ELASTIC ANALYSIS OF TWO-STORY OR TWO-BAY DUPLEX FRAMED SHEAR WALLS SUBJECTED TO SYMMETRICAL LOADS WITH RESPECT TO THE AXES OF THE INTERMEDIATE MEMBERS OF THEIR FRAMES

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1. Introduction

The purpose of this paper is to analyze two-story or two-bay duplex framed shear walls (hereafter referred to as "duplex shear wall"), whose shapes are symmetric with respect to the longitudinal and transversal center lines, to obtain the elastic deformations, plane stresses and two of four partitioned fundamental flexibility matrices from which the stiffness matrix of the duplex shear walls can be derived.

The elastic deformations and stresses of infilled panel walls and frames of the duplex shear walls, which cannot be given by conventional method such as the beam theory and equivalent strut method, can be given by this analysis.

The comparison between the numerical results of the axial force of the column and the vertical displacement of the top of the column of a duplex shear wall by this analysis and those of the conventional equivalent strut method is shown, and the defect of the latter method is pointed out.

The relations between the fundamental flexibility matrices and the stiffness matrix of duplex shear walls were reported by Tomii and Yamakawa¹⁾.

This analysis of duplex shear walls is induced from the analysis^{2),3)} of one-bay-one-story framed shear walls whose edge beams have the same sections but whose edge columns have different sections, where the boundary conditions and analytical method of the left edge column are revised to correspond to the boundary conditions and analytical method of the intermediate beams of the duplex shear walls (See Fig. 1).

This paper deals with the analysis of duplex shear walls subjected to symmetrical loads with respect to the axes of their intermediate beams, where the loads are decomposed into the following two types of fundamental components of plane nodal external forces, moments and distributed loads.

Type II : fundamental components which are symmetric with respect to the longitudinal and transversal center lines of the duplex shear wall

Type III : fundamental components which are antisymmetric with respect to the longitudinal center line of the duplex shear wall and symmetric with respect to the transversal center line of the duplex shear wall

The plane stress analysis of a duplex shear wall assumed to be an isotropic elastic body can be developed by analyzing the quarter part ($x \geq 0$) of the duplex shear wall (the half part of a unit shear

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- $m(e), n(e)$ = Even numbers 2, 4, 6, ... of m and n
 $m(o), n(o)$ = Odd numbers 1, 3, 5, ... of m and n
 M = External moments applied on the center of the connection of the upper edge beam and right edge column
 M_{ai} = Bending moment on a cross section of the intermediate beam
 q_{ae}, q_{be} = Sum of external axial shearing forces (per unit length and with the same eccentric distance from the center axis of the member) distributed on both side faces of the edge beam, that of the edge column
 q_{aie} = Sum of external axial shearing forces (per unit length and with the same eccentric distance from the center axis of the member) distributed on both side faces of the upper half of the intermediate beam
 q_{a2}, q_{b2} = External axial shearing forces (per unit length) distributed on the edge faces of the edge beam and right edge column
 Q_{ai} = Shearing force on a cross section of the intermediate beam
 t = Thickness of the wall
 u, v = Displacements in the wall parallel to x - and y -axes
 u_{ai1} = Axial displacement on the upper face, adjacent to the upper wall, of the intermediate beam
 w_{a1}, w_{b1} = Normal forces (per unit length) distributed on the inside faces, adjacent to the wall, of the edge beam and right edge column
 w_{a2}, w_{b2} = Sum of the external forces perpendicular to the axes of the members, (per unit length) distributed on the edge face and distributed on both side faces of the edge beam and that of the right edge column
 x, y = Longitudinal and transversal coordinates crossing at the center of the wall
 X, Y, X_c = External horizontal and vertical forces applied on the center of the connection of the edge beam and right edge column, and external horizontal force applied on the center of the connection of the intermediate beam and right edge column
 r = Shearing unit strain in rectangular coordinates x and y
 $\varepsilon_{a1}, \varepsilon_{b1}$ = Axial unit elongations on the inside faces, adjacent to the wall, of the edge beam and right edge column
 ε_{ai1} = Axial unit elongation on the upper face, adjacent to the upper wall, of the intermediate beam
 $\varepsilon_x, \varepsilon_y$ = Unit elongations in the wall parallel to x - and y -axes
 $\theta_a, \theta_{ai}, \theta_b$ = Slopes due to flexure of the edge beam, intermediate beam and right edge column, and are positive when their axes rotate clockwise
 ν = Poisson's ratio
 σ_x, σ_y = Normal unit stresses in the wall parallel to x - and y -axes
 τ = Shearing unit stress in the rectangular coordinates x and y

Additional symbols used in this paper are defined where they first appear.

