ON THE ULTIMATE VALUES OF DEFORMATION ANGLE FOR REINFORCED CONCRETE COLUMNS

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1 Introduction

Deformation capability of structural components is one of the most important factors in seismic safety of buildings. Since the Tokachiko earthquake attacked northern Japan in 1968, considerably numerous experiments on reinforced concrete columns have been conducted at the various institutes in Japan, including the synthetic projects initiated from 1972 with the proposal of the Building Research Institute of the Ministry of Construction. The quantitative understandings have, however, not yet been sufficient due to the complicated resistance characteristics of reinforced concrete columns. The aim of this study is to clarify the relationship between the deformation capability of reinforced concrete columns and the various parameters, such as hoop reinforcement ratio, longitudinal reinforcement ratio, shear span ratio or axial load level applied, by using the test results mentioned above.

In the experimental works of the synthetic projects mentioned above, most of column specimens were subjected to about fifty times reversals of horizontal load until failure to obtain the hysteretic loop characteristics at each cycle of a given deflection amplitude. Also in these projects, the ultimate value of deformation angle has been proposed as an index representing the deformation capability of reinforced concrete columns. The value means the useful ultimate or useful limit of deformation angle and was defined as the deformation angle when the first load of cyclic loading at a given deflection amplitude deteriorated less than 80% of maximum load or when the tenth load deteriorated less than 50% of the maximum load as shown in Fig. 1 A and B respectively. This definition is due to the fact that from the deformation angle when the first load deteriorates less than 80% of the maximum load, the load decreases remarkably with the loops unstable and the criterion that loops can be regarded to be unstable when the tenth load deteriorates less than 50% of the maximum load even if the first load is not less than 80% of the maximum load. Thus the ultimate value of deformation angle defined is summarized to correspond to that of the amplitude from which the loops become unstable. It should be also added that amplitudes are discrete quantities given by each investigator although usually given in a regular way with multiples of an unit amplitude.

In this paper the ultimate value defined here is statistically examined, by including the test results of the investigations other than those of the synthetic projects. Total number of the specimens used in this study was about two hundreds and sixty^{(1)~(17)}.

2 Variables influencing the deformation capability of columns

The ultimate deformation angle is influenced by various parameters, such as axial load level, shear span ratio, longitudinal reinforcement ratio etc. as well as hoop reinforcement ratio. In Table 1 are shown the ranges of these parameters, used in the experiments mentioned above. It can be readily presumed that the size of columns, the kind of hoops (spiral, subtie-type or conventional) or the kind

Fig. 1 Load-deflection curves illustrated, regarding the definition for the ultimate values of deformation angle (Ref. 12)

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Table 1 Ranges of each variable in specimens

Shear span ratio a/D	1.0	1.5	2.0	2.5	3.0
Number of specimens	33	39	131	27	33
Axial load ratio η_0	0.056~0.254	0.078~0.274	0.056~0.330	0.084~0.330	0.110~0.284
Long. rein ratio P_t (%)	0.31~0.95	0.34~1.24	0.31~1.275	0.34~1.00	0.34~1.24
Hoop rein ratio P_w (%)	0.20~2.39	0.15~2.12	0.09~2.44	0.18~1.30	0.09~1.15
Concrete strength F_c (kg/cm ²)	210~473	124~277	124~473	205~233	185~292
Steel yield strength σ_y (kg/cm ²)	Long. 3000~4281 Hoop 2960~3930	3332~4180 2960~4635	3220~4281 2798~5200	3660~4230 2890~4110	4020~5180 2890~3770
Section D (cm)	20~50	15~48	20~50	20~45	20~25
Shear span a (cm)	20~50	22.5~72	40~100	50~112.5	60~70

of longitudinal reinforcement bars (plain or deformed), or the number of cycles of reversal loads influence the deformation capability. In this paper, however, only four variables, axial load level, shear span ratio, longitudinal reinforcement ratio as well as hoop reinforcement ratio are considered mainly, in order to obtain fundamental information on the deformation capability.

