

種々な外力を受ける耐震壁の弾性解析における フーリエ級数解の収斂*

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序

著者と富井政英博士はすでに“エアリーの応力関数を用いた耐震壁の弾性解析”を発表してある^{1)~3)}。これらの応力関数は全て三角関数と双曲線関数からなる級数式と代数型の多項式を併用した応力関数を用いて解析してある。

さらにこの応力関数から得られる応力、変形などの式はフーリエ展開されていることからこれらの数値は近似的なものとなっている。本論は、これらの解析を耐震壁に応用するのに先立ってまず計算された数値の誤差について検討を行なうものである。

また作用する外力を

I 縦横軸に関しともに逆対称な外力

II 縦横軸に関しともに対称な外力

III 縦軸に関し逆対称で、横軸に関し対称な外力

に分解し、これらの外力が作用する耐震壁を耐震壁 I, II, III と呼んだ。解析は全て外力の逆対称、あるいは対称条件を考慮して耐震壁の4分の1部分($x \geq 0, y \geq 0$)について解析してある。

既報^{1),2)}に耐震壁の壁板と付帯ラーメンの境界上の連続条件のたて方について詳述してあるが、以下にその概略を述べる。

壁板とはりおよび壁板と柱の各境界において連続条件を満足している四要素（せん断応力、垂直応力、境界線に平行なひずみ度、垂直変位）があった。壁板の周辺における上記四要素のうちの二要素（せん断応力、垂直変位）を付帯ラーメンに与え、付帯ラーメンに関する残りの二要素を算定し、これらを壁板の当該要素との間に連続条件式をたてた。

この連続条件式は有限項のフーリエ展開によって各要素のフーリエ係数を等しく置くという方法によってたてられているのであるから、連続条件は完全には成立しないことになり、この展開誤差は他の要素（応力、変形）にも波及する。本論では耐震壁Iの解析¹⁾におけるフーリエ級数の項数と収斂を、耐震壁IIの解析²⁾におけるフーリエ級数の項数と収斂をそれぞれ §5.1 および §5.2

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に示す。

本論で採用してある耐震壁の形状、断面は図—5.1 に示してあり、既報^{1)~3)}の数値計算例で用いた耐震壁と同じ形状、断面である。

また既報の数値計算例においては、フーリエ級数の項数を10項($r_m, r_n=10$)で計算したときの、壁板内および付帯ラーメンの応力分布、変形などを詳細に示してある。

なお本論中の記号は、文献^{1)~3)}に説明してあるが、本論で始めて出てくる所で説明した。

§5.1 x, y 軸に関してそれぞれ逆対称な外力が作用する場合（図—5.2）の耐震壁の弾性解析におけるフーリエ級数の項数と収斂

壁板と付帯ラーメンの境界上の連続条件

A. 壁板の垂直応力 $t\sigma_y|_{y=b}$ およびはりの垂直応力 w_{a1} をそれぞれ $\sin \xi_m$ に関するフーリエ級数に 5.1 a, 5.2 a 式のように展開し、それぞれの式の右边を等しく置くと壁板とはりの境界上の垂直応力の連続条件式が $m=1, 3, 5, \dots$ の数 r_m 個だけ 5.3 a 式のように作られる。

B. 壁板の垂直応力 $t\sigma_x|_{x=a}$ および柱の垂直応力 w_{b1} をそれぞれ $\sin \eta_n$ に関するフーリエ級数に 5.1 b, 5.2 b 式のように展開し、それぞれの式の右边を等しく置くと壁板と柱の境界上の垂直応力の連続条件式が $n=1, 3, 5, \dots$ の数 r_n 個だけ 5.3 b 式のように作られる。

C. 壁板の境界線方向ひずみ度 $\epsilon_x|_{y=b}$ およびはりの境界線方向ひずみ度 ϵ_{a1} をそれぞれ $\sin \xi_m$ に関するフーリエ級数に 5.4 a, 5.5 a 式のように展開し、それぞれの式の右边を等しく置くと壁板とはりの境界上の境界線方向ひずみ度の連続条件式が $m=1, 3, 5, \dots$ の数 r_m 個だけ 5.6 a 式のように作られる。

D. 壁板の境界線方向ひずみ度 $\epsilon_y|_{x=a}$ および柱の境界線方向ひずみ度 ϵ_{b1} をそれぞれ $\sin \eta_n$ に関するフーリエ級数に 5.4 a, 5.5 a 式のように展開し、それぞれの式の右边を等しく置くと壁板とはりの境界上の境界線方向ひずみ度の連続条件式が $n=1, 3, 5, \dots$ の数 r_n 個だけ 5.6 b 式のように作られる。