2. Airy's Stress Functions

The Airy's stress functions ${}_II F$ and ${}_III F$ composed of serial and polynomial functions for Type II and III components are given by Eqs. 2 and 3.

$$\begin{aligned}
 {}_II F = & \sum_{m(e)} \frac{1}{\cosh \alpha_{am}} (A'_{am} \sinh \eta_m + B'_{am} \eta_m \cosh \eta_m + C_{am} \cosh \eta_m + D_{am} \eta_m \sinh \eta_m) \cos \xi_m \\
 & + \sum_{n(o)} \frac{1}{\cosh \alpha_{bn}} (C'_{bn} \cosh \xi_n + D'_{bn} \xi_n \sinh \xi_n) \sin \eta_n \\
 & + \sum_{n(e)} \frac{1}{\cosh \alpha_{bn}} (C_{bn} \cosh \xi_n + D_{bn} \xi_n \sinh \xi_n) \cos \eta_n \\
 & + \lambda_b^2 C_{40} \xi^4 + \frac{\lambda_b}{2} C_{20} \xi^2 + C_{21} \xi^2 \eta - 3(C_{40} + C_{04}) \xi^2 \eta^2 + \frac{\lambda_a}{2} C_{02} \eta^2 + C_{03} \eta^3 + \lambda_a^2 C_{04} \eta^4 \dots \dots \dots (2)
 \end{aligned}$$

$$\begin{aligned}
\text{III } F = & \sum_{m(o)} \frac{1}{\cosh \alpha_{am}} (A_{am} \sinh \eta_m + B_{am} \eta_m \cosh \eta_m + C'_{am} \cosh \eta_m + D'_{am} \eta_m \sinh \eta_m) \sin \xi_m \\
& + \sum_{n(o)} \frac{1}{\cosh \alpha_{bn}} (A_{bn} \sinh \xi_n + B_{bn} \xi_n \cosh \xi_n) \sin \eta_n \\
& + \sum_{n(e)} \frac{1}{\cosh \alpha_{bn}} (A'_{bn} \sinh \xi_n + B'_{bn} \xi_n \cosh \xi_n) \cos \eta_n \\
& + C_{30} \xi^3 + C_{31} \xi^3 \eta + C_{11} \xi \eta + C_{12} \xi \eta^2 + C_{13} \xi \eta^3 \dots \dots \dots (3)
\end{aligned}$$

where

$$\lambda_a = \frac{b}{a}, \quad \alpha_{am} = \frac{m\pi b}{2a}, \quad \xi = \frac{x}{a}, \quad \xi_m = \frac{m\pi x}{2a}, \quad \xi_n = \frac{n\pi x}{2b}$$

$$\lambda_b = \frac{a}{b}, \quad \alpha_{bn} = \frac{n\pi a}{2b}, \quad \eta = \frac{y}{b}, \quad \eta_n = \frac{n\pi y}{2b}, \quad \eta_m = \frac{m\pi y}{2a}$$

The stress functions $\text{II } F$ and $\text{III } F$ satisfy the following compatibility equation.

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \dots \dots \dots (4)$$

3. Unit Stresses, Unit Elongations and Displacements in the Walls

Normal unit stresses σ_x and σ_y parallel to the x - and y -axes, respectively, shearing unit stress τ in the rectangular coordinates x and y , unit elongations ϵ_x and ϵ_y parallel to the x - and y -axes, respectively, shearing unit strain γ in the rectangular coordinates x and y and the displacements u and v parallel to the x - and y -axes, respectively are given by Eqs. 5~12.

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} \dots \dots \dots (5)$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} \dots \dots \dots (6)$$

$$\tau = -\frac{\partial^2 F}{\partial x \partial y} \dots \dots \dots (7)$$

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{1}{E} (\sigma_x - \nu \sigma_y) \dots \dots \dots (8)$$

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{1}{E} (\sigma_y - \nu \sigma_x) \dots \dots \dots (9)$$

$$\gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\tau}{G} = \frac{2(1+\nu)}{E} \tau \dots \dots \dots (10)$$

$$u = \int \frac{\partial u}{\partial x} dx = \frac{1}{E} \int (\sigma_x - \nu \sigma_y) dx \dots \dots \dots (11)$$

$$v = \int \frac{\partial v}{\partial y} dy = \frac{1}{E} \int (\sigma_y - \nu \sigma_x) dy \dots \dots \dots (12)$$

Fig. 2 Unit stresses in the wall

Here, the unit stresses mentioned in Fig. 2 should take the positive sign and F is the stress function given by Eqs. 2 or 3.

The integration constants in Eqs. 11 and 12 are determined by Eqs. 7, 10, 13 a, 13 b, 14 a and 14 b.