It should be noted herein that columns are in general designed so that flexural yielding precedes before shear failure in calculation. According to the design formula of the Reinforced Concrete Building Code Requirements and Commentary of AIJ (1970 or 1979)^{18)~19)}, the calculated value for flexural yield is obtained as follows when axial load level is lower than that at balanced conditions.

$$M_r = 0.8 a_t \cdot \sigma_y \cdot D + 0.5 \eta_0 (1 - \eta_0) F_c b D^2 \dots \dots \dots (1)$$

in which a_t , σ_y , F_c , b , D and η_0 are the area and yield strength of longitudinal reinforcement, concrete strength, width and depth for concrete section and axial load level applied ($N/F_c b D$) respectively, as shown in Fig. 2. On the other hand the following equation, called modified Ōhno-Arakawa equation²⁰⁾ is commonly used for estimating the shear capacity of reinforced concrete columns.

$$Q_s = \left(0.90 + \frac{\sigma_0}{250}\right) \left\{ k_u k_p \frac{0.115}{M/Qd + 0.115} (180 + F_c) + 2.7 \sqrt{p_w \sigma_y} \right\} \cdot b j \dots \dots \dots (2)$$

in which d , j , M/Q and σ_0 are effective depth, lever arm of the internal couple ($7/8 d$), shear span ratio (a) and axial stress (N/bD) while P_w , σ_y and k_u , k_p are the ratio and yield strength of hoop reinforcement and coefficients dependent on d and p_t , respectively. In the Eq. (2), kg/cm² is needed to use as unit for the σ_0 , F_c and σ_y values and σ_0 is needed not to be greater than 100 kg/cm². The k_u and k_p values are given in Fig. 3 A and B, cited from Ref. 18, respectively. In the tests the equations used to calculate flexural strength for columns specimens can be regarded to have been Eq. (1) but those for shear strength were not always the Eq. (2). Most of specimens were designed for shear strength with Arakawa's minimum equation^{18), 21)}, similar to Eq. (2) but lacking in the item regarding axial stress and with a little modification. It is however, clear that Eq. (2) is more appropriate in estimating the ultimate shear strength of columns because of taking into account the effects of axial load. Thus the levels of shear capacity to flexural (Q_s/Q_m) were calculated for all the specimens, using Eq. (1) and Eq. (2). In Fig. 4 are shown the ranges of the values of Q_s/Q_m for each shear span ratio of 1-3. In Fig. 5 is also illustrated the distribution of the Q_s/Q_m values for the shear span ratio of 2, with tested strength to calculated values (Q_{max}/Q_m) being set as ordinate. It can be concluded from these figures that calculated shear strength is not less than calculated flexural strength for all the columns used in the experiments except several specimens.

3 Derivation of equation for estimating the ultimate values of deformation angle

Fig. 2 Equilibrium in section at ultimate (Ref. 19) **Fig. 3** k_u and k_p values (from Ref. 18)

Fig. 4 Ranges of calculated values of shear to flexural capacity for each shear span ratio

Fig. 5 Distribution of tested to calculated values on maximum loads versus Q_s/Q_m values ($a/D=2.0$)

Fig. 6 Axial load levels versus ultimate values of deformation angle

In Figs. 6 (a)–(c) are plotted the tested values regarding the deformation capability, the ultimate values of deformation angle R_u as defined above, versus the axial load level applied, in the three groups of the same shear span ratio. It can be seen from these figures that R_u values are not less than 10^{-2} rad. if flexural yielding precedes as mentioned above, although each range of the variables dealt with herein should be taken into account. These plotted points are classified according to the values of the ratio of hoop reinforcement to longitudinal (p_w/p_t) in each figure. It is found that the deformation capability values, as a whole, increase with lower values of η_0 and with higher values of p_w/p_t . It should be commented herein that little systematic relationships have been found between deformation capability and hoop reinforcement ratio alone, as shown later. This seems due to the fact that the higher values of shear are developed for the columns with higher values of longitudinal reinforcement, indicating that higher hoop reinforcement is needed to obtain sufficient deformation capability for these columns. In each of Fig. 6 (a)~(c) is shown a line, indicating a lower bound for the R_u values of plotted points. These lines are obtained from the following three assumptions regarding ultimate conditions^{22), 23)}.

- (1) The length of yielding region (critical region) equals to D , depth of section.
- (2) The average curvature in this region is $\epsilon/\eta_0 D$.
- (3) The ultimate concrete compressive strain ϵ is 0.003.

