APPLICATION OF THE PREDICTION OF EXTREME VALUES

(MEAN WIND SPEED AT THE SITE OF HAWAII'S FLOATING CITY)

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I. GUMBEL'S FIRST ASYMPTOTIC DISTRIBUTION

1. INTRODUCTION

The simplest method by which the extreme values of a variate can be predicted is known as that of the First Asymptotic Distribution of Gumbel (1958). Since the theory for deriving the distribution was in large measure previously developed by Fisher and Tippett (1928), the distribution is also identified by the names of these authors. The method can be used to predict both maximum and minimum values of a totality. In this paper, only the former are considered; but minimum values can be predicted by the same procedure, if they are first transformed into reciprocals and at the end reconverted into their original expression. The distribution is of the exponential type and approaches infinity (or zero) as the variate controlling the extrapolation grows indefinitely. Whether the distribution is a reasonable one to employ in a specific application is a matter of judgment. Such a judgment can be exercised by considering how well the inputs fit the distribution and by taking into account any factor which may tend to influence the trend toward the extreme values. While goodness of fit is a statistical problem, the criterion by which it is judged depends on the intended application and no a priori rule can be formulated in its regard. The deviation in trend to extreme is usually a physical problem and its prediction cannot always be made on a purely statistical basis but may require insight into the nature of the phenomenon to be predicted.

2. INPUTS

The prediction of extreme values is made by carrying out certain statistical operations on a set of maximum values of the variate to be predicted. Such a set is obtained either by sampling subsets of a totality (e.g., maximum deviations from specifications of 5 ball bearings from each batch of 100 comprising a total lot of 10,000), or by the sequential measurement of a process over specific intervals of time suitably spaced (e.g., one-minute average wind intensities measured every hour and reported as monthly maxima). The method is the same in either case, but since the environmental phenomena of interest are essentially time-dependent, the exposition that follows is expressed in the pertinent to time functions.

Thus, the inputs required are :

- a) A compilation of set maxima, x
- b) The total number of set maxima, N

3. ORDERING OF INPUTS

When N is large, the (adjusted) set maxima are ordered by magnitude, either ascending or descending. In what follows, the ascending order is used. The generic value of the ranking index of the set maxima is denoted by x_m , thus :

$$x_1 < x_2 < \dots < x_m < \dots < x_N$$

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4. ORDINAL FREQUENCY φ_m

To each set maximum x_m corresponds the ordinal frequency

$$\varphi_m = \frac{m}{N+1}$$

5. INPUT STATISTICS

The mean value of x_m

$$\bar{x} = \frac{1}{N} \cdot \sum_{i=1}^N x_m$$

The variance

$$\sigma^2 = \frac{1}{N} \cdot \sum_{i=1}^N [x_m - \bar{x}]^2$$

The corresponding dispersion

$$\sigma = \sqrt{\sigma^2}$$

6. REDUCED VARIATE y_m

The set maximum x_m is plotted at the position given by the reduced variate

$$y_m = -\ln[-\ln(\varphi_m)]$$

7. SLOPE $1/a$

The slope is calculated by the least square technique as

$$\frac{1}{a} = \frac{\sum_{i=1}^N (x_m - \bar{x}_m)(y_m - \bar{y}_m)}{\sum_{i=1}^N (y_m - \bar{y}_m)^2}$$

8. MODE U

The mode is calculated as

$$U = \bar{x}_m - \frac{\bar{y}_m}{a}$$

9. EXPECTED MAXIMUM VALUE $x(y)$

The regression line of expected maxima is

$$x(y) = U + \frac{y}{a}$$

II. GUMBEL'S THIRD ASYMPTOTIC DISTRIBUTION

1. INTRODUCTION

The First Asymptotic Distribution of Gumbel predicts extreme values of a variate approaching infinity as the return period approaches infinity. To be sure, the variate's approach is lethargic, inasmuch as the extremal expected values of the variate increase with the logarithm of the return period, and in most applications to forecasting this tendency of growth does lead to predictions of reasonable accuracy. This obtains when the observed values, ranked by magnitude and related to a probability of occurrence, crowd the regression line of the expected values. There exist, however, cases for which there is indeed, a marked tendency of the observed values to deviate increasingly from the regression line as the return period increases. Such cases occur when physical causes operate to impose constraints of ever greater severity as the variate tends to grow in magnitude.

