STUDY ON SHRINKAGE STRESS ANALYSIS AND CRACK DEVELOPMENT IN R.C. STRUCTURE WALLs

— I . STRESS ANALYSIS —

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1. Preface

Shrinkage cracks in concrete structures are mainly caused by shrinkage deformation which occurs when concrete hardens and dries and by the frame conditions of structures for restraining such deformation. The surveys by Mr. Hitotsuya and Mr. Ishikawa show that in the case of concrete structures, cracks develop in the order of 24% for wall sides, 21% for roofs, 17% for floors, followed by beams and parapets. In many cases, cracks lead to water leakage, spoil the exterior views of buildings, corrosion of reinforcements and steel frames and promotion of concrete freezing. Of the factors which call for crack control, 37% is for prevention of water leakage, 23.9% for structural reasons, 21.7% for preservation of exterior views and 17.4% for prevention of steel corrosion. Thus, cracks greatly affect the durability of concrete structures. In addition, crack development has a complex cause-effect relationship to characteristics of concrete and all other conditions of structures and is promoted by interaction of different factors.

Many of past studies have been made only for a single material or as part of field surveys. And very few of these studies suggest measures against crack control by considering of such factors as relaxation of restraint force based on an analysis of principal shrinkage stress on the whole wall sides of a building in connection with its length and height. In order to solve the question of crack development due to shrinkage on wall sides, it is important to consider not only concrete materials but also principal stress due to shrinkage and deformation properties concerning the whole building. The following common knowledge is obtained when we conduct a detailed review of past studies concerning crack development due to shrinkage on wall sides.

a. Cracks due to hardening and drying shrinkage

The critical amount of shrinkage which causes cracks is 4×10^{-4} for natural aggregate concrete and 3.8×10^{-4} for artificial lightweight aggregate concrete. In the central span, cracks are vertical. Elsewhere, they are inclined and their inclination increases as they appear closer to the end span. Their inclination is the largest in the extreme end span. Horizontal cracks develop mostly at construction joints. Cracks at the lowest story is remarkably frequent and large compared with the upper stories. Cracks develop more on south sides than on north sides, but no great difference exists between east and west sides. Generally, cracks cause practical damage when their width reaches "0.1~0.5 mm". This kind of crack on wall sides is usually observed 3 to 6 months after

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placing concrete.

b. Cracks due to expansion or construction of concrete resulting from temperature change

Elasticity property of concrete shows a trend similar to change of temperature. Shrinkage of wall sides 2.5 months after placing concrete is 0.35 mm/m for natural aggregate concrete, 0.4 mm/m for 3 type lightweight concrete. Shrinkage of columns is about half that of wall sides with their shrinkage speed being slower. But once a crack develops, its width fluctuates from time to time influenced by outside temperature¹⁴⁾. Temperature expansion modulus of reinforced concrete under sunshine is $1.05\sim 1.2\times 10^{-5}$ for a 10 cm-thick wall and about 1.2×10^{-5} for a 20 cm-thick wall.

c. Cracks due to imperfection and defect in design and construction

⊙ Material, mixing : type and quality of frames, cement type, mixing (especially, W/C ratio, excess of unit water content).

⊙ Construction : imperfect protection of wall sides, imperfect curing and placing methods, mistakes in section and order setting for concrete placing.

⊙ Design : extremely thin walls, insufficient reinforcement steel, mistake in bar-arrangement.

d. Cracks due to sinking of the foundation (omitted)

e. Cracks due to external force (omitted)

f. Cracks due to complex combination of the above-mentioned factors (omitted).

In this study on shrinkage cracks on wall sides caused by drying shrinkage of concrete, we made a model considering all wall sides of concrete structures and analyzed shrinkage stress on all or part of the structures by using the finite element method (F.E.M.). The study was aimed at foreseeing the location, direction and shape of cracks as well as at clarifying their correlation to the actual conditions of cracks in existing buildings. Furthermore, crack control measures conceived from the results of this stress analysis and field studies were incorporated into several buildings from the onset of their designing. Follow-up surveys were made on these buildings to obtain further data. Our report this time deals with our analysis of shrinkage stress on all wall sides of two reinforced concrete buildings (one with horizontally-long walls and the other with vertically-long walls) by means of the finite element method, aimed at foreseeing the location, direction and shape of cracks. Next time, we shall deal with the correlation between analysis results and actual conditions of cracks in existing buildings, and report on the feasibility of stress analysis as a means of controlling cracks.

2. Analysis on shrinkage stress

Shrinkage stress analysis was conducted on a given structure composed of structural members consisting of lineal member and face member. Calculation was made by dividing the continuum with an unlimited degree of freedom, by means of an assumed boundary, into finite elements (lineal element, triangle element, square element etc). We indicated each element force while defining the movement and rotation angle of each panel point as an unknown quantity and the analysis is based on the deformation method obtained from an equation of equilibrium of forces of panel points.