The first assumption was adapted from the the fact that flexural-shear cracks of column specimens are mostly observed to concentrate on the region of the length equal to the depth at ultimate. The second came from the same assumption with that used for the Eq. (1) mentioned above. The depth ratio of neutral axis equals to the value of η_0 as shown in Fig. 2. The third is that commonly used. From these assumptions and by referring to Fig. 7, the following can be derived as an equation estimating the ultimate value of deformation angle for the columns without hoop reinforcement.

$$R_u = \frac{0.003}{\eta_0} \left(1 - \frac{D}{2a} \right) \quad (3)$$

This type of expression will be more generalized later on, by considering the effects of the value of $\sqrt{p_w/p_t}$. In this paper the ranges of the axial load level dealt with are about 0.07–0.33 as shown in Figs. 6 (a)~(c), as well as in Table 1. Further studies

Fig. 7 Relationship between deformation and unit curvature

are needed for the cases of lower values of η_0 , including beams although it has been experimentally reported²⁴⁾ that the coupled resistance by both the compression and tension reinforcement is extremely ductile unless the buckling or tensile rupture occurs in the reinforcement, just as the Eq. (3) suggests. On the other hand, experimental works have been made for the cases of η_0 values higher than 0.33, as the test results²⁵⁾ are illustrated in Fig. 6 (b). In this paper, these high ranges of axial load level is, however, excluded for discussion because of the limited number of test results as well as the limitation of the applicability of Eqs. (1) and (2).

The differences of horizontal distance between the tested values and the lines of Eq. (3) in Figs. 6 (a)~(c) were plotted, using the values of p_w and p_t . The plottings against the values of p_w alone as abscissa have been, first, made but little systematic relationships have been found as illustrated in Figs. 8 (a) and (b). Second, p_w/p_t values have been used as abscissa under the consideration of the effects of shear level developed as explained above. It is found from Figs. 9 (a) and (b) that there exist, to some extent, systematic trends and that most of plotted values lie in each strip indicated in each figure. The p_w/p_t values at the starting of the effects, which are around 1.0 and 0.25 for the shear span ratios 1.0 and 2.0 respectively, can be related to those at the coincidence of flexural and shear capacities, determined by Eqs. 1 and 2. It is also interesting to note that the upper limit of the effects is nearly the same in both the figures although the effect rate due to hoop reinforcement is much faster for the shear span ratio of 2.0 than for that of 1.0. By referring to these figures and using $\sqrt{p_w/p_t}$ values as abscissa, Figs. 10 (a)~(e) have been finally made. The $\sqrt{p_w/p_t}$ values can be regarded to be the equivalent diameter ratios of hoop to longitudinal reinforcement since p_w/p_t values are approximately the ratios of area of both the reinforcement. From Figs. 10 (a)~(e), the intermediate trend between the shear span ratios of 1 and 2 can be traced as that of the shear span ratio of 2, regarding the shear ratios of 2.5 and 3.0.

Each solid line in Figs. 10 (a)~(e) was determined as a simple expression representing the average

Fig. 8 Logarithmic values of the effect due to p_w values alone

Fig. 9 Logarithmic values of the effect due to p_w/p_t values

Fig. 10 Logarithmic values of the effect due to $\sqrt{p_w/p_t}$ values

line for plotted values. The effects of $\sqrt{p_w/p_t}$ consist of three steps for all shear ratios, as follows.

$$Y=0 \dots\dots\dots (4)$$

$$Y=0.8(\sqrt{p_w/p_t}-S) \dots\dots\dots (5)$$

$$Y=0.56 \dots\dots\dots (6)$$

in which S is the $\sqrt{p_w/p_t}$ value at which Eq. (5) starts and can be expressed together with $\sqrt{p_w/p_t}$ value E of the ending point of Eq. (5), as follows for each shear span ratio.

$$S(E)=1.0(1.7) \text{ for } a/D=1.0 \dots\dots\dots (7)$$

$$S(E)=0.75(1.45) \text{ for } a/D=1.5 \dots\dots\dots (8)$$

$$S(E)=0.50(1.2) \text{ for } a/D \geq 2.0 \dots\dots\dots (9)$$

The S and E values for $a/D=1.5$ were determined at the middle between those for $a/D=1.0$ and 2.0 , as shown above. It should be added that Eq. (4) is little used since flexural yielding usually precedes around $\sqrt{p_w/p_t}$ values of S .