The reliable treatment of such problems requires extremal probability distributions somewhat more flexible than the First Asymptotic Distribution. A convenient one is Gumbel's Third Asymptotic Distribution. As against the First, which is a two-parameter distribution (mode and slope), the Third is a three-parameter distribution (asymptote, characteristic value, and exponent). However, the enhanced flexibility introduced by the third parameter is paid for in increased complexity, and a relatively simple and direct determination of parameters, as in the case of the First distribution, is not quite in hand. The way out is to have recourse to a computer and derive the parameters by iteration. A method for accomplishing this is outlined herein.

The cumulative probability corresponding to Gumbel's Third Asymptotic Distribution is given by

$$\varphi(x) \equiv \exp\left(-\left[\frac{W-x_m}{W-V}\right]^K\right)$$

where x is the variable, W its limiting value, i.e., the expected maximum value the variable is likely to attain in the infinitely long run, V its characteristic value and K an exponent. The problem is to derive them from the input data. The inputs are a set of maximum values measured over a specific interval.

The set of maximum values x_m is arranged in ascending order and plotted against the corresponding ordinal frequency

$$\varphi_m = \frac{m}{N+1}$$

where N is the total number of data points. The mean value, variance and dispersion (x_N, σ_N^2, σ) are obtained as for the First Asymptotic Distribution.

The reduced variate z_m when introduced on to the expression

$$Z_m = -\ln\left(\frac{W-x_m}{W-V}\right)^K$$

will plot as a straight line against

$$y_m = -\ln[-\ln(\varphi_m)]$$

Note that in the outline that follows the subscript m is employed to define either an input (x_m) or a value directly related to an input (y_m, φ_m , etc.).

Note also that the characteristic value is that value of x which corresponds to the cumulative probability $\varphi=1/e=0.368$ i.e. to $y=0$. In the First Asymptotic Distribution, such a value is termed the mode.

2. PROCEDURE

2.1 Variance $\sigma_N^2(V, W, K)$

$$\sigma_N^2(V, W, K) = \frac{1}{N} \sum_1^N (Z_m - y_m)^2 = \frac{1}{N} \sum_1^N \left(-K \ln \left[\frac{W-x_m}{W-V} \right] - y_m \right)^2$$

2.2 Exponent K

Using the least square method, the exponent K can be expressed in terms of W and V as

$$K = - \frac{\sum_1^N y_m \ln \left(\frac{W-x_m}{W-V} \right)}{\sum_1^N \left\{ \ln \left(\frac{W-x_m}{W-V} \right) \right\}^2}$$

Then the variance σ_N^2 is the function of only V and W as

$$\sigma_N^2(V, W) = \frac{1}{N} \sum_1^N \left[-K(V, W) \ln \left(\frac{W-x_m}{W-V} \right) - y_m \right]^2$$

2.3 Parametric Sequence of the Characteristic Value V

The characteristic value V is obtained by iteration. A new characteristic value V_{p+1} is related to its previous value V_p through recurrence relation

$$V_{p+1} = V_p + \left[-\frac{1}{2} \right]^{C_p} \cdot \Delta V$$

where p is an iteration index and ΔV is an arbitrary incremental value.

Note that the characteristic value V corresponds to the mode U of the First Distribution. Therefore, an approximate characteristic value V_0 is obtained by letting $V_0 = \text{Mode of the First Distribution}$.

The exponent C_p appearing in the recurrence relation denotes the number of times the inequality sign changes in step 2.6. The initial value of C_p is 0.

2.4 Parametric Sequence of the Asymptote W

The asymptote W is also obtained by iteration. A new value of the asymptote W_{q+1} is related to the previous value W_q through the recurrence relation

$$W_{q+1}(V) = W_q(V) + \left[-\frac{1}{2} \right]^{C_q} \cdot \Delta W$$

where q is a sequential index and ΔW is an arbitrary incremental value whose selection should be

based, if possible, on physical reasoning.

The exponent C_q denotes the number of times the inequality sign changes in Step 2.5. The initial value of C_q is zero.