The equations defining the displacements due to the rigid-body motion of duplex shear walls are given by Eqs. 13 a, 13 b, 14 a and 14 b.

For Type II components

$$u|_{x=0}=0 \dots \dots \dots (13 a)$$

$$v|_{y=-b}=0 \dots \dots \dots (13 b)$$

For Type III components

$$u|_{x=0}=0 \dots \dots \dots (14 a)$$

$$v|_{y=-b}=0 \dots \dots \dots (14 b)$$

4. Stresses, Displacements, Etc. in the Beams and Columns of the Frame

M. Tomii and H. Hiraishi reported the equations defining stresses, elongations and displacements in edge beams and edge columns of framed shear walls by considering shearing deformation of the beams and columns⁴⁾.

These equations can be applied to analyze the edge beams and edge columns of duplex shear walls, but these equations must be revised to analyze the intermediate beams of the duplex shear walls as follows.

The forces and moments on a small segment $D_{ai} \times dx$ of the intermediate beam are shown in Fig. 3.

In the intermediate beam, deflection due to shear, bending moment and shearing force on a cross section are zero, and axial force N_{ai} on a cross section is given by Eq. 15, and are positive as shown in Fig. 3.

$$N_{ai} = -2 \int t\tau|_{y=-b} dx - 2 \int q_{aie} dx \dots \dots \dots (15)$$

The N_{ai} caused by Type III components is antisymmetric with respect to the longitudinal center line of duplex shear wall and the integration constant in Eq. 15 is zero. The integration constant for Type II components is set as an unknown factor and can be determined together with other unknown integration constants and coefficients by solving the simultaneous equations for conditions shown in Table 1.

Axial unit elongation ε_{ai1} and axial displacement u_{ai1} on the upper and bottom face, adjacent to the walls, of the intermediate beam are given by Eqs. 16 and 17, and u_{ai1} is positive in x -direction.

$$\varepsilon_{ai1} = \frac{1}{E} \frac{N_{ai}}{A_{ai}} \dots \dots \dots (16)$$

Fig. 3 Forces on a small segment $D_{ai} \times dx$ of the intermediate beam

Table 1 The equations defining the unknown coefficients and unknown integration constants

The equations for conditions			The numbers of the equations	
			Components Type II	Components Type III
Continuity conditions of normal forces on the boundary between the wall and frame	Wall -- Right edge column	$t\bar{\sigma}_x _{x=a} = \bar{w}_{b1}$	$r_{n(o)} + r_{n(e)} + 1$	$r_{n(o)} + r_{n(e)} + 1$
	Wall -- Upper edge beam	$t\bar{\sigma}_y _{y=b} = \bar{w}_{a1}$	$r_{m(e)} + 1$	$r_{m(o)}$
Continuity conditions of unit elongations on the boundary between the wall and frame	Wall -- Right edge column	$\bar{\varepsilon}_y _{x=a} = \bar{\varepsilon}_{b1}$	$r_{n(o)} + r_{n(e)} + 1$	$r_{n(o)} + r_{n(e)} + 1$
	Wall -- Upper edge beam	$\bar{\varepsilon}_x _{y=b} = \bar{\varepsilon}_{a1}$	$r_{m(e)} + 1$	$r_{m(o)}$
	Wall -- Intermediate beam	$\bar{\varepsilon}_x _{y=-b} = \bar{\varepsilon}_{a1}$	$r_{m(e)} + 1$	$r_{m(o)}$
Continuity condition of vertical displacement on the boundary between the wall and frame	Wall -- Intermediate beam	$v _{y=-b} = 0$	$r_{m(e)} + 2$	$r_{m(o)} + 2$
The equations assuming the connections to be rigid-zones	Right edge column -- Upper edge beam	$\theta_b _{y=b} - \theta_a _{x=a} = 0$	1	1
	Right edge column -- Intermediate beam	$\theta_{az} _{x=a} = 0$	1	1
Equilibriums for forces in the x and y directions and moments on the connections of the beams and columns	Right edge column -- Upper edge beam	$\Sigma X = 0$ $\Sigma Y = 0$ $\Sigma M = 0$	1 1 1	1 1 1
	Right edge column -- Intermediate beam	$\Sigma X = 0$	1	0
Additional equations :			Total number of the equations	
$u _{x=0} = 0$ (For Type II components)			$4r_{m(e)} + 2r_{n(o)}$	$4r_{m(o)} + 2r_{n(o)}$
$u _{y=0} = 0$ (For Type III components)			$+2r_{n(e)} + 13$	$+2r_{n(e)} + 9$

note : the symbol-refers to the Fourier expansion

where

$r_{m(o)}$: number of $m(o) = 1, 3, 5, \dots$, $r_{n(o)}$: number of $n(o) = 1, 3, 5, \dots$,