By using the Eq. (3) and considering that the effects of Eq. (4)~(6) are logarithmic values, the following can be obtained as an expression for estimating the ultimate values of deformation angle,

including the effects of $\sqrt{p_w/p_t}$ values.

$$R_u = \frac{0.003 \cdot 10^Y}{\eta_0} \left(1 - \frac{D}{2a}\right) \dots \dots \dots (10)$$

However this means an expression for the average lines about plotted points as shown in Figs. 10 (a) ~ (e). By shifting the average lines in these figures vertically downwards as shown with dotted lines each, the following two expressions can be obtained as conservative one and very conservative one respectively.

$$R_u = \frac{0.002 \cdot 10^Y}{\eta_0} \left(1 - \frac{D}{2a}\right) \dots \dots \dots (11)$$

$$R_u = \frac{0.0015 \cdot 10^Y}{\eta_0} \left(1 - \frac{D}{2a}\right) \dots \dots \dots (12)$$

The total number ratios of the specimens which lie below the Eq. (11) and Eq. (12)

are about 14.8% and 4.6% respectively as shown in Table 2 with each shear span ratio. It is interesting to note that the coefficient values, 0.002 and 0.0015, of these expressions are those of strain at maximum stress for ordinary concrete cylinders, instead of ultimate strain 0.003 and that the term of 10^Y can be also regarded as a factor increasing these three strains, depending upon the values of $\sqrt{p_w/p_t}$.

It should be added herein that the variables other than the four main variables examined in this paper, such as type of hoops (spiral, subtie, closed-type or conventional), level of shear capacity to flexural, difference of definitions for the ultimate values of deformation angle (first cycled-80% or tenth-cycled-50% explained in the Introduction), kind of longitudinal and hoop reinforcement (plain or

Table 2 Numbers and percentages of the specimens having R_u values below Eqs. (11) and (12)

		Shear span ratio					Total Num.	Total Num ratio (%)
		1.0	1.5	2.0	2.5	3.0		
Equation (11)	Num.	6	5	21	5	2	39	14.8
	(%)	(18.2)	(15.6)	(16.0)	(18.5)	(6.6)	263	
Equation (12)	Num.	1	3	7	1	0	12	4.6
	(%)	(3.0)	(7.7)	(5.3)	(3.7)	(0)	263

Fig. 11 Logarithmic values of the effect due to the $\sqrt{p_w/p_t}$ values, classified by the types of hoops ($a/D=2.0$)

Fig. 12 Logarithmic values of the effect due to the $\sqrt{p_w/p_t}$ values, classified by the levels of Q_s/Q_m values ($a/D=2.0$)

Fig. 13 Logarithmic values of the effect due to the $\sqrt{p_w/p_t}$ values, illustrated by the definition of the ultimate values of deformation angle ($a/D=2.0$)

Fig. 14 Distribution of ultimate values of deformation angle versus Q_s/Q_m values

deformed) or number of reversals of loads are also examined to find the causes of spread distributions in Figs. 10 (a)~(e), as illustrated in Figs. 11~13. However, definite conclusions have not been drawn regarding the effects of these variables on the ultimate values of deformation angle, the examinations still being continued. In Figs. 14 (a)~(e), are also plotted the direct values of ultimate deformation angle using Q_s/Q_m values as abscissa. It can be seen from these figures that there exist little systematic relationships. The Q_s/Q_m value is only a factor to determine the border of failure mode, flexural or shear. The deformation characteristics seem to be predominated by the resistance mechanism of the one side. In other words, Q_s/Q_m value seems to be insensible to the ultimate values of deformation angle even if there exist its effects.

Furthermore, deformation characteristics are essentially more affected by various uncertain factors than strength. Thus it is concluded herein that for seismic design point of view, the Eq. (12) is recommended to use.