以上の壁板と付帯ラーメンの垂直応力（5.1 a~5.2 b 式の左边）の連続条件をフーリエ級数の項数 $r_m=r_n=5$,

20, 45 についてそれぞれ図—5.3 a, 5.4 a, 5.5 a に示し、壁板と付帯ラーメンの境界上の境界線方向ひずみ度(5.4 a~5.5 b 式の左辺)の連続条件をフーリエ級数の項数 $r_m=r_n=5, 20, 45$ についてそれぞれ図—5.3 b, 5.4 b, 5.5 b に示してある。

フーリエ展開における誤差を検討するために、有限フーリエ展開誤差の2乗平均を Bessel の不等式で 5.7 a~5.10 b 式のように示し、垂直応力のフーリエ展開誤差を図—5.6 a に、境界線方向ひずみ度のフーリエ展開誤差を図—5.6 b に示した。図—5.7 a, 5.7 b にははりおよび柱の応力 (M, N, Q) の収斂状態を図—5.8 には付帯ラーメンの変位の収斂状態を示した。

§ 5.2 x, y 軸に関してそれぞれ対称な外力が作用する場合 (図—5.9) の耐震壁の弾性解析におけるフーリエ級数の項数と収斂

壁板と付帯ラーメンの境界上の連続条件

A. 壁板の垂直応力 $t\sigma_y|_{y=b}$ およびはりの垂直応力 w_{a1} をそれぞれ $\cos \xi_m$ に関するフーリエ級数に 5.11 a, 5.12 a 式のように展開し、それぞれの式の右辺を等しく置くと、壁板とはりの境界上の垂直応力の連続条件式が $m=2, 4, 6, \dots$ の数 r_m 個と定数項を等値する条件式 1 個、合計 r_m+1 個だけ 5.13 a 式のように作られる。

B. 壁板の垂直応力 $t\sigma_x|_{x=a}$ および柱の垂直応力 w_{b1} をそれぞれ $\cos \eta_n$ に関するフーリエ級数に 5.11 b, 5.12 b 式のように展開し、それぞれの式の右辺を等しく置くと壁板と柱の境界上の垂直応力の連続条件式が $n=2, 4, 6, \dots$ の数 r_n 個と定数項を等値する条件式 1 個、合計 r_n+1 個だけ 5.13 b 式のように作られる。

C. 壁板の境界線方向ひずみ度 $\epsilon_x|_{y=b}$ およびはりの境界線方向ひずみ度 ϵ_{a1} をそれぞれ $\cos \xi_m$ に関するフーリエ級数に 5.14 a, 5.15 a 式のように展開し、それぞれの式の右辺を等しく置くと壁板とはりの境界線方向ひずみ度の連続条件式が $m=2, 4, 6, \dots$ の数 r_m 個と定数項を等値する条件式 1 個、合計 r_m+1 個だけ 5.16 a 式のように作られる。

D. 壁板の境界線方向ひずみ度 $\epsilon_y|_{x=a}$ および柱の境

界線方向ひずみ度 ϵ_{b1} をそれぞれ $\cos \eta_n$ に関するフーリエ級数に 5.14 b, 5.15 b 式のように展開し、それぞれの式の右辺を等しく置くと壁板と柱の境界線方向ひずみ度の連続条件式が $n=2, 4, 6, \dots$ の数 r_n 個と定数項を等値する条件式 1 個、合計 r_n+1 個だけ 5.16 b 式のように作られる。

以上の壁板と付帯ラーメンの垂直応力(5.11 a~5.12 b 式の左辺)の連続条件をフーリエ級数の項数 $r_m=r_n=5, 20, 45$ についてそれぞれ図—5.10 a, 5.11 a, 5.12 a に示し、壁板と付帯ラーメンの境界上の境界線方向ひずみ度(5.14 a~5.15 b 式の左辺)の連続条件をフーリエ級数の項数 $r_m=r_n=5, 20, 45$ についてそれぞれ図—5.10 b, 5.11 b, 5.12 b に示した。