$r_{m(e)}$: number of $m(e) = 2, 4, 6, \dots$, $r_{n(e)}$: number of $n(e) = 2, 4, 6, \dots$.

$$u_{ai1} = \int \epsilon_{ai1} dx = \frac{1}{E} \int \frac{N_{ai}}{A_{ai}} dx \dots\dots\dots (17)$$

Deformations caused by Type II components are symmetric with respect to the center of the intermediate beam, and the integration constant in Eq. 17 is zero. The integration constant in Eq. 17 for Type III components is determined by considering the continuity condition $u_{ai1}|_{x=a} = u|_{x=a, y=-b}$, where $u|_{x=a, y=-b}$ is given by Eq. 11.

5. Equations for Conditions Defining the Unknown Coefficients and Unknown Integration Constants

The equations for conditions which are necessary to define the unknown coefficients in the stress functions ${}_II F$ and ${}_III F$ given by Eqs. 2 and 3 and the unknown integration constants in the derivation processes of the equations are mentioned in Table 1.

The numbers of the simultaneous equations for conditions are $4r_{m(e)} + 2r_{n(o)} + 2r_{n(e)} + 14$ for Type II components and $4r_{m(o)} + 2r_{n(o)} + 2r_{n(e)} + 10$ for Type III components. These numbers are equal to the total numbers of the unknown coefficients in the stress functions ${}_II F$ and ${}_III F$ and the unknown integration constants in the derivation processes of the equations for Type II and III components, respectively. Therefore, the simultaneous equations for Types II and III components can be solved.

6. Numerical Examples

The authors deal with the elastic analysis of a duplex shear wall (See Fig. 4) whose unit shear wall has the same definitions as those of a shear wall appearing as a design example in the Commentary of A.I.J. (Architectural Institute of Japan) Building Code Requirements for Reinforced Concrete.

The nodal external forces applied to the duplex shear walls are shown in Fig. 5 and their numerical results of normal stresses σ_x and σ_y in the wall, axial force N and bending moment M in the beams and columns of the frame, shearing unit stress τ in the wall and shearing force Q on a cross section of the beams and columns of the frame are mentioned in Fig. 6~9.

Poisson's ratio ν is assumed to be $1/6$.

The stress distribution of the duplex shear wall subjected to the gravitational loads can be obtained by adding the stress distributions of the duplex shear wall subjected to the vertical nodal components of Type II as shown in Fig. 5 (a) (See Figs. 6 and 7) to the stress distributions of the duplex shear wall subjected to the symmetrical loads with respect to the longitudinal center line of the duplex shear wall and antisymmetrical loads with respect to the transversal center line of the duplex shear wall (Type IV)⁵⁾.

In the case when the duplex shear wall is subjected to the nodal forces of Type III as shown in Fig. 5 (b), which are considered to be one of the main components of external forces during earth-

Fig. 4 Definitions of the duplex shear wall adopted as the numerical examples

Fig. 5 The nodal external forces applied to the duplex shear walls adopted as the numerical examples

Fig. 6 The normal stresses σ_x and σ_y in the wall (unit : P/th) and the axial force N in the frame (unit : P) [Type II]

Fig. 7 The shearing unit stress τ in the wall (unit : P/th) and the shearing force Q on a cross section of the frame (unit : P) [Type II]

Fig. 8 The normal stresses σ_x and σ_y in the wall (unit : P/th) and the bending moment M in the frame (unit : Ph) [Type III]

Fig. 9 The shearing unit stress τ in the wall (unit : P/th) and the shearing force Q on a cross section of the frame (unit : P) [Type III]

quakes, the plane at the boundary between the intermediate member and wall remains to plane approximately (See Fig. 8).

By applying the unit fundamental components of Types II and III (See Fig. 10) to the duplex shear wall, the elements of the partitioned matrices of Types II and III of the fundamental flexibility matrix of the duplex shear wall¹⁾ can be given as the nodal displacements of the node i or the node k (See Fig. 11) and are mentioned in Eqs. 18 and 19 for the three numbers ($r_{m(o)}=r_{m(e)}=r_{n(o)}=r_{n(e)}=5, 15, 25$) of the terms of the serial stress functions.

