ALIGNMENT CHARTS FOR THE EFFECTIVE LENGTH FACTOR IN A FRAMED COLUMN INCLUDING SHEAR DEFORMATION

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§1. Introduction

The elastic buckling load of a frame is an useful parameter for the prediction of the frame collapse load. In assessing the actual failure load the theoretical elastic buckling load of an overall frame is frequently obtained for the theoretical studies. However, it is scarcely obtained for the practical design of a frame, because that laborious computations are needed. This doesn't mean no need to consider the overall frame instability. It may be guaranteed by taking into account of the effective length of the each framed column for the practical design.

There are several proposals^{1)~4)} to estimate the approximate effective length of the framed column. So-called alignment charts, which are suggested in the CRC Guide⁵⁾ and are used as the basis for the AISC specifications, seem to be the simplest.

Recently, it is noticed^{6),7)} to estimate a frame stiffness more exactly in the dynamic analysis of multi-story steel frames. As often pointed out, the shear deformations of members in a frame affect the overall frame stiffness considerably. The effects of shear deformations of members, as a matter of course, make the frame stiffness reduce. The decrease of the frame stiffness corresponds to make effective lengths of each framed columns longer for the practical design.

In this paper, alignment charts for the effective length factor in a framed column including the shear deformation are proposed and illustrated a few examples to show the procedure to find the effective length factor and the effects of the shear deformations. Proposed alignment charts may be useful to consider the frame instability more precisely.

§2. Buckling criteria of the column in a frame

2-1 Basic assumptions

- 1) Columns in a frame are perfectly elastic and prismatic.
- 2) In the frame buckling the restraining moments from the beams at the joint may be balanced to the columns in proportion to their nominal stiffnesses.
- 3) Adjacent columns are assumed nearly identical in the shear-deformation parameter ϕ , which is expressed as $\phi = \frac{12EI}{GA_s l^2}$ where EI and GA_s are the flexural and shear rigidities and l is the length of the member, respectively.
- 4) It may be assumed that a frame buckles totally when all columns of it buckle simultaneously and that the buckling criterion of the frame may be considered in a replaced subassemblage of a column and two beams connected at the both ends of the column.
- 5) The effective length factor is expressed as

$$K = \frac{l_k}{l} = \sqrt{\frac{P_e}{P_{cr}}}$$

where $P_e = \frac{\pi^2 EI}{l^2}$, the Euler load of the pin-ended column, and P_{cr} is the buckling load of the subas-

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semblage.

6) The nonlinear stiffness matrix of a beam element including the shear deformation may be given⁸⁾ as

$$[K] = \begin{bmatrix} \alpha \frac{EI}{l} & \beta \frac{EI}{l} & r \frac{EI}{l^2} & -r \frac{EI}{l^2} \\ & \alpha \frac{EI}{l} & r \frac{EI}{l^2} & -r \frac{EI}{l^2} \\ & & \delta \frac{EI}{l^3} & -\delta \frac{EI}{l^3} \\ \text{sym.} & & & \delta \frac{EI}{l^3} \end{bmatrix} \dots \dots \dots (1)$$

where the coefficients are

$$\begin{aligned}\alpha &= \frac{z(\sin z - a \cdot z \cdot \cos z)}{2 - 2 \cos z - a \cdot z \cdot \sin z}, & \beta &= \frac{z(a \cdot z - \sin z)}{2 - 2 \cos z - a \cdot z \cdot \sin z} \\ \gamma &= \frac{a \cdot z^2(1 - \cos z)}{2 - 2 \cos z - a \cdot z \cdot \sin z}, & \delta &= \frac{a^2 \cdot z^3 \cdot \sin z}{2 - 2 \cos z - a \cdot z \cdot \sin z}\end{aligned}$$

when a compressive axial force P is presented, in which

$$a=1-\frac{P}{GA_s}=1-\frac{\pi^2}{12}\phi\frac{P}{P_e}, \quad z=\pi\sqrt{\left(\frac{P}{P_e}\right)\cdot\frac{1}{\left(1-\frac{\pi^2}{12}\phi\frac{P}{P_e}\right)}}$$

and

$$\alpha = \frac{4+\phi}{1+\phi}, \quad \beta = \frac{2-\phi}{1+\phi}, \quad r = \frac{6}{1+\phi}, \quad \delta = \frac{12}{1+\phi}$$

when no axial force exists.