2.5 Criterion for the Asymptote Value, $W(V_p)$

If $\sigma_{N^2}(V_p, W_{q+1}) < \sigma_{N^2}(V_p, W_q)$, the sign change count is

$$C_{q+1} = C_q$$

If $\sigma_{N^2}(V_p, W_{q+1}) > \sigma_{N^2}(V_p, W_q)$, the sign change count is

$$C_{q+1} = C_q + 1$$

Repeat calculation from $C_q=0$ until $C_q=3, 4$ or 5 and find the minimum value of $\sigma_{N^2}(V_p, W_q)$. The selection of $C_q=3, 4$ or 5 depends upon the nature of the problem.

2.6 Criterion for the Characteristic Value V (for all possible values of W)

If the minimum value of $\sigma_{N^2}(V_{p+1}, W)$ is smaller than that of $\sigma_{N^2}(V_p, W)$, the sign change count is

$$C_{p+1} = C_p$$

If the minimum value of $\sigma_{N^2}(V_{p+1}, W)$ is larger than that of $\sigma_{N^2}(V_p, W)$, the sign change count is

$$C_{p+1} = C_p + 1$$

Repeat calculation from $C_p=0$ until $C_p=3, 4$, or 5 and find the smallest value of $\sigma_{N^2}(V_p, W)$ for all possible combinations of V and W by iteration. The selection of $C_p=3, 4$ or 5 also depends on the nature of the problem.

The exact solution of V , W and K corresponds to the smallest value of $\sigma_{N^2}(V, W)$. Note that the value of exponent K is a function of V and W , for which the variance $\sigma_{N^2}(V, W)$ is smallest, and is calculated for each step of V_p and W_q .

III. MEAN WIND SPEED AT THE SITE OF HAWAII'S FLOATING CITY

1. INTRODUCTION

This part deals with long-term forecasts of mean wind intensity at a site some three miles off Waikiki selected as a possible location for Hawaii's Floating City.

To predict the long-term trend of mean wind speed at this site, wind data observed at the Honolulu Airport during the most recent 29 years were used.

The prediction of the extreme values of the mean wind speed was made by the following two methods;

- 1) Prediction based upon Gumbel's First Asymptotic Distribution, and
- 2) Prediction based upon Gumbel's Third Asymptotic Distribution.

2. PREDICTION OF EXTREME VALUES BASED UPON GUMBEL'S FIRST ASYMPTOTIC DISTRIBUTION

Table A in Appendix A lists the frequency of occurrence of parametric intervals of wind intensity both as a function of wind direction and independently thereof, the second set of values being obtained by simple summation over direction for each intensity interval. It is the second set that is of interest in the context of this study. For convenience of reference, the pertinent data are reproduced in Table 1. As some small fractions are omitted in calculating the percentage of probability density and cumulative probability, the calculated summation of the probability density is less than

Table 1 Percentage Frequency of Wind Intensity
at the Honolulu Airport

Speed Interval (Knots)		Frequency of Occurrence of Wind Intensity			
Beaufort	U_a	Probability Density		Cumulative Probability	
		%	0/00	%	0/00
0 (Calm)	(0 - 0.5)	4.7	47	4.7	47
1 - 3	(0.5 - 3.5)	8.3	83	13.0	130
4 - 6	(3.5 - 6.5)	16.4	164	29.4	294
7 - 10	(6.5 - 10.5)	30.1	301	59.5	595
11 - 16	(10.5 - 16.5)	29.3	293	88.8	888
17 - 21	(16.5 - 21.5)	9.4	94	98.2	982
22 - 27	(21.5 - 27.5)	1.5	15	99.7	997
28 - 33	(27.5 - 33.5)	0.1	1	99.8	998
		99.8	998		

100 percent, namely 99.8 percent. In this calculation, the number of interval of observation was replaced by 998.

The wind intensities are reported in integer values of speed (knots) and the groupings by intervals reflect this. If a reported wind intensity U_a is interpreted to mean a speed ranging from

$$U_a - 0.5 \leq U_a \leq U_a + 0.5 \quad (\text{knots})$$

the intervals become continuous. Such a plausible interpretation is mathematically convenient. The resulting intervals of wind intensity are listed in column 2 of Table 1.