4 Comparisons with the expression suggested in the Reinforced Concrete Building Code Requirements and Commentary of AIJ (1971 or 1979)

In the commentary of Article 15 of Sect. (pp. 166-1979 ed.) the following expression is suggested to estimate the ultimate values of deformation angle for reinforced concrete columns, by changing the symbols into those of this paper,

$$R_u = \frac{1}{3} \frac{0.8 s \epsilon_c}{\eta_0 - 0.08} \left(\frac{a}{D} \right) \dots \dots \dots (13)$$

This was derived by assuming the unit curvature of the critical section as follows²⁶⁾.

$$\frac{1}{\rho_0} = \frac{s\varepsilon_c}{x_1 - d_{c1}} = \frac{0.8 s\varepsilon_c}{\eta_0 - 0.08} \quad (14)$$

in which $s\varepsilon_c$, x_1 and d_{c1} are ultimate compression steel strain, neutral axis ratio obtained by assuming that concrete stress block $0.8 F_c b x_1 D$ equals to axial load N , under the consideration that both the compression and tension steels yield at ultimate and compression steel depth ratio assumed as 0.1, respectively. Thus Eq. (13) can be obtained by assuming curvatures distribution along column length as linearly.

In Fig. 15 are compared both the trends of Eq. (3) and Eq. (13) in which the ultimate compression steel strain is assumed as 0.003, for $a/D=2.0$ and 3.0. It is found that both the Eqs. give the similar R_u values above the axial load level of about 0.2 but that the trends become quite different below the axial load level of 0.1. In Fig. 15 are also shown the tested values cited from the Code. The definition used as ultimate value of deformation angle, for these tested values is quite different from that of this paper and the reversals of load for specimens only within several cycles. In these tests, the ultimate value of deformation angle was defined as that at final failure, commonly at the buckling of compression steel. It is found from Fig. 15 that Eq. (3) can be lower bound even for the tested values from the Code. In the Eq. (13) the effects of hoop reinforcement ratio etc. are not formulated quantitatively, different from the Eqs. (10)~(12) and it is doubtful that strain at the buckling of compression steel can be determined definitely. Thus it is concluded that the Eq. (10) and Eq. (12) are much more appropriate as average and minimum expressions in estimating the ultimate values of deformation angle, respectively than the Eq. (13) based on the insufficient data at that time.

Fig. 15 Comparison of Eq. (3) and Eq. (13)

5 Conclusions

Based on the studies reported herein, the following conclusions can be drawn.

1) Equations were proposed for estimating the ultimate values of deformation angle for reinforced concrete columns subjected to multi-reversals of horizontal load, through the analyses of the tested results by various investigations, by taking into account the effects of hoop and longitudinal reinforcement ratios, shear span ratio as well as axial load level. The equations have been also compared with that suggested in the Code of AIJ.

2) Further researches are needed to clarify the effects of the various parameters other than those dealt with herein, such as, kind of hoops (spiral, subtie, or conventional hoop), etc., and also to extend the equations for columns with axial load levels outside the range of 0.07~0.33, which is dealt with in this paper.

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7 References

- 1) Arakawa, T., Takeda, H., Yonezawa, T., and Matsuka, Y.; Abstracts of the Annual Convention of Architectural Institute of Japan, Oct. 1974, pp. 1339-40, Oct., 1975, pp. 1111-12, 1117-18 and Oct. 1976, pp. 1415-16
- 2) Fukada, Y.; Abst. of the Ann. Con. of AIJ, Oct. 1975, pp. 1119-20 and Oct. 1976, pp. 1413-14
- 3) Fukuzawa, R., Chiba, Y., Kondo, N., and Yanagishita, K.; Abst. of the Ann. Con. of AIJ, Oct., 1974, pp. 1311-12
- 4) Higashi, Y., Okubo, Z., and Otsuka, M.; Abst. of the Ann. Con. of AIJ, Oct. 1975, pp. 1113-14
- 5) Hirosawa, M., Nakazawa, M., Shimizu, Y., Takahashi, H. and Kumagai, K.; Abst. of the Ann. Con. of AIJ, Oct. 1977, pp. 1501-02 and Sep. 1978, pp. 1747-48
- 6) Kano, Y., Endo, T. and Kato, H.; Abst. of the Ann. Con. of AIJ, Oct. 1973, pp. 1425-26
- 7) Kokusho, S., Hirosawa, M., Fukuhara, M., Hayashi, S., Yanagishita, K., Iwashita, S., Yamaguchi, M., and Nagaoka, T.; Abst. of the Ann. Con. of AIJ, Oct. 1973, pp. 1393-94, 1401-02, 1423-24