(1) Premise of analysis

a. This analysis can be feasible when deformation of a structure is extremely small compared with the size of the structure. It is based on the theory of infinitesimal deformation.

b. Deformation of a structure and external force always have a proportional relationship. That is to say, calculation is made within the range of elasticity.

c. The analysis is based on the deformation method obtained from an equilibrium equation of forces existing between external forces and a structure with the movement and rotation angle of each panel point defined as an unknown quantity.

d. A section vertical to the long axis of lineal members can maintain its plane even after deformation.

e. The structure is assumed to consist of panel points, each considered to be an infinitely small hard ball, and lineal, face and solid members linking the panel points.

(2) Deformation method

A define absolute co-ordinate system (X, Y, Z) was set as a basis for the entire structure (Fig. 1). In addition a partial co-ordinate system (x, y, z) peculiar to elements of lineal members IJ was set. where E : modulus of elasticity, G : shear elastic modulus ($E/2(1-\nu)$), A : area of section, l : length of linear member, ν : Poisson's ratio I_X, I_Y, I_Z : geometrical moment of section.

In the partial co-ordinate system, the relationship between unit stress and displacement of elements is shown by setting X axis in long axis direction IJ for lineal member element and X axis in direction IJ for face member element and y axis in direction IK within the element. We selected a displacement function by which displacement conditions within the element was determined as a function of displacement of panel point. Hence, the force of panel points concentrated on the end of element was determined. The relations-

hip between this force (f) and displacement of panel point (d) is shown below. Assuming that $f_1^i \sim f_6^i$ represent the force of section of panel point I for lineal member IJ , and $d_1^i \sim d_6^i$ represent displacement of panel point I for lineal member IJ , the relationship between force of section and displacement of lineal member IJ is shown as in Fig. 2. This can be represented by eq 1 and 2, when shown in the partial co-ordinate system. Fig. 2 can be summed up as follows.

$$\begin{Bmatrix} f^i \\ f^j \end{Bmatrix} = \begin{pmatrix} K & I \\ M & N \end{pmatrix} \begin{Bmatrix} d^i \\ d^j \end{Bmatrix} \cdots (1)$$

$$\{f\}_{n \times 1} = (K)_{n \times m} \{d\}_{n \times 1} \cdots (2)$$

Where n : number of panel points for each element, K : rigidity-determinant showing the hardness of element.

Furthermore, eq 3 and 4 can be given as follows when transforming unit stress and displacement into absolute co-ordinate system.

$$\begin{Bmatrix} F^i \\ F^j \end{Bmatrix} = \begin{pmatrix} K & L \\ M & N \end{pmatrix} \begin{Bmatrix} D^i \\ D^j \end{Bmatrix} \cdots (3)$$

$$\{F\}_{n \times 1} = (K)_{n \times m} \{D\}_{n \times 1} \cdots (4)$$

Unit stress of each element following the co-ordinate transmission can be evaluated by using eq. 5 from the balance between a combination of stresses of element end concentrated on panel point I and load vector (P_i) acting on that panel point. " D " in eq. 5 can be obtained from a single simultaneous equation for all panel points.

By substituting this result into eq. 3 and 4 for transforming the co-ordinate, the force of panel point of element can be obtained.

$$(\sum K_{ii})D_i + \sum_{j=1, j \neq i}^n K_{ij}D_j + P_i = 0 \cdots (5)$$

Fig. 1 Partial Co-ordinate System

$$\begin{Bmatrix} f_1^i \\ f_2^i \\ f_3^i \\ f_4^i \\ f_5^i \\ f_6^i \\ f_1^j \\ f_2^j \\ f_3^j \\ f_4^j \\ f_5^j \\ f_6^j \end{Bmatrix} = \begin{pmatrix} \frac{EA}{l} & & & & & & & & & & & \\ & \frac{12EI_z}{l^3} & & & & & & & & & & \\ & & \frac{6EI_z}{l^2} & & & & & & & & & \\ & & & \frac{12EI_z}{l^3} & & & & & & & & \\ & & & & \frac{6EI_y}{l^2} & & & & & & & \\ & & & & & \frac{12EI_y}{l^3} & & & & & & \\ & & & & & & \frac{GI_x}{l} & & & & & \\ & & & & & & & \frac{4EI_y}{l} & & & & \\ & & & & & & & & \frac{4EI_z}{l} & & & \\ & & & & & & & & & \frac{EA}{l} & & \\ & & & & & & & & & & \frac{12EI_z}{l^3} & \\ & & & & & & & & & & & \frac{6EI_z}{l^2} \\ & & & & & & & & & & & & \frac{12EI_z}{l^3} \\ & & & & & & & & & & & & & \frac{6EI_y}{l^2} \\ & & & & & & & & & & & & & & \frac{12EI_y}{l^3} \\ & & & & & & & & & & & & & & & \frac{GI_x}{l} \\ & & & & & & & & & & & & & & & & \frac{4EI_y}{l} \\ & & & & & & & & & & & & & & & & & \frac{4EI_z}{l} \end{pmatrix} \begin{Bmatrix} d_1^i \\ d_2^i \\ d_3^i \\ d_4^i \\ d_5^i \\ d_6^i \\ d_1^j \\ d_2^j \\ d_3^j \\ d_4^j \\ d_5^j \\ d_6^j \end{Bmatrix} \quad \text{Symmetry}$$

