FLAT SLAB STIFFENED WITH ONE-WAY CURVED BEAM SYSTEM

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INTRODUCTION

The Flat slab is generally used for the floor and roof of warehouse as well as garage. When the distance between the columns supporting the flat slab is considerably long, it is more economic to stiffen it with ribs (or beams). The Gatti wool Factory (in Rome) and the Palace of Labor (in Torino), designed by P.L. Nervi and the Taiseiki Temple (in Fujinomiya), by Professor S. Aoki, are well known as the example of the slab structure reinforced by curved beams. All these slabs are reinforced by the curved beams in the principal directions in bending of isotropic flat slab. The curved beams discussed herein may be obtained by the author's method^{(1),(2)}, which is based upon the fact that the streamlines agree with the direction of primary beam and that the potential lines with that of secondary beam, under assumption that the primary beams alone carry the loads to these supports. Although an example of the flat slab was discussed in previous monograph, the present paper will be reported on the one-way curved beam system in the case of the flat slab, from both theoretical and experimental view points.

1. ANALOGY BETWEEN FLAT SLAB AND FLUID WITH OUTFLOW AND INFLOW POINTS

The complex potential of two-dimensional stationary stream with a outflow point at the origin is expressed by

$$f(z) = \frac{m}{2\pi} \log z \quad \dots\dots\dots (1)$$

where m is the quantity of the fluid streaming out from the origin. Let us assume the outflow point at the coordinates of $x=a, y=a$ and $x=-a, y=-a$, and the inflow point at those of $x=a, y=-a$ and $x=-a, y=a$, as shown in Fig. 1. The potential in this case may be obtained by applying Eq.1 to those points and summing these results as follows :

$$f(z) = \frac{m}{2\pi} \{ \log(z-a-ia) + \log(z+a+ia) - \log(z-a+ia) - \log(z+a-ia) \} \quad \dots\dots\dots (2)$$

Let the distances from an arbitrary point to these four points be r_1, r_2, r_3 and r_4 , respectively and let the angles between these lines and the x axis be $\theta_1, \theta_2, \theta_3$ and θ_4 , respectively. Then, Eq. 2 can be rewritten in the following from :

$$f(z) = \frac{m}{2\pi} \log \frac{r_1 e^{i\theta_1} r_2 e^{i\theta_2}}{r_3 e^{i\theta_3} r_4 e^{i\theta_4}}$$

The real part of this equation is given by

$$u = \frac{m}{2\pi} \log \frac{r_1 \cdot r_2}{r_3 \cdot r_4}$$

Fig. 1 Field of two-dimensional fluid with two outflow and inflow points, respectively.

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(1) G. Matsui, "Pattern of One-Way Curved Beam System and Its application", Trans. A.I.J., No. 241, March, 1976.

(2) G. Matsui, "One-Way Curved Beam System with the Regular Polygonal Plates", "Trans. of A.I.J. No. 251, January 1977.

By transforming the above equation into the Cartesian coordinates (x, y) , we obtain

$$u = \frac{m}{4\pi} \log \frac{(x^2 + y^2)^2 + 4a^4 - 8a^2xy}{(x^2 + y^2)^2 + 4a^4 + 8a^2xy} \quad (3)$$

Also, using the polar coordinates (r, θ) , we obtain

$$u = \frac{m}{2\pi} \log \frac{r^4 + 4a^2(a^2 - 2r^2 \sin \theta \cos \theta)}{r^4 + 4a^2(a^2 + 2r^2 \sin \theta \cos \theta)} \quad (3)'$$

It is assumed that the column supporting the flat slab is infinitely continuous in every direction of it. However, in this case the area enclosed by the lines connecting four points will be close to that of flat slab. Let us now assume that the radius of capital in the flat slab, R , is very small. The shearing forces along the outer circumferential boundary in the capital are equal to aw at the intersection of column row line with its boundary and $\sqrt{2}aw$ at the intersection of diagonal line. Under the assumption that the shearing forces between these lines change in the shape of trigonometric function, the shearing forces can be given by

$$(Q_\alpha)_R = \left\{ 1 + \frac{(\sqrt{2}-1)}{2} (1 - \cos 4\theta') \right\} aw$$

where θ' is the angle between the arbitrary line from the centre of capital and the column row line. The mean shearing force is equal to

$$\frac{4}{\pi R} \int_0^{\pi/4} \left\{ 1 + \frac{(\sqrt{2}-1)}{2} (1 - \cos 4\theta') \right\} aw R d\theta' = \frac{1+\sqrt{2}}{2} aw$$

Accordingly, making use of

$$m = \frac{1+\sqrt{2}}{2} aw$$

as the value of m in the case of fluid, Eq. (3) and Eq. (3)' represent the shearing forces in the flat slab.