- 8) Nakata, S., and Sato, K. ; Abst. of the Ann. Con. of AIJ, Oct. 1976, pp. 1411-12
- 9) Omori, S., Takahashi, T., Ishii, K., and Watanabe, S. ; Abstr. of the Ann. Con. of AIJ, Oct. 1974, pp. 1313-14, 1341-42
- 10) Ozaki, M., Taga, A., Eguchi, K., Maeda, Y. and Fujisawa, M. ; Abst. of the Ann. Con. of AIJ, Oct. 1975, pp. 1089-90
- 11) Saito, T., Yosida, S. and Nishigaki, T. ; Abst. of the Ann. Con. of AIJ, Oct. 1973, pp. 1423-24
- 12) Sgano, S., Tobata, Y., Hoshino, N., Yamaguchi, I., and Oi, Y. ; Abst. of the Ann. Con. of AIJ, Oct. 1973, pp. 1391-92
- 13) Takada, S., Nagai, Y., Yamagata, Y., Karatsu, T., Sakakibara, H., Inoue, T. and Moriwaki, Y. ; Abst. of the Ann. Con. of AIJ, Sep. 1979, pp. 1253-54
- 14) Takeda, T., Nakayama, T., Yoshioka, K., Koike, K. and Kikuchi, T. ; Abst. of the Ann. Con. of AIJ, Oct. 1974, pp. 1307-08, Oct. 1975, pp. 1087-88 and Oct. 1977, pp. 1495-96
- 15) Tanaka, R. ; Abst. of the Ann. Con. of AIJ, Sep. 1978, pp. 1767-68
- 16) Wakabayashi, M., Sasaki, R., Minami, K. and Asai, Y. ; Abst. of the Ann. Con. of AIJ, Sep. 1978, pp. 1375-76
- 17) Yamamoto, K. ; Abst. of the Ann. Con. of AIJ, Oct. 1976, pp. 1391-92
- 18) Architectural Institute of Japan ; Reinforced concrete buildings code requirements and commentary (1979)
- 19) Sugano, S. ; Concrete journal of Concrete Institute of Japan, Vol. 11, No. 2, Feb. 1973, pp. 1-9
- 20) Ōno, K., Shibata, T., Hattori, T., Imano, S., Owada, S., and Saito, Y. ; Abst. of the Ann. Con. of AIJ, Oct. 1972, pp. 1091-92
- 21) Arakawa, T. ; Concrete journal of Concrete Institute of Japan, Vol. 8, No. 2, Jul. 1970, pp. 11-20
- 22) Corley, W.G. ; Journal of ASCE ST 5, Oct. 1966, pp. 121-146
- 23) Mattock, A.H. ; Journal of ASCE ST 2, Apr. 1967, pp. 519-522
- 24) Kent, D.C., and Park, R. ; Journal of ASCE ST 7, Jul. 1971, pp. 1969-90
- 25) Higashi, Y., Okubo, Z., and Iso, K. ; Abst. of the Ann. Con. of AIJ, Oct. 1977, pp. 1497-98
- 26) Miyatake, T., and Sugita, K. ; Proc. of the Kanto Branch Symposium of AIJ, May. 1964, 1964, pp. 93-96

8 Appendix

ultimate values of deformation angle used in the References (1)~(17)

App. Table 1 (a)