The observed mean wind intensity when plotted against the reduced variate

$$y^* = -\ln[-\ln(\varphi(\bar{U}_a))]$$

have the trend of the broken line in Figure 1. Here $\varphi(\bar{U}_a)$ is the cumulative frequency of the observed intensity in Table A. The trend of the broken line in Figure 1 is for the raw observational data corrected for the consequence of intermittent sampling duration.

The straight line in Figure 1 shows the calculation results (see Table 2) based upon Gumbel's First Asymptotic Distribution. The line has its mode $u=7.2274$ (knots) at $y=0$ and slope $1/a=4.3290$.

Fig. 1 Input Data for Mean Wind Intensity

Fig. 2 Expected Maximum Value of Mean Wind Intensity vs. Return Period (Gumbel's First Asymptotic Distribution)

Table 2 Calculation Results Based on Gumbel's First Asymptotic Distribution

Return Period at 50 yr. Intervals	Reduced Variate	Expected Max. Mean Wind Speed
50	7.452	39.486 (knots)
100	8.145	42.486
150	8.550	44.241
200	8.838	45.486
250	9.061	46.452
300	9.243	47.242
350	9.397	47.909
400	9.531	48.487
450	9.649	48.997
500	9.754	49.453
550	9.849	49.866
600	9.936	50.242
650	10.016	50.589
700	10.091	50.910
750	10.159	51.208
800	10.224	51.488
850	10.285	51.750
900	10.342	51.997
950	10.396	52.231
1000	10.447	52.454

There is a 50 percent probability that these expected values will be exceeded. To gain an idea of the validity of these results, the confidence bands for the numbers $n=\pm 1, \pm 2$ of standard deviations of the dispersion of actual and predicted values of x were calculated. There are shown in Figure 1.

The return period corresponding to a generic value of the reduced variate y when expressed in terms of the interval of observation is given by

$$T_r = \exp(y) + \frac{1}{2}$$

To express the return period in years, modify the expression to

$$T_r = \frac{1}{c} \left[\exp(y) + \frac{1}{2} \right]$$

where c is the number of intervals of observation in a year. For the mean wind intensity, the replaced total number of interval of observation is 998 during $T=29$ years. Therefore,

$$C = \frac{998}{29}$$

The line of predicted values in terms of the return period in years is shown in Figure 2. The expected maximum value of the mean wind intensity corresponding to a return period of 100 years is 42.49 knots.

3. PREDICTION OF EXTREME VALUES BASED UPON GUMBEL'S THIRD ASYMPTOTIC DISTRIBUTION

In the prediction of extreme values based upon Gumbel's First Asymptotic Distribution there is a marked tendency of the observed values to deviate increasingly from the predicted line as the return period increases. The reliable treatment of such problems requires extremal probability distributions somewhat more flexible than the First Asymptotic Distribution.

Of the available alternatives, Gumbel's Third Asymptotic Distribution was applied to the prediction of the extreme values of mean wind speed.

This distribution has the three parameters :

- 1) Asymptotic values W
- 2) Characteristic value V
- 3) Exponent value K

and the reduced variate z_m is expressed as

$$z_m = -\ln \left(\frac{W - x_m}{W - V} \right)^K$$

which corresponds to the reduced variate y in the First Distribution. These parameters are calculated by using the same data for the First Distribution as

$$W = 40.06 \quad (\text{knots})$$

$$V = 7.33 \quad (\text{knots})$$

$$K = 6.29$$

The return period corresponding to the reduced variate z_m when expressed in terms of the interval of observation is in the same form as that of the First Distribution by using the reduced variate z .

$$T_r = \exp(z) + \frac{1}{2}$$

The return period in years is also given by

$$T_r = \frac{1}{c} \left[\exp(z) + \frac{1}{2} \right]$$

where c is the same value as that of the First Distribution.

Using the relationship between the reduced

Fig. 3 Expected Maximum Value of Mean Wind Intensity vs. Return Period (Gumbel's Third Asymptotic Distribution)

Table 3 Calculation Results Based on Gumbel's Third Asymptotic Distribution

Return Period at 50 yr. Intervals	Reduced Variate	Expected Max. Mean Wind Speed
50	8.282	31.294 (knots)
100	8.975	32.209
150	9.380	32.700
200	9.668	33.029
250	9.891	33.274
300	10.074	33.468
350	10.228	33.628
400	10.361	33.763
450	10.479	33.880
500	10.584	33.983
550	10.680	34.074
600	10.767	34.157
650	10.847	34.231
700	10.921	34.300
750	10.990	34.363
800	11.054	34.421
850	11.115	34.475
900	11.172	34.526
950	11.226	34.573
1000	11.278	34.618

variate Z and x , the predicted values of x is expressed by the return period in years.