Therefore, the shearing forces in the column row line of $x=a$ are obtained from Eq. (3) as follow :

$$Q_\alpha = -\frac{(1+\sqrt{2})}{8\pi} aw \log \frac{(a^2 + y^2)^2 + 4a^2(a^2 + 2ay)}{(a^2 + y^2)^2 + 4a^2(a^2 - 2ay)} \quad (4)$$

The shearing forces in the diagonal line of $\theta = \pi/4$ can be found from Eq. (3)' as follows :

$$Q_\alpha = -\frac{(1+\sqrt{2})}{8\pi} aw \log \frac{r^4 + 4r^2(a^2 + r^2)}{r^4 + 4r^2(a^2 - r^2)} \quad (4)'$$

The results for computation of Q_α are shown in Fig. 2 (a) and (b).

To verify this theory, we have performed the experiment that stretched a membrane over the square frame with side length of 30 cm $\times$ 30 cm which, in the case of $a=7.5$ cm, was lifted at two outflow points and dropped at two inflow points (where the difference in height between these two points is equal to 2 cm, respectively). Photo. 1 demonstrates the contour lines of this membrane which have been obtained from Moire method. Hence, it is seen that the shearing force Q_α is equal

Fig. 2 (a) Shearing forces in column row line.

Fig. 2 (b) Shearing forces in diagonal line.

Photo. 1 Moiré fringes of contour lines in the flat slab supported by small column capital.

Fig. 3 The primary and secondary beams of one-way beam system in the flat slab under the assumption that the column capital is a point.

to zero at the middle point of column row line. The values of shearing force which have been evaluated from this experiment are well agree with the theoretical values, as plotted by solid points in Fig. 2. The plan of the primary and secondary beams in one-way curved beam system, which have been found from the contour lines obtained, is shown in Fig. 3, where the solid line denotes the primary beam and dashed line the secondary beam.

2. FLAT SLAB WITH SKEW COLUMN ROW LINES

The column row lines in flat slab are in general orthogonal. However, they are skew in the flat slab whose column rows are moved only a in the lateral direction, as shown in Fig. 4. In this case, by applying the stream line of Karman's vertex street to the potential line, we obtain the following shearing force Q_a :

$$Q_a = \frac{m}{2\pi} \log \frac{\cosh \frac{\pi}{a}(y-a) - \cos \frac{\pi}{a}x}{\cosh \frac{\pi}{a}(y+a) + \cos \frac{\pi}{a}x} \quad \dots\dots\dots (5)$$

Fig. 4 Field of fluid corresponding to the flat slab with skew column row lines.

where $m = (3 \times 5/4 + 2 \times \sqrt{5}/2 + 1)aw/6 = 1.16aw$. Fig. 5 expresses the shearing forces in the y axis in Fig. 4 which have been computed by Eq. (5). Analogously, the shearing forces in a skew line connecting the columns, *i.e.*, the line connecting t, e points of outflow and inflow, are represented in Fig. 6.

Fig. 5 Shearing forces in the y axis in Fig. 4.

Fig. 6 Shearing forces in the line between outflow and inflow points in Fig. 4.

Photo. 2 Contour lines in the flat slab with skew column row lines.

Fig. 7 The primary and secondary beams in the flat slab with skew column row lines.

Photo. 2 shows the contour lines in this case which have been obtained by Moiré method. These experimental values are considerably close to the theoretical ones, as plotted by solid points in Fig. 5 and Fig. 6. However, in order that the theoretical values might be obtained for the case of two column row lines and otherwise that the experimental values for the case of three column row lines, there is a little discrepancy between both values. Fig. 7 demonstrates a plan of the primary and secondary beams, which has been evaluated by these results.

3. SHEARING FORCES NEAR AND IN THE COLUMN CAPITAL

In the theoretical consideration mentioned previously, the radius of the column capital was taken as a point under the assumption that it is so much small. However, it is never the point in practice. Let us hence discuss with regard to the difference between the real structure and this assumption. The contour lines near and in the column capital are so much similar to the family of concentric circle that Eq. (1) may be assumed to satisfy approximately, as seen in Photo. 1. Since the column row and diagonal lines correspond to the primary beams, the shearing forces are equal to $(a-R)w$ at the intersection between the column row line and the circumferential line of column capital and $(\sqrt{2}a-R)w$ at the intersection between the diagonal line and it. If it is assumed that the shearing forces in the area of the column capital change in the form of trigonometric function, we obtain the shearing forces in the circumferential line of column capital as follows :

$$(Q_\alpha)_{r=R} = \left(\frac{1+\sqrt{2}}{2}a - R \right) w - \left(\frac{\sqrt{2}-1}{2} \right) aw \cos 4\theta$$