	No.	D (cm)	2a (cm)	Fc (kg/cm ²)	N (ton)	Pt (%)	Pw (%)	Ru (x10 ³)	Hoop Type	Shear Equ.
Ref(1) (a)	1		50	203	125	0.34	0.21 0.26 0.28 0.63	1.08		Ohn.
	25	25	75	250	0.61	0.34 0.41 0.43 0.52 0.54	0.71 0.86 1.20 1.33 1.40	3.54	Su.	"
	26		100	209	125	0.34	0.18 0.21 0.22	1.54		"
	43	25	125	246	313	0.96	0.32 0.25	4.39	Co.	"
	44		100	164	0.61	0.37 0.40	0.59 0.57	3.53	Co.	"
	55	25	125	210	328	0.61	0.25 0.36	2.40	Co.	Ara.
	56		100	185	164	0.61	0.25 0.36	2.40	Co.	"
	61	25	125	217	328	0.96	0.53 0.77	4.52	Co.	"
	62		75	124	0.57	0.45 0.92	1.76 0.81	2.55	Su.	Ara.
	74	25	100	153	164	1.02	1.10 0.64 0.69	6.08	Co.	"
Ref(2) (a)	75		100	204	0.64	0.71 1.02	1.03	3.00	Su.	"
	89	25	100	261	408	0.92 1.02	0.71 1.02	10.8	Co.	"
Ref(3)	90		75		0.34	0.15 0.20	0.49 0.51	1.95		"
	100	25	193	328	0.61	0.25 0.29 0.45	0.60 0.98 1.08	5.85	Co.	"
Ref(4)	101		100		0.34	0.27	0.61	3.10		"
	111	25	100	240	18.8	0.61	0.36	1.22	Co.	"
Ref(5) (a)	112		75		0.95	0.51	1.27	7.40		"
	117	25	100	262	164	0.31 0.64 0.96	0.19 0.44 0.54	0.88 3.04 8.01	Co.	Cod.
	118		144	46.1	0.38	0.41	0.72	2.97		Ara.
	122	48	192	227	69.1 92.1	0.59 0.88	0.58 0.98	3.15	Co.	"
Ref(6)	123		100		0.10	0.19	1.36	0.81		Ara.
		50	210	65.6	0.34	0.27	1.42		Co.	"
					0.61	0.42	1.81		"	"
	137		200	131.3	0.95	0.53 0.74 0.85	2.33 2.44	3.26	"	"

Co. Conventional Hoop Cl. Closed Hoop Sp. Spiral Hoop
 DSp. Double Spiral Hoop Su. Sub Tie Hoop We. Weld Hoop
 Ara.: Arakawa minimum Equation. Cod.: Code Equ. Ohn.: Modified
 Ohno-Arakawa Equ.

App. Table 1 (b)

	No.	D (cm)	2a (cm)	Fc (kg/cm ²)	N (ton)	Pt (%)	Pw (%)	Ru (x10 ³)	Hoop Type	Shear Equ.
Ref(7) (a)	138		20			0.23	0.78	1.67		Ara.
	150	20	60	218	164	0.53	0.31 0.42	2.39		"
	151		75	240		0.41	0.09	1.02	Co.	"
	167	25	150	270	18.8	1.24	0.22	1.42	Co.	"
	168		50		16.4	0.34	0.09	0.61		"
		25	100	244.5		0.61	0.18 0.27	1.22 1.27	Co.	"
	182				32.8	0.45 0.95	2.23 2.44	6.40		"
	183		50	140	164	0.31	0.20	0.48		"
	187	25	100	180	32.8	0.64	0.21	0.61	Co.	"
	188		50	270	164	0.96	0.30	0.94		"
Ref(9) (a)	202	25	100	322	32.8	0.82	0.27	1.24	Su.	"
	203		50	272	164	1.12	0.15	6.18	Co.	"
	207	25	100	329	32.8	1.12	0.27	1.22	Sp.	"
	208		100	206.4	16.4	0.34	0.07	1.12	Co.	Ara.
Ref(10)	209	25	50	473	164	0.34	0.21	0.58		"
	216		100		32.8	0.95	0.29 0.40	0.91 1.12	Co.	"
Ref(12)	217	25	100	165	218	0.61	0.27	2.30	Co.	"
	222					0.41		~250		"
Ref(13)	223	35	175	206	78.6	0.64	0.40	1.30		"
	229			87.7	1.00	0.79		2.00	D.Sp.	"
	230	25	50	273.6	164	0.61	0.45	1.09		"
	233		100	328	32.8	0.56	0.23	~350	Cl.	"
Ref(14) (a)	234	40	160	287.2	480	0.61	1.30	1.41	Sp.	Cod.
	241		120	314.5	128.0	0.99	1.38	1.67	Su.	"
	242	40	240	800	0.61	0.95	1.77	3.61	We.	"
	245		160	277	128.0	0.99	1.11	2.07		"
	246	25	150	292	22.8	0.34	0.21	0.54	Co.	Ara.
	249			45.6	~0.95	0.39	1.15	~316		"
Ref(16)	250	15	45	192	9.40	1.13	0.42	1.98	Sp.	Cod.
	252			~230	~10.6			~300		"
Ref(17)	253	30	120	183.1	41.6	0.42	0.75	1.40	We.	"
	263			47.3	0.66	0.85 0.98	1.62 1.69	5.88	Sp. Su.	" "

鉄筋コンクリート柱の変形限界値について (梗概)