2-2 Criterion for an unbraced frame buckling (Sway columns)

Provided that the rotations at the far ends of restraining beams are equal in magnitude and same in sense to the joint rotations at the column end as shown in Fig. 1, which yield $\theta_e = \theta_b = \theta_f$ and $\theta_c = \theta_a = \theta_d$. The modified slope-deflection equations for a subassemblage are, therefore, given as follows. Of course, the effects of the shear deformations in the beams are considered.

$$\begin{aligned} M_{ac} &= \frac{EI_{bLL}}{l_{bL}} \cdot \frac{6}{1 + \Phi_{bLL}} \cdot \theta_a \\ M_{ad} &= \frac{EI_{bRL}}{l_{bR}} \cdot \frac{6}{1 + \Phi_{bRL}} \cdot \theta_a \\ M_{be} &= \frac{EI_{bLU}}{l_{bL}} \cdot \frac{6}{1 + \Phi_{bLU}} \cdot \theta_b \\ M_{bf} &= \frac{EI_{bRU}}{l_{bR}} \cdot \frac{6}{1 + \Phi_{bRU}} \cdot \theta_b \\ M_{ab} &= \frac{EI_c}{l_c} \cdot \{ \alpha \cdot \theta_a + \beta \cdot \theta_b - \gamma \cdot (v_b - v_a) / l_c \} \\ M_{ba} &= \frac{EI_c}{l_c} \cdot \{ \beta \cdot \theta_a + \alpha \cdot \theta_b - \gamma \cdot (v_b - v_a) / l_c \} \end{aligned}$$

..... (2)

Using the assumptions, the equilibrium of the moments at the top and bottom joints of the column yields

$$\left. \begin{aligned} M_{ab} + \frac{I_c/l_c \cdot (M_{ac} + M_{ad})}{I_c/l_c + A I_c/A' l_c} &= 0 \\ M_{ba} + \frac{I_c/l_c \cdot (M_{be} + M_{bf})}{I_c/l_c + B I_c/B' l_c} &= 0 \end{aligned} \right\} \dots (3)$$

Considering the equilibrium of the lateral forces in the column, since no lateral force

Fig. 1 Sway buckling mode

The equation is slightly modified with a parameter, a , from the buckling condition not including the shear deformations of the members. However, the parameter G is expressed a nominal stiffness ratio only because that the effects of the shear deformations in the beams are just cancelled through its derivation.

2-4 Correction factor into beam stiffness

The factors by which the beam stiffness may be multiplied when the boundary conditions at the far end of the beam are definitely known are described in Table 1.

§ 3. Alignment charts

The buckling conditions obtained from Eqs.

(5) and (7) may be conveniently expressed in well-known nomographic forms with the shear-deformation parameter ϕ_c of the column. However, it may be very laborious to use so many nomographs prepared for each ϕ_c even if possible. Consequently, condensed nomographic forms having the shear-deformation parameter ϕ_c are proposed as shown in Fig. 3-a and 3-b for the unbraced frame buckling and the braced frame buckling, respectively.

Strictly speaking, as far as the braced frame buckling is concerned the ordinate of G -scale in the nomograph is slightly shifted between the terminals in accordance with parameter ϕ_c . However, in the nomograph shown in Fig. 3-b G -values are plotted for the case of $\phi_c=0$. Consequently K -values are modified for the column restrained equally in both ends. In the case of the greatest ϕ_c in Fig. 3-b the error is estimated only about 2% at the most for a column with a hinged and fixed boundary at the both ends.

§ 4. Numerical examples

At first, the procedure to find the effective length factor on the proposed alignment chart is illustrated in Fig. 4. Given G_A and G_B are 1.5 and 0.5 respectively and ϕ_c is 0.25, the effective length factor K for a no sway column is found to be 0.88.

Next, consider the example of a two story frame shown in Fig. 5, which is illustrated in the CRC Guide. The beams and columns in the frame have various boundary conditions as a illustration and columns in the upstairs are permitted to sway and columns in the downstairs are prevented to sway. All columns are oriented webs parallel to the plane of the drawing. All columns are assumed to be braced in the weak direction. Section properties of the members in the frame are shown in Table 2. The procedures of computations are the same as the illustration of the CRC Guide and its proposals to design practice are also adopted.

Table 2 Section properties of the members

Member	Designation	Moment of Inertia, I (in. ⁴)	Shear Area, A_s (in. ²)	Shear Parameter ϕ
B1	12W 27	204.1	2.65	0.085
B2	12W 27	204.1	2.65	0.069
B3	14W 30	289.6	3.54	0.196
B4	14W 30	289.6	3.54	0.090
B5	14W 30	289.6	3.54	0.074
C1	6W 15.5	30.3	1.29	0.042
C2	8W 31	109.7	2.05	0.096
C3	6W 15.5	30.3	1.29	0.070
C4	6W 15.5	30.3	1.29	0.038
C5	6W 15.5	30.3	1.29	0.038
C6	8W 31	109.7	2.05	0.087

Table 3 Comparison of the effect due to the shear deformation

Column	Neglected shear deformation		Considered shear deformation		Rate of Increase (%)
	K	l	K	l	
C1	1.06	11'-8"	1.07	11'-10"	1.0
C2	1.18	13'-0"	1.22	13'-5"	3.4
C3	1.89	16'-1"	1.90	16'-2"	0.5
C4	0.86	9'-11"	0.88	10'-2"	2.3
C5	0.65	7'-6"	0.67	7'-9"	3.1
C6	0.78	9'-0"	0.82	9'-5"	5.1

The comparisons of the effect due to the shear deformation in the members are summarized in Table 3. In this example, as mentioned in the CRC Guide, it is not intended to illustrate a certain actual structure so that the shear deformations are not so remarkable. However, the column of $\phi_c=0.087$ increased about 5% in the buckling length.