Fig. 2 Relationship between Force of Panel Point (f) and Displacement of Panel Point (d) (Rigid Joint of Both End)

where P_i : external vector acting on panel point

(3) Assumed conditions

When analyzing this problem, the following three conditions are assumed.

Assumption 1 to analyze frame within the range of elasticity

Assumption 2 internal unit stress (σ) of concrete due to stiffness of frame

$$\sigma = K \cdot E_c \cdot \varepsilon \quad (0 < E_c < 1) \quad (6)$$

where K : restraint modulus concerning frames (column, beam, wall members), E_c : elastic modulus of concrete, ε : unit strain arising when load of Δt (Table 1 and 2) is given.

Assumption 3 relaxation rate of shrinkage restraint (k) of concrete internally restrained by reinforcement.

$$k = 1 / (1 + n \cdot \mu)$$

where n : elastic modulus ratio, μ : reinforcement ratio

Shrinkage strain of concrete appears to be smaller than it actually is according to reinforcement ratio and the value k is necessary because the relaxation of shrinkage strain is taken into consideration. This stress analysis was carried out by using an assumed amount for free shrinkage strain of each member. All frames were expressed by means of several panel point co-ordinates, taking into consideration section shape, bearing condition and boundary restraint condition. Stress was determined through the combination of lineal member and face member against the force in the face.

3. Model building

3.1 Analysis 1 Building with horizontal-long walls

(1) Main features of the building

- 1) Construction : ramen structure (R.C. structure, one-storied, partially two-storied), The size of building : total length of spans ; about 80 m, total length of girders ; about 140 m (each span and girder : 10 m), Eaves height : about 8 m, Roof : steel trussed roof (folded plate).

Fig. 3 Building of Horizontally-Long Wall (Plane)

Fig. 4 Section Size of Members

The plan and section size of members are shown in Fig. 3 and 4.

- 2) Wall system Wall : retaining wall type, external wall : 150 mm, vertical-horizontal axis D10-@ 200 mm, single bar, Bar of opening : 2-D16 diagonally-reinforced, Finishing : architectural for both exterior and interior walls.

- 3) Foundation Continuous P.C. piles and independent footing.

- 4) Design strength 180 kg/cm² (natural aggregate concrete)

(2) Analyzed Model

Analysis was conducted on a practical building. Two models, with different assumed boundary conditions were used.

a. Model 1

One part of the exterior end was taken from the above-mentioned building. The upper and exterior ends were kept unrestrained but the horizontal direction of the lower end was fixed in consideration of the influence of other members. But the remaining part was kept by rollers on the

Fig. 5 Analysis Model

Table-1 Condition on the Stress Analysis due to Shrinkage

kind	part	section width cm	section area cm ²	arrangement of bar	section area of bar cm ²	μ	β	* temperature $\times$ K	Δt °C
column	capital	60x45	2700	D19-12	34.2	1.26	0.76	-65x0.76	-49
	base of column	60x45	2700	D22-14	54.3	1.94	0.43	-65x0.43	-28
	stud	30x25	750	D16-4	7.92	1.06	0.8	-65x0.8	-52
beam	W1	60x45	2700	D19-6	17.1	0.63	0.37	-65x0.37	-24
	W2	15x100	1500	D10-200	3.57	0.42	0.37	-65x0.37	-24

Note. 1. $F_c=210\text{kg/cm}^2$ (long period) 2. $\mu=0.4$ (section calculation) 3. $\beta=0.76$ (section calculation) 4. $\mu=1.0$ (input) 5. $\mu=1.0$ (input) 6. $\mu=1.0$ (input)

assumption that the building was not discontinued (Fig. 5(a)).

b. Model 2

One part of the central section of the same building was taken and the space between column and stud (one half of the span) was divided into meshes of "12×12" in height and width.

Assumed boundary conditions are that the building extends continuously to the right and left with the analyzed model situated in the center. As a result, Vertical direction of both ends was kept by rollers and the upper end was kept unrestrained and for the lower end the horizontal direction was fixed (Fig. 5(b)).

(3) Input Input factors are as follows.