Here, the nodal external forces and nodal displacements in the positive x - and y -directions mentioned in Fig. 12 have positive signs. The clockwise nodal moments and clockwise nodal rotational angles have positive signs.

$$\left\{ \begin{array}{c} u_{II}^* \\ u_{II}^{**} \\ v_{II}^* \\ h\theta_{II}^* \end{array} \right\} = \frac{1}{Et} \left[\begin{array}{cccc} 0.2972 & -0.0018 & -0.0718 & 0.5643 \\ 0.2972 & -0.0018 & -0.0720 & 0.5614 \\ 0.2972 & -0.0018 & -0.0720 & 0.5613 \\ -0.0018 & 0.3503 & 0.0271 & -0.2032 \\ -0.0018 & 0.3503 & 0.0271 & -0.2020 \\ -0.0018 & 0.3503 & 0.0271 & -0.2020 \\ -0.0721 & 0.0271 & 0.2546 & -0.4377 \\ -0.0720 & 0.0271 & 0.2547 & -0.4346 \\ -0.0720 & 0.0271 & 0.2547 & -0.4345 \\ 0.5614 & -0.2018 & -0.4336 & 7.1940 \\ 0.5610 & -0.2019 & -0.4340 & 7.1733 \\ 0.5610 & -0.2019 & -0.4340 & 7.1725 \end{array} \right] \left\{ \begin{array}{c} 4 X_{II}^* \\ 2 X_{II}^{**} \\ 4 Y_{II}^* \\ \frac{4 M_{II}^*}{h} \end{array} \right\} \dots\dots\dots (18)$$

$$\left\{ \begin{array}{c} u_{III}^{**} \\ v_{III}^* \\ h\theta_{III}^* \end{array} \right\} = \frac{1}{Et} \left[\begin{array}{ccc} 0.8686 & 0.1871 & -0.8959 \\ 0.8683 & 0.1871 & -0.8898 \\ 0.8683 & 0.1871 & -0.8896 \\ 0.1874 & 0.2848 & -0.5175 \\ 0.1872 & 0.2848 & -0.5130 \\ 0.1871 & 0.2848 & -0.5128 \\ -0.8904 & -0.5121 & 7.1788 \\ -0.8894 & -0.5123 & 7.1531 \\ -0.8893 & -0.5124 & 7.1522 \end{array} \right] \left\{ \begin{array}{c} 2 X_{III}^{**} \\ 4 Y_{III}^* \\ \frac{4 M_{III}^*}{h} \end{array} \right\} \dots\dots\dots (19)$$

where

The values in the top row of each element are derived by taking $r_{m(o)}=r_{m(e)}=r_{n(o)}=r_{n(e)}=5$.

The values in the middle row of each element are derived by taking $r_{m(o)}=r_{m(e)}=r_{n(o)}=r_{n(e)}=15$.

The values in the bottom row of each element are derived by taking $r_{m(o)}=r_{m(e)}=r_{n(o)}=r_{n(e)}=25$.

$4 X_{II}^*, 2 X_{II}^{**}, 4 Y_{II}^*, 4 M_{II}^*, 2 X_{III}^{**}, 4 Y_{III}^*, 4 M_{III}^*$ =Representative components of the fundamental components of Types II and III of the nodal external forces, which are given as the total of the absolute values of the independent fundamental components and take the sign of the components of the node i (* is attached) or the node k (** is attached) (See Fig. 10)

$u_{II}^*, u_{II}^{**}, v_{II}^*, \theta_{II}^*, u_{III}^{**}, v_{III}^*, \theta_{III}^*$ =Representative components of the fundamental components Types II and III of the nodal displacements which are the fundamental components of the node i or the node k (See Fig. 11)

When the serial stress functions with twenty-five terms are applied, the absolute values of errors, with respect to the symmetry of the elements of the partitioned fundamental flexibility matrices of Types II and III, are less than $0.0005/Et$ (0.115%) and $0.0004/Et$ (0.078%), respectively. This accuracy for symmetry indicates the reliability of this analysis.

The values of the elements of the partitioned fundamental flexibility matrices derived by using the serial stress function with fifteen terms are nearly equal to those values derived by using the serial stress

Note: The values in parentheses are treated as the dependent fundamental components

Fig. 10 Fundamental components Types II and III of nodal external forces

Fig. 11 Fundamental components Types II and III of nodal displacements

Fig. 12 Positive directions x and y of the fundamental components of nodal external forces and nodal displacements

Table 2 The comparison of the axial forces and the vertical displacements for the upper column by the conventional equivalent strut method and the analytical method

External force	Axial force(unit:P)		Vertical displacement of the top of the upper column(unit:P/Et)
	top	bottom	
Type II	0.866	0.608	1.019
	0.805	0.805	1.011
	7%	32%	1%
Type III	0.882	0.748	1.139
	1.000	1.000	1.279
	13%	34%	12%

The values in the top row are derived by the analytical method.

The values in the middle row are derived by the conventional equivalent strut method.