フーリエ展開における誤差を検討するために、有限フーリエ展開誤差の2乗平均を Bessel の不等式で 5.17 a~5.20 b 式のように示し、垂直応力のフーリエ展開誤差を図 5.13 a に、境界線方向ひずみ度のフーリエ展開誤差を図 5.13 b に示した。図 5.14 a, 図 5.14 b にははりおよび柱の応力 (M, N, Q) の収斂状態を、図—5.15 には付帯ラーメンの節点の変位 (u_{ac}, v_{bc}, θ) の収斂状態を示した。

§ 5.3 結 論

壁板と付帯ラーメンの境界上で連続条件に用いた要素(垂直応力、境界線方向ひずみ度)を積分することによって求められる要素(はりの応力、耐震壁の変形)はフーリエ級数の項数を 10 項として計算すればかなりの精度(3桁)で得られる。なおフーリエ級数の項数が 5 項と 10 項の場合の応力 (M, N, Q) 変形などは 45 項の場合と比べて 2% 程度の差がある。連続条件に用いた要素(垂直応力、境界線方向ひずみ度)に関する有限項のフーリエ展開による部分和の誤差の2乗平均はフーリエ展開項数が 20 項のとき 2% 以内となる。

以上は耐震壁 I, II についてのフーリエ級数の項数と収斂であるが、耐震壁 III は耐震壁 I, II の組合せと考えられるので検討を要しないと思われる。

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ELASTIC-PLASTIC ANALYSIS OF THREE-DIMENSIONAL CONCRETE STRUCTURE BY THE FINITE ELEMENT METHOD (APPLICATION TO AXYSIMMETRIC PRESTRESSED CONCRETE PRESSURE VESSEL)

by **OSAMU ISOBATA**, Research Laboratory of Shimizu Construction Co., LTD, Member of A.I.J.

An orthotropic elasto-plastic material in which elastic modulus and Poisson's ratio may be defined based on those of uniaxial stress-strain characteristics is assumed for the concrete.

Three-dimensional stress-strain relationship matrix $[D_{ep}]$ is derived from that of the anisotropic elastic body and expressed in the form;

$$[D_{ep}] = [A_p]^T \{ [D] + [H_p] \} [A_p]$$

Where $[D]$ is the stress-strain relationship matrix of the homogeneous elastic body. $[A_p]$ and $[H_p]$ are related to the changes of elastic tangent modulus and Poisson's ratio respectively.

The element stiffness matrix may be obtained by using $[D_{ep}]$ instead of $[D]$.

As an application of this method to the axisymmetric body, ultimate pressure analysis of the prestressed concrete pressure vessel is described by using the technic of elastic stress analysis by the F.E.M..

In the numerical procedure of the computer program, it is assumed that when a principal tensile strain exceeds the critical tensile strain and tensile stress remains in the same direction, the element is able to crack and stresses on the crack are released.

When a compressive strain exceeds the critical compressive strain the element stiffness is treated to fail completely.

The numerical results are discussed with the 1/40th scale model test results, and it is assured that the method is valid to consider the behaviors of the PCPV beyond the elastic range.

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CONVERGENCE OF FOURIER SERIES SOLUTIONS IN THE ELASTIC ANALYSES OF THE SHEAR WALLS SUBJECTED TO THE VARIOUS EXTERNAL PLANE FORCES

(See Page 15)

by **IKUO TOKUHIRO**, Assistant Prof. of Kyushu Institute of Design, Member of A.I.J.

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TENTATIVE PROPOSAL FOR DYNAMIC EARTHQUAKE RESISTANT DESIGN OF TALL REINFORCED CONCRETE CHIMNEYS

by **KAZUYOSHI UCHIDA**, Research Engineer of Muto Institute of Structural Mechanics, Inc. Member of A.I.J.

In 1966 the author found through the dynamic response analysis that tall reinforced concrete chimneys designed by the Japanese seismic code are very strong in their bottom portion but extremely weak in their upper portion. Then in order to renew this absurdity, the author tried to establish a tentative proposal of the earthquake resistant design method on which effect of the dynamic behavior is reflected.

Namely, such parameters as chimney heights, chimney proportions, foundation conditions, damping assumptions and earthquake waves are selected for investigating the vibration characteristics of tall chimneys by means of the direct earthquake response analysis. Thus, a tentative method for manual calculation of dynamic earthquake forces acting on chimneys is proposed on the basis of the analyzed vibration characteristics.

The main formulas recommended in this paper are as follows :

- 1) Determination of chimney proportion
- 2) First natural period
- 3) Coefficients of both base shear and base overturning moment
- 4) Distribution of both shear and bending moment along chimney height