- Co-ordinate of panel points (omitted)
- Bearing conditions and boundary restraint conditions
- Wall and frame sections (Table 1, Fig. 4)
- Load (input)

In view of characteristics of the applied program, this was made by converting stress caused by drying shrinkage to temperature stress caused by temperature drop, which was pointed out by Ignacio Martin. Shrinkage strain was assumed to be 6.5×10^{-4} , the amount for ordinarily-used concrete (three months old). The method of converting shrinkage strain to temperature strain is as follows.

Ex.

$$-1 \times 10^{-4} = 10^{-5} \text{C} \times T^{\circ}\text{C} \quad \therefore T = -10^{\circ}\text{C}, \quad -6.5 \times 10^{-4} / 1.0 \times 10^{-5} = -65^{\circ}\text{C}$$

where 1. (-) : shrinkage

2. expansion modulus of concrete at normal temperature : $1.0 \sim 1.1 \times 10^{-5} / ^{\circ}\text{C}$

3. free shrinkage / expansion modulus : $1.0 \times 10^{-5} / ^{\circ}\text{C}$.

As a result, it was assumed that there was a temperature drop of "-65°C" on concrete wall sides.

Calculation of temperature stress was made, taking into consideration internal restraint (the amount of reinforcement) caused by reinforcement in each section. On the other hand, it was assumed that concrete under the ground (foundation) did not contract.

foundation : 0°C other part : -65°C

The result of converting unit shrinkage strain of internally restrained concrete in each section to unit temperature strain is shown in Table 1.

(4) Output

Principal unit stress (hereafter called unit shrinkage stress) and deformation of walls and frames are shown in Fig.-6 to 9. Unit shrinkage stress was analyzed within the range of elasticity.

Stress analysis on the assumption that creep coefficient of concrete is 3.0, a 1/3 value of unit shrinkage stress of concrete can be possibly compared to unit stress at the onset of crack development due to shrinkage.

(5) Analysis results and comments

1) Model 1 (end span)

Principal unit stress of end span gradually increased in the walls of lower stories ($80 \sim 90 \text{ kg/cm}^2$) and decreased in the walls of the upper stories. Walls closest to underground beams registered the

Fig. 6 Principal Stress [kg/cm²], Scale of stress [Normal Shear Principal] Horizontally-Long Wall Building

Fig. 7 Displacement Scale of Displacement Horizontally-Long Wall Building

Fig. 8 Principal Stress [kg/cm²] Horizontal-Long Wall Building

Fig. 9 Displacement Horizontal-Long Wall Building

maximum unit stress in the center (141 kg/cm²) and smaller stress towards the end.

Principal stress caused by shrinkage was found to be shaped like $\backslash /$. Principal stress (unit shrinkage stress) became inclined against the horizontal direction toward the unrestrained end, and gradually took a horizontal direction toward the restrained stop. For example, when it was assumed that the stud was fixed by other members, unit shrinkage stress of these members showed a larger value than that of the stud side. Therefore, if the stud restrains the wall just as assumed, it is considered to have little effect on the control of cracks due to shrinkage. For these reasons, it is estimated that cracks which appear on the lower part of the extreme exterior span in the free end become diagonal ($\backslash /$). And they become vertical as they appear closer to the center. Particularly, shrinkage stress on the end span is considered to become smaller because restraint of frames dewindles.

In addition, the foundation in a humid state under the ground and underground beams are predicted to have less shrinkage strain. On the contrary, since the upper stories are subjected to uniform contraction due to drying, they are affected by the underground part and restrained by shrin-

kage strain. Especially, shrinkage strain of the first story is substantially restrained by underground influences.

The degree of restraint is considered to gradually decrease in upper stories. Thus, internal unit stress in R.C. walls varies according to boundary conditions, but cracks may develop when this unit stress exceeds shrinkage stress. Especially, cracks shaped like $\backslash /$ develop when deformation in the unrestrained end of the extreme exterior span surpass that of continuous walls, making principal stress something like $/ \backslash$. In this connection Dr. Tsunoda pointed out that cracks develop at an angle of about 45° . But the results obtained from our calculation indicate that this angle varies in accordance with length/hight ratios and stories of the building. That the difference of stress in the extreme exterior end of the building with horizontally-long walls between upper and lower stories is smaller ($10 \sim 100 \text{ kg/cm}^2$) than that of the building with vertically-long walls ($4 \sim 118 \text{ kg/cm}^2$) seems to result from the fact that the distance restrained by the foundation is small. It also depends on the difference of section ratio between the foundation and upper story members (wall, column, beam).

It seems that cracks develop from lower parts toward upper parts because shrinkage is restrained to a greater degree by underground beams.

At the same time, it can be said that the number of cracks is the largest in the lowest part and decreases as they appear closer to the upper-most part. This is clearly shown by the results of the field studies conducted by Messrs. Tsunoda, Asahina, Unno, Takai, Sugiyama, Nagai, Iwama and the authors.