The values in the bottom row are the error for the values by the analytical method.

functions with twenty-five terms.

Therefore, these values converge well enough by adopting the serial stress functions with fifteen terms.

The axial forces in the column of the boundary frame determined by the conventional equivalent strut method and analytical method mentioned above are shown in Table 2 for the external forces of Type II and III (See Fig. 5).

The maximum error for the axial force is 34% at the bottom of the upper column. The maximum

error for the vertical displacement at the top of the upper column is 12% for Type III.

Therefore, the conventional equivalent strut method cannot be used to determine the axial force of the column of the boundary frame or the vertical displacement of the top of the upper column of the boundary frame of the duplex framed shear wall.

In the equivalent frame of the conventional equivalent strut method, the axial and flexural rigidities of the beams are assumed to be infinite.

The columns are hinged to those beams, and the struts are hinged to the frame diagonally (See Fig. 13).

Here, the cross-sectional area of the struts in the equivalent frame is determined by assuming that the shearing rigidity of the wall, where the horizontal cross-sectional area is lt , is equal to that of the equivalent frame, and is given by Eq. 20.

And the cross-sectional area of the columns in the equivalent frame is determined by assuming that flexural rigidity of the horizontal I-shaped cross-section of the shear wall is equal to the flexural rigidity of the equivalent frame with struts, and is given by Eq. 21.

$$A_b = \frac{G}{E} \cdot \frac{t(h^2 + l^2)^{1.5}}{2\kappa lh} \quad (20)$$

where A_b = Cross-sectional area of the struts in the equivalent frame

h = Distance from center to center of the edge beam and intermediate beam of the duplex shear wall

l = Distance from center to center of the edge columns of the duplex shear wall

κ = Shear factor for shearing deformation

$$A_c = \frac{4}{l^2} I_{wc} + A_{wc} + \frac{t l'^3}{6 l^2} \quad (21)$$

where A_c = Cross-sectional area of the columns in the equivalent frame

I_{wc} = Geometrical moment of inertia of the edge columns of the duplex shear wall

A_{wc} = Cross-sectional area of the edge columns of the duplex shear wall

l' = Clear span of the boundary frame

The numerical results were computed by the digital computer (FACOM M-190, 15 effective) of the Kyushu University Computer Center.

7. Conclusions

Following the analysis of single framed shear walls, exact elastic analysis method of duplex shear walls symmetric with regard to both the longitudinal and transversal center lines was investigated by using Airy's stress function.

The elastic analysis of duplex shear walls whose shapes are symmetric with respect to the longitudinal and transversal center lines subjected to antisymmetrical loads with respect to the axes of the intermediate members of their frames can be induced by revising the boundary conditions and analytical method of the left edge columns of one-bay-one-story framed shear walls whose edge beams have the same sections but whose edge columns have different sections so as to correspond to the boundary conditions and analytical method of the intermediate beams of the duplex framed shear wall.

Thus, the stresses in the walls, beams and columns which cannot be given by the equivalent strut method can be obtained by this analysis.

From these numerical results the following things are observed.

In the case when the duplex shear wall is subjected to the nodal forces of Type III, which are considered to be one of the main components of external forces operative during earthquakes, the plane at the boundary between the intermediate member and wall remains to plane approximately.

It was numerically investigated how the values of the elements of the partitioned fundamental

Fig. 13 The conventional equivalent frame with struts

flexibility matrices of the duplex shear walls of Type II and III converge with the increase in the number of terms of the stress function when this analytical method is applied. It is pointed out, as a result, that in calculating the values of the elements of the partitioned fundamental flexibility matrices of the duplex shear walls, sufficient accuracy is obtained by taking about fifteen terms of the serial stress function.

It was also pointed out that the conventional strut method cannot be used to determine precisely the axial force of the column of the boundary frame or the vertical displacement of the top of the upper column of the boundary frame in the duplex framed shear wall.

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付帯ラーメンの中間部材の材軸に関して 対称な外力を受ける2連耐震壁の弾性解析 (梗概)

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1. 序

本論の目的は、慣用のビーム法やブレース置換法では求めることができない2連耐震壁の壁板内、付帯ラーメン部材のそれぞれの弾性応力や変形の解析解をはじめ、多連耐震壁の付帯ラーメンの節点に関する剛性マトリックスを求めるための基本的要素となる、中心軸に関し、左右、上下とも対称な形状の2連耐震壁の基本たわみ性マトリックスの4個の部分マトリックスのうちの2個の部分マトリックスの解析解などを求めることにある。このような2連耐震壁の基本たわみ性マトリックスと剛性マトリックスの関係は、既に富井、山川により発表されている¹⁾。また、本解析および慣用のブレース置換法により算出された2連耐震壁の柱の軸力および柱頭の鉛直変位の数値計算結果を比較し、慣用のブレース置換法の欠陥を指摘した。