The line of predicted values (see Table 3) based upon the Third Distribution in terms of the return period in years is shown in Figure 3, and compared with that of the First Distribution. The expected maximum value of the mean wind intensity corresponding to a return period of 100 years is 32.21 knots.

4. CONCLUSIONS

It is justifiable question whether the wind strength measured at the Honolulu Airport can be assumed to represent validly that obtaining at the site of Hawaii's Floating City. Since relevant measurements are lacking, the question cannot be answered objectively. But the condition that the

Table A Percentage Frequency of Main Wind Direction and Intensity from Hourly Observations at the Honolulu Airport

Direction	Range of Mean Wind Intensity in Knots (5 min. observation taken every hour)									Percentage of time wind blows from indicated direction	Mean wind speed (knots)	Maximum value of mean wind speed(knots)
	1 1-3	2 4-6	3 7-10	4 11-16	5 17-21	6 22-27	7 28-33	8 34-40	9 41-47			
N	1.5	1.8	1.1	.3	.0	.0				4.8	5.7	16
NNE	.8	1.2	.9	.5	.1	.0				3.6	7.0	21
NE	1.2	2.9	6.4	7.8	2.7	.5	.0	.0	.0	21.5	11.2	27
ENE	.6	3.0	9.9	12.6	4.2	.6	.0	.0		30.9	11.8	27
E	.7	1.9	5.3	4.6	1.6	.2	.0			14.3	10.7	27
ESE	.2	.4	.6	.3	.1	.0	.0	.0	.0	1.5	8.9	21
SE	.2	.4	.8	.7	.1	.0	.0	.0		2.2	9.8	21
SSE	.1	.3	.8	.6	.2	.0	.0	.0	.0	2.2	10.1	21
S	.2	.6	1.3	.6	.1	.0	.0			2.9	9.0	21
SSW	.1	.2	.6	.3	.0	.0	.0			1.3	9.2	16
SW	.1	.2	.6	.3	.1	.0	.0			1.3	9.3	21
WSW	.1	.1	.2	.2	.1	.0	.0			0.6	10.3	21
W	.2	.2	.2	.1	.0	.0	.0			0.8	7.3	16
WNW	.2	.3	.1	.0	.0	.0	.0			0.7	5.3	10
NW	1.4	1.6	.7	.1	.0	.0				33.9	5.1	16
NNW	.8	1.2	.6	.1	.0	.0	.0			2.8	5.5	16
Variable										95.3	9.7	
Calm										4.7		
%	8.3	16.4	30.1	29.3	9.4	1.5	.1	.0	.0	100.0		

Beaufort Scale

Source U.S. Air Force, Data Processing Division/ Asheville, N.C. 28801

Total number of Observations
224,409

airport is just off the sea and that the site planned for the floating city is but ten miles distant from the airport tower should support the belief that any difference in wind data should not be of serious importance.

From the calculation results shown above, the predicted values of the mean wind speed based upon Gumbel's Third Asymptotic Distribution seem to be more reasonable than those based upon Gumbel's First Asymptotic Distribution.

5. ACKNOWLEDGEMENT

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APPENDIX A Wind Data

Table A contains a listing of mean wind intensities which have been observed at the Honolulu Airport during 29 years. There are averages of five minute observations taken every hour.

The data were obtained through the courtesy of the Honolulu Bureau of the U.S. Weather Service for predicting the environment at the site selected for the floating community.