However, it is supposed that the growing process of shrinkage stress after crack development is different form that precedes crack development.

The amount of deformation registered 25 mm in the vertical direction in the upper-most story (in the fifth story in this case) just as in the central span and gradually decreased in lower stories. The deformation angle and amount of deformation were the largest in the extreme exterior span of the free end and the smallest in the central span. The amount of deformation in the extreme exterior span was 15.6 mm in the horizontal direction. The deformation was inward as a whole and tended to increase in upper stories.

2) Model 2 (central span)

Shrinkage stress of walls in each story registered an approximately uniform $140 \sim 144 \text{ kg/cm}^2$ and the angle of principal stress due to shrinkage was approximate to 0° or 180° against the horizontal. Consequently, it seems that studs hardly influence the decrease in shrinkage stress. Since cracks develop in the direction vertical to that of principal shrinkage stress, it seems that vertical cracks develop in the central span. This has been clearly shown by the results of field studies conducted by Messrs. Tsunoda, Asahina, Hino, Nagai, Iwama and the authors of this report. Shrinkage stress within the range affected by restraint relaxation is slightly larger in the lower part of walls than that of in the upper part. Accordingly, cracks are considered to develop from the lower part toward the upper part of a wall side. Dr. Tsunoda pointed out that this was due to a quantitative difference of shrinkage between concrete placed under the ground and concrete drying in the atmosphere.

It is difficult to give an quantitative indication concerning the location of vertical cracks, but there are not a few cases in which cracks develop vertically in the central span because of defects in the structure. On the one hand, it is considered that the rate of restraint due to the frame was about 100% (rate of restraint ≈ 1) for shrinkage stress in the central span with less effect of restraint relaxation and increased uniformly prior to crack development. Shrinkage stress at that time was $140 \sim 144 \text{ kg/cm}^2$, 37~68% larger than stress ($45 \sim 90 \text{ kg/cm}^2$) in the lower part of the extreme exterior end.

Whereas stress in the central span of the building with horizontally-long walls registered an approximately uniform $140 \sim 144 \text{ kg/cm}^2$, stress (analysis 2) for the building with vertically-long walls

was $4\sim 80\text{ kg/cm}^2$ with the figure decreasing in upper stories. This depends on the length (L)/height (H) ratio of the building and it is considered that the larger the L/H ratio is, the larger and more uniform the unit stress is.

Deformation in the central span is smaller than that of underground beams, and this seems to create a situation where the wall side is fixed almost completely.

It is considered that, even if a girder has almost the same section as the underground beam, there exists a considerable difference in restraint between them depending on shrinkage strain and shrinkage speed. The amount of deformation was larger closer in upper stories and registered 31.2 mm at the top story (eaves height : 8 m).

3.2 Analysis 2 Building with vertically-long walls

(1) Main features of the building

- 1) Construction : ramen structure (R.C., structure five-storied), height : 25.70 m, Size of building draughty through of beam spans : 33.60 m (each span 5.6 m, 6 spans), total length of girder

Fig. 10 Plan

Fig. 11 Section (External Wall)

Fig. 12 Design Condition of Building Supposed

spans : 42.00 m (each span 6.0 m, 7 spans), Slab : large wall side being draughty through all stories without any opening. The plan, elevation and section sizes of members are shown in Fig. 10 to 12.

- 2) Wall system Bar-arrangement design is presented in Fig. 12. Internal and external walls : both architectural concrete finishing.
- 3) Foundation P.C. piles, independent footing.
- 4) Longitudinal joints on the exterior wall sides of all stories were filled with urethane seal after being cut in pitch of 1.6~2.0 m and in gutter of $20\times 20\text{ mm}$.
- 5) Design compressive strength 210 kg/cm^2 (natural aggregate concrete), $n=22$.

(2) Analysis model

This building had walls with the ratio of the length of a girder to height set at 1 : 2. Columns, beams, furring strips were divided into linear members and the wall was divided into members $3\text{ m}\times 3\text{ m}$ and $3\text{ m}\times 4\text{ m}$ in size. Stress analysis was conducted on one half of the entire wall divided symmetrically into two in the center. Assumed boundary conditions were set on the basis of existing buildings.

Footing of the foundation was supported by pins and its symmetric part was given on input of bearing conditions horizontal to the center of shrinkage stress.

The extreme exterior end of the building was kept free and the other end was kept by rollers on the assumption that the building was not discontinued (omitted).