本解析は、中心軸に関し上下対称で左右柱断面の異なる単独耐震壁の弾性解析^{2),3)}において、左柱の境界条件と解析法を2連耐震壁の中間ばりの境界条件と解析法に修正するような方法を採用した。

本解析では、2連耐震壁に作用する一般的な外力に関し、付帯ラーメンの中間部材の材軸に関して対称な外力成分をとり上げ、この外力成分をさらに2通りの基本外力成分に分解し、それぞれの基本外力成分を次のように呼び、逆対称、対称条件を考慮して、2連耐震壁の4分の1部分(単位の耐震壁の $x \geq 0$ の部分)について解析を行なった(図1参照)。

Ⅱ型の外力：中心軸に関し、左右、上下とも対称な外力
Ⅲ型の外力：中心軸に関し、左右逆対称、上下対称な外力

解析には次の仮定を用いた。すなわち

1. 壁板は2次元弾性体とする。
2. 壁板と付帯ラーメン部材は密着固定している。
3. 付帯ラーメン部材の接合部(図1の斜線部分)は

剛域とみなす。

4. 付帯ラーメン部材には、せん断変形を考慮した初等曲げ理論⁴⁾を適用する。

Ⅱ型の外力は任意に与えられるが、Ⅲ型の外力は任意に与えられず、水平方向の力のつりあい式である1式を満足しなければならない。ここで、 x および y 方向の外力ならびに時計廻りのモーメントを正とする。

2. エアリーの応力関数

Ⅱ型およびⅢ型の外力に対して、級数型応力関数と代数型応力関数を各々2, 3式のように選んだ。これらの応力関数は4式の適合条件を満足する。

3. 壁板内の応力度、ひずみ度、変位

x, y 軸に平行な垂直応力度 σ_x, σ_y およびせん断応力度 τ , x, y 軸に平行な垂直ひずみ度 ϵ_x, ϵ_y およびせん断ひずみ度 γ , x, y 方向の変位 u, v は、それぞれ、5, 6, 7, 8, 9, 10, 11, 12 式から求められる。

4. 付帯ラーメン部材の応力、変位など

2連耐震壁の外ばりと柱の応力、ひずみ度、変位は、富井、平石により示された単独耐震壁の柱とはりの応力、ひずみ度、変位の式⁴⁾を用いて表わすことができるが、中間ばりに関しては、各式を修正する必要がある。

中間ばりの微小部分 $D_{ai} \times dx$ に作用する応力を図3に示す。

Ⅱ型Ⅲ型の外力が作用する場合は、中間ばりにはせん断変形がなく、曲げモーメントとせん断力は零である。中間ばりの軸力 N_{ai} は15式により与えられる。

Ⅲ型の外力が作用する場合、軸力 N_{ai} が y 軸に関して逆対称であり、15式の積分定数は零となる。Ⅱ型の外力に対しては、この積分定数は未知積分定数となり、表1に示す連立方程式を解くことによって、他の未知係数や未知積分定数とともに決定される。

5. 未知係数および未知積分定数を決める条件式

2, 3式に示した応力関数 ΠF および $\text{III} F$ に含まれる未知係数と諸式の誘導過程で生じる未知積分定数を決定する条件式(連立方程式)を表1に示す。条件式は、壁板と外ばりおよび右柱の境界上の垂直応力のつりあい条件式、壁板と付帯ラーメン部材の境界上の境界線方向

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ひずみ度の連続条件式、壁板と中間ばりの境界での鉛直変位の連続条件式、付帯ラーメン部材の接合部が剛域であるための条件式、付帯ラーメン部材の接合部における軸方向力およびモーメントのつりあい条件式からなっている。条件式の数は、Ⅱ型の外力に関しては $4r_{m(o)} + 2r_{n(o)} + 2r_{n(e)} + 14$ 個、Ⅲ型の外力に関しては $4r_{m(o)} + 2r_{n(o)} + 2r_{n(e)} + 10$ 個である。これらの条件式の個数は、Ⅱ型およびⅢ型のそれぞれの外力に関して、 ${}_{II}F$ および ${}_{III}F$ の未知係数ならびに諸式の誘導過程における未知積分定数の総数に等しい。これより、表1に示す連立方程式を解くことができる。

6. 数値計算例

日本建築学会鉄筋コンクリート構造計算規準・同解説の設計例の単独耐震壁と同一形状の単位耐震壁からなる、図4に示す2層耐震壁に対し、数値計算を行なった。

図5に示すような節点外力を受ける2層耐震壁の壁板の垂直応力度 σ_x, σ_y 、せん断応力度 τ 、付帯ラーメン部材の軸力 N 、曲げモーメント M およびせん断力 Q を、図6～9に示した。ここで、ポアソン比 ν は $1/6$ と仮定した。