極値予測の応用について

——ハワイ海上都市予定海域の平均風速——

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1. はじめに

現在、建築構造物の設計法として、弾性計算法が簡単なため広く使われているが、最近の構造工学の傾向として、塑性計算法または、構造物の崩壊を対象とした計算法が発達してくると、荷重も過去に生じた最大荷重を使用するのでは不十分で、将来ある適当な期間に起り得る最大期待値を対象として、構造設計する方がより合理的であると考えられる。

極値統計理論の目的は、観測極値を解析し、さらに起り得べき極値を予想することであり、現在、気象学、水文学、航空学、材料学、船舶工学等の諸分野で広く利用されている。本文では、極値統計理論として、広く知られている Gumbel の第1及び第3漸近分布について、簡単に説明すると共に、第3漸近分布の解法が数学的に難解で実用的でないため、筆者らが開発したコンピューター計算に適した独自の解析法を紹介し、その応用例として、ハワイ海上都市予定海域における平均風速の長期的予測について報告する。

2. Gumbel の第1漸近分布

観測しようとする量を統計学では、特に変量というが、変量の極値を予測する最も簡単な方法は、Gumbel の第1漸近分布として知られている。この方法で、最小極値と最大極値の両方を予測することができるが、ここでは、最大極値だけを取り扱うことにする。

第1漸近分布は指数タイプのもので、この分布が適用できるかどうかは、如何に観測データがこの分布に当てはまるかによって判断される。漸近分布の適用に当たっては、単に、計算から答もだすだけでなく、実際の現象そのものの性質に対する考察及び分析に基づいて、適用を判断することが重要である。第1漸近分布の理論については、本文に簡単に説明されているので、ここでは省略する。

3. Gumbel の第3漸近分布

観測データを用いて、頻度曲線を画くと、一般には、正規分布のように左右対称とならず、一番高い山の位置

が全体の中央より、右に偏った非対称形分布を示す場合がある。このような場合に、第1漸近分布で極値を予測すると、再現期間の大きい方で観測値と予測値が一致しなくなる。このような変量の分布に対して、極値を予測する方法に Gumbel の第3漸近分布が知られている。第1漸近分布は、再現期間が無限大となると変量の極値も無限大となるが、第3漸近分布では、極値はある有限値に近づいていく。

第1漸近分布は、正規確率紙で画くと直線となるので、2つのパラメーター（勾配とモード）を求めたが、これに対して、第3漸近分布では3つのパラメーター（漸近値 W 、特性値 V 及び指数値 K ）を求めることになり、第1漸近分布のように簡単に計算できない。従って、ここではコンピューターを使用した繰り返し計算法により3つのパラメーターを求める方法を紹介する。

Gumbel の第3漸近分布では、第1漸近分布で使用した順位頻度 $\varphi(x) = m/(N+1)$ の代りに、次式を使用する。

$$\varphi(x) = \exp\left(-\left[\frac{W-x_m}{W-V}\right]^K\right)$$

ここで、 x は変量値、 W はある有限値で、再現期間が無限大になるにつれて、 x の予測値はこの値に漸近する。また、特性値 V は第1漸近分布の場合のモードに相当するものである。指数値 K は正の実数である。

第1漸近分布の場合の変換変数 $y_m = -\ln[-\ln(\varphi_m)]$ に相当する第3漸近分布の変換変数は、

$$Z_m = -\ln\left(\frac{W-x_m}{W-V}\right)^K$$

で示される。

[計算手順]

計算として、次のような分散 $\sigma_N^2(V, W, K)$ を考える。

$$\sigma_N^2(V, W, K) = \frac{1}{N} \sum_{i=1}^N (z_m - y_m)^2$$

特性値 V 、漸近値 W 及び指数値 K は、最小の値を与える分散値 $\sigma_N^2(V, W, K)$ から求められる。まず $\sigma_N^2(V, W, K)$ を V 及び W のみの関数 $\sigma_N^2(V, W)$ に変形させる。即ち $\partial \sigma_N^2 / \partial K = 0$ より

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$$K(V, W) = \frac{-\sum_{m=1}^N y_m \ln\left(\frac{W-x_m}{W-V}\right)}{\sum_{m=1}^N \left\{ \ln\left(\frac{W-x_m}{W-V}\right) \right\}^2}$$

従って、分散値 σ_{N^2} は、次のように V と W のみの関数となる。

$$\sigma_{N^2}(V, W) = \frac{1}{N} \sum_{m=1}^N \left[-K(V, W) \cdot \ln\left(\frac{W-x_m}{W-V}\right) - y_m \right]^2$$