(3) Input Factors of Input were as follows

- a. Co-ordinate of panel points
- b. Bearing condition and boundary restraint

Table-2 Condition on the Stress analysis due to Shrinkage

part	section	width cm	area cm ²	arrangement of bar	k	* temperature x K	Δt °C
C ₁	base of column	75x150	11250	D25-44 D16- 2	0.80	-8x0.82	-6.6
C ₂	capital	75x150	11250	D25-25 D22-12	0.80	-8x0.82 -48x0.82	-6.6 -39.4
FB	underground beam	40x120	4800	D22-3	0.875	-8x0.875	-7.0
B ₂	beam of top story	84x230	19320	D22-16	0.934	-48x0.934	-44.8
B ₁	beam	84x60	5040	D22-10 D19- 6	0.805	-48x0.805	-38.6
W	wall	100x18	1800	D10-8100 6φ-8100	0.892	-48x0.892	-42.8

Note 1. $F_c=210\text{ kg/cm}^2$, 2. $n=22$ (Section Calculation, 3. K : Release Rate of Restraint, 4. $K=1/(1+nu)$, 5. * $K (-E_f\times 10)$

Fig. 13 Principal Stress (kg/cm²) Vertically-Long Wall Building

Fig. 14 Displacement Vertically-Long Wall Building

conditions

c. Wall and frame sections (Table 2, Fig. 12)

d. Load (input) Like analysis 1, this was carried out by converting stress due to drying shrinkage to temperature stress.

Studies previously conducted have indicated that cracks develop about three months after placing concrete. Consequently, shrinkage of reinforced concrete was assumed to be 0.8×10^{-4} for the foundation and 4.8×10^{-4} for other part (shrinkage registered when cracks begin to develop in practical buildings). The method for converting shrinkage strain to temperature stress was the same as that used in Analysis 1. The results are shown in Table 2.

(4) Output

Principal unit stress (called unit shrinkage stress) and the amount of deformation of the wall are shown in Fig. 13 and 14.

(5) Analysis results and comments

In the case of multi-story buildings, unit shrinkage stress increased up to the second story and gradually decreased in upper stories. The direction of stress due to shrinkage in the end span tended to show larger inclinations against the horizontal (direction) at points closer to the free end and approximated to 0° or 180° against the horizontal (direction) in the central part. Therefore, principal stress tended to be shaped like / \.

Also, shrinkage stress in the end span on the lower part of the building was $54 \sim 118 \text{ kg/cm}^2$, about 20% larger than that of the central span ($56 \sim 80 \text{ kg/cm}^2$). The figure for the top story was $2.5 \sim 5.0 \text{ kg/cm}^2$. In comparison, shrinkage stress in the central span was $56 \sim 80 \text{ kg/cm}^2$ on the lowest part (first story), $30 \sim 45 \text{ kg/cm}^2$ on the second story and $14 \sim 24 \text{ kg/cm}^2$ on the third story. These were, respectively, 20% less, 30% larger and 68% larger than the corresponding figures for the end span. However, shrinkage stress in the top story registered $2.5 \sim 5.0 \text{ kg/cm}^2$ for both end span and central span.

Thus, that shrinkage stress of concrete becomes smaller in upper stories can be explained by the fact that transmission of restraint force weakens gradually as the distance from the foundation increases. On the other hand, shrinkage stress represents results of analysis within the range of elasticity.

But it is considered that a $1/3$ value of unit shrinkage stress can be compared to stress at the onset of crack development due to shrinkage if stress analysis is conducted on the assumption of creep coefficient of concrete = 3.0. This shows that shrinkage stress on and above the third story is smaller than tensile strength ($F_t = 1/10 \sim 1/13$) of concrete.

And, it can be said that crack development decreases in the upper stories. It can also be said that cracks which develop on the lower part up to 2 spans from the end assume a shape like \ / while those on the central span are vertical.

As mentioned previously, this is supposed to result from the fact that strain of concrete caused by restraint of the foundation becomes larger than the critical amount of shrinkage strain for crack development. Shrinkage stress within the range of effective restraint relaxation is larger in the lower story of a wall than in the upper story. Therefore, it is considered that crack develop from the lower story of a wall toward the upper story.

This is clearly shown by the results of field studies conducted by Messrs, Tsunoda Asahina, Hino, Nagai, Iwama and the authors of this report. Meanwhile, it can considerably expected that cracks can be concentrated by adopting joints. This is related to the fact that stress increases in defective parts caused by joints in concrete section. But, location and depth of joint cuts require a comprehensive study from the stand points of designing and construction work. Deformation increased as it moved from the lowest story toward the uppermost story. Deformation in the central span was 45 mm in the vertical (direction) on the top story (fifth story). In the horizontal (direction), it became smaller as it moved from the end toward the center, approximating to 0° in the center. In comparison, deformation in the end span on the top story was 45 mm in the vertical (direction) and 45 mm in the horizontal (direction).

It was also found that deformation angles became larger in the end span.

4. Conclusion

Results of shrinkage stress analysis clarify the following points.