図5(a)に示すⅡ型の鉛直節点外力成分が作用する場合の応力状態(図-6, 7参照)と既報²⁾で報告したⅣ型の鉛直節点外力成分が作用する場合の応力状態とを重ね合わせることによって、鉛直荷重時の応力状態を得ることができる。また、図5(b)に示すような水平荷重時の成分であるⅢ型の節点外力が作用する場合、壁板内の中間ばりとの境界付近では平面保持に近い応力状態(図8参照)となっていることがわかる。

図10に示す単位の基本節点外力を与えると、図11に示す i および k 節点の変位として2連耐震壁の基本たわみ性マトリックスのⅡ型およびⅢ型の部分マトリックスの各要素が得られる¹⁾。級数型応力関数の項数 ($r_{m(o)} = r_{m(e)} = r_{n(o)} = r_{n(e)}$) を5項、15項および25項として計算した場合のⅡ型およびⅢ型の部分基本たわみ性マトリックスを18, 19式に示す。

ここで、節点外力および節点変位は、図12に示す x, y 軸の方向を正とし、節点モーメントおよび節点回転角は、時計廻りの方向を正とする。

25項の級数型応力関数を用いた場合、Ⅱ型およびⅢ型の部分基本たわみ性マトリックスの対称性に関する誤差の絶対値は、それぞれ $0.0005/Et$ (0.115%) および $0.0004/Et$ (0.078%) 以下である。この対称性の正確さが本解析の信頼性を示している。

また、これらのマトリックスの各要素の値は15項の場合と25項の場合、その差が極めて小さい。これより、15項の級数型応力関数を用いれば、基本たわみ性マトリックスの各要素の値は十分収斂すると思われる。

本解析により得られた付帯ラーメン部材の柱の軸力および上部柱の柱頭の鉛直方向変位を、慣用のブレース置換法により得られたものとともに表2に示す。なお外力は図5に示すⅡ型およびⅢ型外力である。表2より、上部柱の柱脚部分の軸力で最大34%、上部柱の柱頭の鉛直変位で最大12%の誤差を生じている。ゆえに、慣用のブレース置換法は2連耐震壁の付帯ラーメン柱の軸力および上部柱の柱頭の鉛直変位の算出には、実用設計上使用することができない。

耐震壁に関する慣用のブレース置換法は、耐震壁を剛なはりとしてこれにピン接合された柱で構成された骨組に両端ピンの筋違を両対角線に挿入した筋違入骨組に置換する方法である(図13参照)。ここで筋違の断面積は、筋違入り骨組のせん断剛性が、耐震壁の壁板部分(水平断面積 ld) のせん断剛性と等しくなるように20式で与えられ、柱の断面積は、筋違入骨組の曲げ剛性と耐震壁のⅠ型水平断面の曲げ剛性が等しくなるように21式で与えられる。

数値計算には、九州大学大型計算機センター(FACOM M-190; 有効桁数15)を用いた。

7. 結 論

単独耐震壁に引続き、左右、上下とも対称な形状の2連耐震壁に対しても、フーリエ級数型の応力関数を使った厳密な弾性解析法の研究を行った。その結果、このような2連耐震壁に、その付帯ラーメンの中間部材の材軸に関して対称な外力が作用する場合の解析は、既に著者らが発表した「中心軸に関し上下対称で左右柱断面の異なる単独耐震壁の弾性解析法^{2), 3)}」において、左柱の境界条件と解析法を2連耐震壁の中間ばりの境界条件と解析法に置換するような修正を行なうことによって可能となることを明らかにした。

本解析による壁板、付帯ラーメン部材の応力の数値計算例を示した。この解析例では、水平荷重時の成分であるⅢ型の節点外力が作用する場合、壁板内の中間ばりとの境界付近では平面保持に近い応力状態となっていることを明らかにした。

また、2連耐震壁を含む有壁ラーメンのマトリックス構造解析に必要な2連耐震壁の剛性マトリックスをこの基本たわみ性マトリックスから求める¹⁾ため、Ⅱ型およびⅢ型の部分基本たわみ性マトリックスの要素を算定する場合には、応力関数の項数を15項もとれば、十分な精度が得られることを指摘した。

慣用のブレース置換法は2連耐震壁の付帯ラーメン柱の軸力や上部柱の柱頭の鉛直変位の算出には実用設計上使用することはできないことを明らかにした。

注: 文中の式、図表および引用文献は英論文を参照されたい。