§ 5. Conclusion

Fig. 3-b Extended alignment chart
(Sidesway prevented)

Fig. 3-a Extended alignment chart
(Sidesway permitted)

Fig. 4 Procedure for the effective length factor (Sidesway prevented)

shear-deformation parameter ϕ_c . The effect seems to be much more sensitive for the no sway buckling mode than for the sway mode. The proposed extended alignment charts may be useful to consider the frame instability more precisely.

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Fig. 5 Model frame in the CRC Guide

It is easy to obtain the effective length of a framed column including the shear deformation by using the proposed extended nomographs herein.

At first sight the effect due to the shear deformation is apparent in these nomographs as a function of the

せん断変形を考慮した有効座屈長さ係数図表 (梗概)

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骨組の弾性座屈荷重は骨組の実際の崩壊荷重を予測する重要なパラメーターの1つである。しかし、実施設計の段階においては骨組の弾性座屈荷重を求めることは殆んどない。これは骨組の安定性を考えないということではなく、骨組の中の柱材の有効座屈長さを考えることにより、全体骨組の安定性が一応保証されるためである。

最近、高層骨組の振動解析に際し、骨組の剛性を正確に評価しようとするところから、部材のせん断変形を考慮することが行なわれていて、その影響がかなり大きいことがわかっている。したがって、部材のせん断変形の影響で骨組の弾性座屈荷重も低下するであろうことは予想される。本論文では、全体骨組座屈を骨組を構成する部分構造の座屈問題に置換し、部材にせん断変形を考慮する場合の、部分構造の中の柱材の有効座屈長さ係数を求める nomograph を提案する。これはアラインメントチャートと呼ばれているものに、部材のせん断変形の効果を表わすパラメーター $\phi \left(= \frac{12EI}{GA_s \cdot l^2} \right)$ を導入し、拡張したものである。

上述のように、本論文では全体骨組を1本の柱と、その上下端でそれぞれ2本の剛接されたはりから構成する部分構造に置換し、その座屈を考えるが、水平移動形、水平移動拘束形の座屈モードに対し、それぞれの座屈条件式を誘導する。柱材の節点力と節点変位の関係は、先に、筆者が提案したせん断変形を考慮する座屈撓角法基本式を使う。座屈条件式を誘導する際に、はりのせん断変形も当然考慮している。

まず、水平移動形座屈条件式はせん断変形を考慮しな

い場合とその形が同じであるが、柱-はり剛度の比 G を計算する際、はりのせん断変形による効果を含めねばならない。一方、水平移動拘束形座屈条件式の場合は、せん断変形を考慮しない条件式にパラメーター a 注(1) が若干附加された形となるが、はりのせん断変形効果は G の計算式には表われてこない。これらの座屈条件式をもとに、オイラー荷重に対する有効座屈長さ係数を求める nomograph を作成した。せん断変形パラメーター ϕ のそれぞれの値に対して、厳密には、グラフをそれぞれ求めるべきであるが、簡便の為、1つのグラフにまとめた。その結果、特に水平移動拘束形のグラフでは、実際は ϕ のそれぞれの値に対して G 目盛が異なってくるが、提案したグラフでは $\phi=0$ に対してのみ G の値を目盛り、それに対応する有効座屈長さ係数 K の値は、柱の上下の G の値が等しい場合の値をプロットした。このため、 G 値の、柱の上下における差が大きくなるにつれて K 値は誤差を生ずることになるが、それでも高々2%で実用上差し支えない範囲である。計算例として、CRC Guide注(2)の骨組モデルを使い、せん断変形を考慮する場合と考慮しない場合の、有効座屈長さ係数を比較計算した。この骨組程度の比較的軽微な柱材でも約5%の影響量がでている。

ここで提案する、部材のせん断変形を考慮する有効座屈長さ係数を求める拡張された nomograph は、骨組の安定性を、より正確に評価する場合に有効であろう。

注(1) $a = 1 - \frac{P}{GA_s}$, P は柱の圧縮軸力, GA_s はせん断剛性。

注(2) Johnston B.G., "The Guide to Design Criteria for Metal Compression Members", 2nd ed., John Wiley & Sons, Inc., 1966.

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