- a. These analysis results can help estimate magnitude of principal stress due to shrinkage on concrete wall sides and predict location, direction and shape of cracks.
- b. The magnitude of shrinkage stress obtained by this method was analyzed according to elastic theory. It showed a larger value than a true unit stress of concrete. The magnitude of this stress may be evaluated as an indicator concerning the danger of crack development.

For these reasons, in order to prevent or reduce cracks due to shrinkage, it is desirable to make a stress analysis of the entire building or frames as means of foreseeing the possible location and direction of cracks. At the same time, a comprehensive study is necessary on ways to reduce shrinkage of concrete, bar-arrangement, wall thickness and suitable positions of contraction joints.

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R. C. 造壁面の収縮による応力解析とひびわれ発生に関する研究

— I. 応 力 解 析¹⁾ —

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1. ま え が き

コンクリート構造物の収縮ひびわれはコンクリートの硬化、乾燥するさいの収縮変形とそれを拘束している建物の架構状態などに影響されて発生することが多い。

一家、石川氏の調査²⁾によると、コンクリート構造物のひびわれ発生は壁面が24%と最も多く、屋根板が21%、床板が17%、はり、パラペットの順となり、ひびわれ発生が漏水、美観の損傷、鉄筋・鉄骨の発錆、コンクリートの凍害助長などを引き起こす原因となっていることも少なくない。このうちで、特にひびわれ防止を必要とする要素は漏水の防止が37%、構造上が23.9%、外観上が21.7%、鉄筋防錆が17.4%の順になっている。

このように、ひびわれ発生はコンクリート構造物の耐久性に大きな影響を与えるものである。また、ひびわれ発生はコンクリートの性質および構造物のあらゆる条件に左右される複雑な因果関係にあり、諸要因の相乗作用に起因されている。これら収縮ひびわれに関する既往研究は狩野博士^{3)~5)}をはじめとして、大野^{6),7)}、近藤⁸⁾、佐治⁹⁾、田中¹¹⁾、仕入^{4),10),13)}、向井¹²⁾各博士などの数多くの研究者によって検討され、収縮低減方法についても“材料・調合、施工、構造”の各方面からまとめられている。これらを総括すると、ひびわれ発生の防止または低減するための終局的な方策は“収縮量の低減、ひびわれの分散およびひびわれの集中化”することであると結論している。しかし、これらの研究の多くは単一部材もしくは実態調査の一部によるもので、建築物を長さ、高さから壁面全体をとらえて収縮による主応力を解析し、拘束力の緩和などを考慮してひびわれ防止を対策したものが少ない。

このことから、壁面の収縮ひびわれ発生を究明するためにはコンクリートの材料的な考慮だけでなく、建物全体としての収縮による主応力と変形性状を把握することが重要である。

壁面の収縮ひびわれ発生に関する既往研究を仔細に検討すると以下のような知見が得られている。

a. 硬化・乾燥収縮によるもの

ひびわれ発生の限界収縮量は川砂・川砂利コンクリートが 4×10^{-4} ^{4),6),27)}、人工軽量骨材コンクリートが 3.8×10^{-4} ²⁷⁾である。ひびわれ発生の位置は中央部スパンで垂直ひびわれ、端部スパンへ移行するにつれて傾斜し、最外端スパンのひびわれが最も傾斜角度が大きい^{14),15)}。また、水平ひびわれは打継ぎに多い²⁰⁾。最下層のひびわれは上層に比べて著しく多く、かつ大きい。方位別のひびわれは北面より南面に多いが、東・西面では大差ない。ひびわれ幅は通常の場合“0.1~0.5 mm”で実害となる¹⁴⁾。壁面のひびわれはコンクリート打設後3カ月あるいは6カ月後に認められる¹⁴⁾。

b. 温度変化の伸縮によるもの^{22),23)}

コンクリートの伸縮性状は気温と相似な傾向を示している。2.5 カ月後の壁の収縮量は川砂・川砂利コンクリートが0.35 mm/m、3種軽量コンクリートが0.4 mm/m、4種軽量コンクリートが0.5 mm/mとなる²¹⁾。柱の収縮量は壁に比べて約半分であり、収縮速度も遅い²¹⁾。しかし、ひびわれは一度発生すると外界温度の影響を受けて、その幅は常に変動している¹⁴⁾。日照を受けるRC造の温度膨張係数は壁厚10 cmで“ $1.05 \sim 1.2 \times 10^{-5}$ ”、20 cmで“ 1.2×10^{-5} 内外”の値を示している²³⁾。

c. 設計および施工の不備と欠陥によるもの

- ・材料・調合 骨材種類と品質、セメント種類、調合（特に、水セメント比、単位水量の過剰）^{3),4),5),10),13)}
- ・施 工 壁面保護の不備、養生方法、打込み方法の不備、打設区画と打込み順序の誤り^{9),10),11)}
- ・設 計 壁厚の過薄、補強鋼材の不足、配筋方法の誤り^{9),10),12)}