Let us rewrite the shearing forces in the following form :

$$Q_\alpha = A_0 \frac{\log\left(\frac{r}{a}\right)}{\log\left(\frac{R}{a}\right)} + A_4 r^{-4} \cos 4\theta$$

where A_0 and A_4 are constant. If the coefficients A_0 and A_4 are determined from the boundary conditions in $r=R$, the shearing forces lead to

$$Q_\alpha = \left(\frac{1+\sqrt{2}}{2}a - R \right) w \frac{\log\left(\frac{r}{a}\right)}{\log\left(\frac{R}{a}\right)} - \frac{(\sqrt{2}-1)}{2} aw \left(\frac{R}{r} \right)^4 \cos 4\theta \dots \dots \dots (6)$$

The values of the first term of Fq. (6), which were calculated for the various ratios of R to a , are shown in Fig. 9 (a), (b) and (c).

The experiment with membrane has been performed for the purpose of comparison with the results of this calculation,

Fig. 8 Area near and in the column capital.

Fig. 9 (a) Shearing forces near and in the column capital in case of $R=a/10$.

Fig. 9 (b) Shearing forces near and in the column capital in case of $R=a/5$.

Fig. 9 (c) Shearing forces near and in the column capital in case of $R=a/2.5$.

Photo. 3 Contour lines in the flat slab in case of $R=a/10$.

Photo. 4 Contour lines in the flat slab in case of $R=a/5$.

Photo. 5 Contour lines in the flat slab in case of $R=a/2.5$.

as shown in Photo. 3, 4 and 5. In Fig. 9, the experimental values in the column row line are expressed by the solid points and these values in the diagonal line by the hollow points. Since the contour lines are assumed to be the concentric circle, the values of shearing forces under this assumption are relatively different at the area far from the column capital, but these values are well agree near and in the column capital. The numerical values of the last term of Eq. (6) are graphically shown in Fig. 10. In order that the variable r^4 is included in the denominator of the last term of Eq. (6), the values of shearing forces decrease rapidly as the place is far from the edge of column capital, and hence there is little influence for

Fig. 10 Influence due to change of shearing forces along the circumferential edge of column capital.

Fig. 11 (a) The primary and secondary beams in the case of $R=a/10$ when the column capital is large.

Fig. 11 (b) The primary and secondary beams in the case of $R=a/5$ when the column capital is large.

Fig. 11 (c) The primary and secondary beams in the case of $R=a/2.5$ when the column capital is large.

the shearing forces in the entire area of the flat slab.

4. FLAT SLAB STIFFENED WITH ONE-WAY CURVED BEAM SYSTEM

When the column capital is small, the potential lines of fluid in which the outflow and inflow points exist alternately, show the direction of the secondary beam while the stream lines the direction of the primary beam. Eq. (6) holds the solution near and in the column capital. The value of the second term in this equation decreases rapidly at the place being far away the column capital. Therefore, even if it is assumed that the shearing force Q_a is constant along the boundary of the column capital, there is little difference between exact values and those.

Fig. 11 (a), (b) and (c) demonstrate the primary and secondary beams of one-way curved beam system which have been obtained from Photo. 3, 4 and 5.

ADDITIONAL REMARKS

The total amount of bending moments in the flat slab supported by the bay of $2a$ has been compared in both cases where the column row lines are orthogonal and skew. Let us consider the following conditions; $a=7.5$ m, $c=2.96$ m⁽³⁾ and $w=1$ t/m. From the previous monograph the sum of bending moments in the area a of the flat slab whose column row lines are orthogonal can be found as

$$Y_1=518.3 \text{ tm}\cdot\text{m}$$

While, under the same conditions the sum of them in the flat slab with the skew column row lines can be obtained by

Fig. 12 (a) Bending moments in the primary beam of the flat slab with skew column row line.

Fig. 12 (b) Bending moments in the secondary beam of the flat slab with skew column row line.

(3) In the previous monograph⁽¹⁾ the constant c is not equal to 2.50 m but 2.96 m.

Fig. 13 Field of fluid corresponding to the flat slab in which the column row lines form the regular triangle.

Fig. 14 A family of primary and secondary beams of the flat slab in which the column row lines form the regular triangle.

Fig. 15 (a) Bending moments in the primary beam of the flat slab with the column row lines forming the regular triangle.

Fig. 15 (b) Bending moments in the secondary beam of the flat slab with the column row lines forming the regular triangle.

$$Y_2 = 520.3 \text{ tm} \cdot \text{m}$$

The difference between both values is only 0.4 percent.