次に、適当な V 及び W の値を決めて、 $\sigma_{N^2}(V, W)$ の値を計算し、 V 及び W の値を少しずつ変化させて、その都度 $\sigma_{N^2}(V, W)$ を計算する。最初を選ぶ V 及び W の値を、 V_p 及び W_q とすると、次の計算に選ぶ V 及び W の値は、次式によって決められる。

$$V_{p+1} = V_p + \left[-\frac{1}{2} \right]^{C_p} \cdot \Delta V$$

$$W_{q+1} = W_q + \left[-\frac{1}{2} \right]^{C_q} \cdot \Delta W$$

ここに、 ΔV 及び ΔW は V 及び W に対する任意の増加量である。また、 C_q は最初の値を 0 とし、次のような条件によって、1, 2, 3 と増加する正の整数である。即ち、 $\sigma_{N^2}(V_p, W_{q+1}) < \sigma_{N^2}(V_p, W_q)$ ならば、 $C_{q+1} = C_q$ 、 $\sigma_{N^2}(V_p, W_{q+1}) > \sigma_{N^2}(V_p, W_q)$ ならば、 $C_{q+1} = C_q + 1$ とする。同様に、 C_p も 0 より 1, 2, 3 と増加する正の整数で、 $\sigma_{N^2}(V_{p+1}, W) < \sigma_{N^2}(V_p, W)$ ならば、 $C_{p+1} = C_p$ 、 $\sigma_{N^2}(V_{p+1}, W) > \sigma_{N^2}(V_p, W)$ ならば、 $C_{p+1} = C_p + 1$ とする。 C_p 及び C_q の値をどこまで計算するかによって、計算精度が異なってくるが、本計算では、 $C_p = 5$ 、 $C_q = 5$ まで計算することにした。

このようにして、 V 及び W の値を変化させながら、 $\sigma_{N^2}(V, W)$ の最小値をコンピューター計算でさがし、最小値を与える V 及び W の値を近似的に求めることができる。従って、 V 及び W から K の値も求められる。

4. ハワイ海上都市予定海域の平均風速

Gumbel の第 1 及び第 3 漸近分布の応用例として、オアフ島ワイキキから約 3 マイル沖のハワイ海上都市に予定されている海域の平均風速の長期的予測を取り扱ったものである。

この海域の平均風速の傾向を予測するため、最近 29 年間のホノルル空港で観測された平均風速のデータが用いられた。

Gumbel の第 1 漸近分布による極値予測

最近 29 年間のホノルル空港で観測された平均風速のデータが、附 A の表—A に示されている。ホノルル空港は、ワイキキと同じく、オアフ島の南側海面に接しており、海上都市予定海域より数マイル離れているが、平均風速の傾向としては、ほぼ類似であると推定される。

この表—A に示された値は、1 時間おきに 5 分間観測

した平均風速で、9 段階の風速に区分して、各風向き毎に百分率で示されている。また、表の最下欄には、風速によって区分された各段階毎の百分率が示されている。

これから、確率密度及び累積確率が計算され、表—1 に示されている。この計算には、小数点以下 2 桁目を省略したため、確率密度の和は 99.8% と、100% より小さい値となっている。実際の観測データの個数が非常に多いので、ここでは、データの個数を 998 個に置きかえて計算を進めている。表—A の風速はノットの整数値で示されているので、整数値で示された風速 U_a は $U_a - 0.5 \leq U_a \leq U_a + 0.5$ の範囲にあると考えられる。従って、風速段階を表—1 の 2 列目に示したように書き換えると、連続したものになり、数学的にも取り扱い易い。

観測平均風速を用いて、変換変数 y^* をプロットすると、図—1 のような破線になる。これは、観測値の傾向を示すものである。同じ図—1 の太い直線は、第 1 漸近分布で求めた計算値である。この直線のモード U は 7.2274 ノット、勾配 $1/a$ は、4.3290 である。実際の観測値が、この直線（予測値）を越える確率は、50% である。観測値と予測値の分散から標準偏差を計算し、予測値を中心として、標準偏差 $\pm \sigma$ 及び $\pm 2\sigma$ だけ離れた領域を示す Confidence Bands が計算され、図—1 の曲線群で示される。