d. 基礎、地盤の沈下によるもの（詳略）^{25),26)}e. 外力によるもの（詳略）²⁴⁾

f. 以上の合成組合せによるもの（詳略）

本研究はコンクリートの乾燥収縮に起因する壁面の収縮ひびわれについて、コンクリート構造物の全壁面を考慮したモデル化をおこない、有限要素法によって建物全

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体あるいは一部について収縮による応力解析して、ひびわれ発生位置、方向、形態を予測し、既存建物のひびわれ発生状況との相関性を明らかにした。さらに、応力解析と実態調査から得られたひびわれ防止対策は2, 3の建物に設計当初より考慮し、引続いてその建物について、追跡調査した結果をまとめたものである。今回はRC造（横長壁の建物、縦長壁の建物）の壁面を有限要素法を用いて収縮による応力解析して、ひびわれ発生位置、方向、形態を予測したものである。次回は解析結果と既存建物のひびわれ発生状況との相関性を明らかにし、ひびわれ防止対策の一例と応力解析の信憑性を記述する。

2. ま と め

これらの解析結果から、以下のことが明らかとなった。

(1) 収縮による応力解析について

- a. この解析結果はコンクリート壁面の収縮主応力度の大小とひびわれ発生位置、方向ならびに形態を予測することができるものと思う。
- b. この方法で得られた収縮応力度の大きさは終局時まで弾性範囲で解析したものであり、コンクリートの真の応力度より大きい値を示した。この応力度の大小はひびわれ発生危険性を示す指数として評価することができるものとする。

(2) モデル建物について

1) 横長壁の建物

(a) 端部スパン

収縮応力度は壁の下部 ($80 \sim 90 \text{ kg/cm}^2$) ほど大きくなり、上部へ移行するにつれて減少していた。地中ばりに接した壁は中央部で最大の収縮応力度 (141 kg/cm^2) を示し、端部へ近づくにつれて減少していた。ゆえに、拘束係数は端部スパンで小さく、中央部スパンほど大きくなるという。このことから、収縮による主応力の方向は八字型となり、ひびわれ発生は端部スパンの下部で斜めひびわれ（逆八字型）、中央部へ移行するにつれて垂直ひびわれになるものと思う。一方では端部スパンで架構の拘束が少なくなるため、収縮応力が小さくなるからという。また、ひびわれは地中ばりで収縮を拘束する割合が大きいため、下部から上部へ発生していくものとする。さらに、横長壁建物の間柱は収縮ひびわれ防止にそれほど影響しないものと判断される。

変形量は中央部スパンと同じように上層へ行くにつれて大きく、最上層の垂直方向で 25 mm となった。変形

角および変形量は最外端スパンほど大きくなり、中央部スパン移行するほど小さい。特に、最外端スパンの変形量は水平方向で 15.6 mm となった。

(b) 中央部スパン

各層における部材の収縮応力度は $140 \sim 144 \text{ kg/cm}^2$ で、全壁面ほぼ均一となり、主応力の方向は水平方向に対して零度または 180 度に近似していた。したがって、ひびわれは垂直方向に発生するものと思う。このことは角田博士^{14), 15)}によっても、地盤下にあるコンクリートと大気中で乾燥するコンクリートとの収縮量の差によるものと指摘されている。拘束緩和の影響が小さい中央部スパンでは上部から下部までの架構による拘束度が約 100% (拘束度 $\div 1$) に近似しており、ひびわれの入るまで収縮応力が均一に大きくなるものといえる。

変形量は上層へ行くにつれて大きくなり、最上層で 31.2 mm の値を示した。

2) 縦長壁の建物

多層階の場合、収縮応力度は特に、2層まで大きくなり、上層へ行くにつれて減少していた。端部スパンの主応力の方向は自由端に近いほど水平方向に対して傾斜角度を持ち、中央部へ移行するにつれて零度または 180 度に近似して、八字型を示した。また、コンクリートの収縮応力度が上層ほど小さくなるのは基礎部分から遠くなるにつれて拘束力の伝達が次第に小さくなったものと考ええる。

ひびわれは端部2, 3スパンまで下部で逆八字型に、中央部スパンで垂直に発生するものと思う。これは基礎部分の拘束によって受けたコンクリートがひびわれ発生限界の収縮ひずみより大きくなるからと推察される。

また、ひびわれは壁面の下層から上層に向かって発生するものといえる。

以上のことから、収縮ひびわれ発生を防止または低減するためには建物全体あるいは架構を応力解析して、ひびわれ発生位置、方向を予測しながらコンクリート自身の収縮量の低減、配筋計画、壁厚、伸縮目地の位置などを総合的に検討することが望ましい。

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