In Fig. 13 shown are the bending moments in the flat slab whose columns have the equidistant bay $2a'$ in all directions and where the column row lines form a regular triangle. In this case, it is assumed to be $a' = (\sqrt{2}/4\sqrt{3})a$ so that the governing area in a column may be all equal. Fig. 15 (a) and (b) show the bending moments in the primary and secondary beams, in which the sum in the area $a \times \sqrt{2}a'$ of this flat slab is given by

$$Y_3 = 482.9 \text{ tm} \cdot \text{m}$$

This value is less 6.8 percent than that of Y_1 .

Photo. 6 Contour lines in the flat slab in the case when the column row lines form the regular triangle.

一方向曲線梁で補強のフラットスラブ (梗概)

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フラットスラブは倉庫や車庫のように荷重の大きい場合によく用いられるが、柱間隔が大きいときにはリブ(梁)をつけた方が経済的である。ネルビー氏、青木繁教授による曲線梁は有名であるが、それらはすべて等方性板の主曲げ方向に曲線梁が設けられている。本論文の曲線梁は著者の方法、すなわち大梁のみが荷重を伝達するものとする。流体の流線が大梁、ポテンシャル線が小梁に一致することにより求めた曲線梁である。前論文にてフラットスラブの一例についてのべたが、本論文では一般的に理論および実験により研究した結果を報告する。

1. フラットスラブと流出点、流入点のある流体との相似

原点に流出点のある二次元定常流の複素ポテンシャルは(1)で示される。第1図のように2個の流出点、2個の流入点があるときは(2)式が求まる。この実数部は(3)又は(3)'式となり流体のポテンシャルを表しているが、 m に適切な値を入れると、フラットスラブの場合のせん断力を表す。柱列線上のせん断力は(4)、対角線上のそれは(4)'となる。これらを図示したのが第2図(a)、(b)である。この場合の薄膜の等高線をモアレ法で求めたのが写真1であり、その結果を第2図に黒点でプロットしている。第3図は実験より得られた曲線梁である。

2. 柱列線が斜交する場合

第4図のように柱列をずらした場合、柱列が2列であるとカルマンの渦列に相当し、(5)が得られる。第5図、第6図はその計算値である。写真2は膜実験でその結果を前図に黒点でプロットしているが、2列と3列の差のため多少違っている。後者による曲線梁を第7図に示している。

3. 柱頭近傍の検討

上記では柱頭の径が非常に小さいものとしているが、この点について検討する。柱頭の大きさを考慮した解として(6)を得る。(6)の第1項について R と a との比を変えて計算した結果を第9図(a)、(b)、(c)に示す。

これに対応して行った実験が写真3、4、5である。実験値の柱列線上の点を黒丸で、対角線上の点を白丸で第9図にプロットしているが、柱頭付近では近似している。(6)の第2項について数値計算したのが第10図である。この項は柱頭周をはなれると急激に減少するので全体に及ぼす影響はきわめて少ない。

4. フラットスラブの曲線梁補強

以上要するに、柱頭の小さい場合には流出点流入点の交互に存在する流体のポテンシャル線が小梁方向を流線が大梁方向を示す。柱頭近傍の解として(6)式が成り立つ、またその第2項は柱頭からはなれると急激に減少するので、柱頭境界で $Q_a = \text{const}$ としても全体的には変らない。得られた曲線梁は第11図(a)、(b)、(c)である。

付 記

フラットスラブの柱が間隔 $2a$ で配置され、柱列線が直交している場合、斜交している場合の比較を総曲げモーメント量によって行なった。 $a=7.5\text{ m}$, $c=2.96\text{ m}$, $w=1\text{ t/m}^2$ とすると、前論文より柱列線が直交するときには $a \times a$ の範囲で

$$Y_1 = 518.3\text{ tm} \cdot \text{m}$$

であった。同じ条件で柱列線が斜交しているときのOne-Way Curved Beam Systemの大梁と小梁の曲げモーメント図を第12図(a)、(b)に示す。この場合には

$$Y_2 = 520.3\text{ tm} \cdot \text{m}$$

であり、その差は0.4%にすぎない。

次に第13図のようにフラットスラブの柱が全ての方角に等しい間隔 $2a'$ で並び、柱列線が正三角形を形づくるときの総曲げモーメント量を求めた。写真6にモアレの実験写真を、第14図に写真6より求められたOne-Way Curved Beam Systemの大梁、小梁群を示す。この場合は、柱一本当りの支配面積が同じものとなるように $a' = (\sqrt{2}/4\sqrt{3})a$ として計算した。第15図(a)、(b)に大梁、小梁の曲げモーメント図を示す。総曲げモーメント量は $a' \times \sqrt{3}a'$ の範囲で

$$Y_3 = 482.9\text{ tm} \cdot \text{m}$$

となり、 Y_1 に比べて6.8%少なくなっている。

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