第 1 漸近分布から計算された変換変数 y に対応する再現期間を年単位で示すと、 $T_r = 1/c[\exp(y) + 1/2]$ となる。

ここで、 c は 1 年当りの観測データの個数で、この平均風速の計算では、観測期間 29 年間のデータの個数を 998 個に置きかえているので、 $c = 998/29$ となる。

表—2 には、50 年より 1000 年までの再現期間に、対応する最大平均風速の予測値が計算されている。これを図に示すと、図—2 のような直線となる。

再現期間 100 年では、最大平均風速の予測値は、42.49 ノットである。

Gumbel の第 3 漸近分布による極値予測

第 1 漸近分布では、再現期間の大きい所で、観測値と予測値がよく一致しない傾向が表れている。

従って、第 1 漸近分布より適合性のよい第 3 漸近分布が、この平均風速の極値予測に適用された。第 1 漸近分布の場合には、モードと勾配の 2 つのパラメーターを計算したが、第 3 漸近分布では、次のパラメーターを計算することになる。

- 1) 漸近値 W , 2) 特性値 V , 3) 指数値 K

また、第 1 漸近分布の変換変数 y に相当する第 3 漸近分布の変換変数 z_m は次式で示される。

$$z_m = -\ln\left(\frac{W-x_m}{W-V}\right)^K$$

これらの3つのパラメーター (W, V, K) を第1漸近分布で使った同じ観測データを用いて、計算すると次のようになる。

- 1) $W=40.06$ (ノット)
- 2) $V=7.33$ (ノット)
- 3) $K=6.29$

ここで、特性値 V は第1漸近分布のモードに近い値を示していることがわかる。また、指数値 K が大きくなると漸近値 W も増大し、第3漸近分布は第1漸近分布に近づくことになる。

第3漸近分布での変換変数 x と再現期間 T_r との関係は、第1漸近分布の場合と全く同様である。

第3漸近分布による50年より1000年までの再現期間に対応する最大平均風速の予測値が表-3に計算されている。これを曲線にプロットしたものが、図-3に示さ

れている。この曲線は再現期間が増大するにつれ、漸近値 W に近づいて行く。

第3漸近分布で予測値を計算すると、再現期間100年では、最大平均風速の予測値は32.21ノットとなり、第1漸近分布で計算された予測値42.49ノットとはかなり違った値を示していることがわかる。

5. む す び

以上、Gumbelの第1及び第3漸近分布を用いて、ハワイ海上都市予定海域の平均風速の長期的予測が計算されたが、図-1に示された観測データの傾向から判断して、第3漸近分布による最大平均風速の予測の方がより適切なものと考えられる。また、同様な応用で、十分な観測データが得られれば、海洋構造物に対する波浪の予測等、建築構造物の設計理論として、広く利用できるものと考えられる。

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EFFECT OF COARSE AGGREGATE ON FRACTURE OF CONCRETE

(Part 1 : Model Analysis)

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The purpose of this investigation is to examine the effect of the strength and surface condition of coarse aggregate on the fracture of concrete under uniaxial compression. The two-phase model specimens consisted of a cylindrical coarse aggregate and mortar matrix were used to simplify the study. Three kinds of compressive strength and five kinds of surface condition were provided for coarse aggregates.

The following conclusions were obtained by this investigation.

- 1) Regardless of the surface roughness of aggregate, the fracture process of model concrete with an aggregate whose compressive strength(F_{ca}) is smaller than that of mortar matrix(F_{cm}) was as follows; (1) initiation of bond crack between aggregate and mortar matrix, (2) tensile cracking of mortar matrix and aggregate parallel to the longitudinal axis, (3) local compressive failure of aggregate and (4) failure of model concrete.
- 2) The fracture process of model concrete with an aggregate whose strength is larger than that of mortar matrix was as follows; (1) initiation of bond crack, (2) tensile cracking of mortar matrix, (3) local compressive failure of mortar matrix at the vicinity of aggregate and (4) failure of model concrete.
- 3) The compressive strength(F_{cc}) of model concrete increases with the increase of the value of F_{ca} in the range of $F_{ca}/F_{cm} < 1$, but never increases in the range of $F_{ca}/F_{cm} > 1$, regardless of the surface roughness of coarse aggregate.

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THE PREDICTION OF EXTREME VALUES

(Theory and application)

(See page 13)

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