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鉄筋コンクリート柱の水平力に対する変形能を明らかにすることは、耐震安全上並びに設計上解決すべき重要な課題の一つである。本論文では、既往の諸実験結果を用いて、柱の変形限界値に関する一つの推定式を提案している。

1968 年の十勝沖地震以来、鉄筋コンクリート柱の水平抵抗に関する実験的研究は数多くなされ、特に 72 年から開始された“総合プロジェクト”の一連の実験は、とりあつかった試験体の数が多いことと同時に、各試験体に、破壊に至るまでに計 50 回程度の正負繰返しの水平力を加えているの特徴的である。またこれらの研究では最大耐力に達したあと、それ以降での振幅における第 1 回目の荷重または第 10 回目の荷重が、最大耐力の 80% または 50% 以下になる時の変形を変形限界値と定義している (図-1 参照)。本論文でもこの定義に従ったものを柱の変形限界値と考え、その値が帯筋比をはじめとして、主筋比や軸圧比あるいはせん断スパン比によってどのように変化するかを調べた。取扱った試験体の総数は総合プロジェクト以外のものも含め、計 263 体である。これら試験体に関する各種変数の範囲は表-1 に示す通りである。柱の変形限界値は、帯筋比、主筋比、軸圧比、せん断スパン比だけでなく、柱の寸法や、同じ帯筋比でも、帯筋の種類 (スパイラルやサブタイや在来型) によって、また同じ主筋比でも、主筋の種類 (異形と丸鋼) 等によって影響をうけることが考えられる。しかし本論文では、大きな傾向を把握することを目的としているので、帯筋比、主筋比、軸圧比、せん断スパンの 4 つの変数の影響をみることに限定した。通常柱は曲げ降伏型に設計されるが、これら試験体でも原則としてそのように設計されている。用いられた、曲げ耐力計算式は、すべて (1) 式と考えてよい。一方せん断耐力計算式は規準式または荒川下限式あるいは修正大野荒川式である。3 つのうち修正大野荒川式が軸力を考慮したせん断耐力を評価できるので、この式にもとづいてすべての試験体のせん断耐力を計算し、(1) 式の曲げ耐力計算値と比較した。図-4 はその比率の範囲を示したもので、図-5 はその分布状態を例示したものである。結局、本論文でとり上げた試験体は殆んどすべて計算上曲げ耐力以上のせん断耐力を有していることがわかった。図-6

の各図は各柱の変形限界値をせん断スパン比毎に示したものである。縦軸には軸圧比がとってある。各図には実験値の下限値にほぼ相当する右下りの直線が示してあるが、それらは図-2 と図-7 及び本文に示す 3 条件にもとづいて導いた (3) 式を表わしている。またこれらの図のプロット値はそれぞれ p_w/p_t (帯筋比/主筋比) 値で分類してある。なお p_w 値だけの分類も試みたが、後に図-9, 10 に関連して、図-8 に示すように、 P_w 値そのものの効果については、法則的な傾向を見出すことができなかった。すなわち主筋比が大きい柱は、同一の変形能をうるためには、より多くの帯筋を用いる必要があることを示している。図-6 の各プロット値の理論線 (右下りの直線) との水平距離はいわば p_w/p_t 値の効果をあらわすものであるが、それらを縦軸にプロットしたのが、図-10 である。横軸はいつでも $\sqrt{p_w/p_t}$ 値をとっている。実験値は各図に示す各折れ線 (実線) のまわりに分布しており、それら各折れ線は (4)~(9) 式に示すように表現できる。この (4)~(6) 式は対数値であるから、それを考慮して (3) 式を拡張すると、 $\sqrt{p_w/p_t}$ 値の効果を入れた (10) 式がえられる。この式によって実験値の平均的な値を推定することができる。しかし耐震設計等、安全率を見込む立場からは、図-10 の各図に示すように、各折れ線 (実線) をそのまま下方にスライドさせた折れ線 (点線) を用いることが望ましい。これらは (11) および (12) 式で表わされ、実験値総数に対する不合格率はそれぞれ 14.8% および 4.6% である。なお上記 3 式は、降伏域でのコンクリート限界歪の与えた値の大小だけが異なるだけで、 $\sqrt{p_w/p_t}$ 値による効果はすべて同一で、それによってコンクリートの限界歪が、同じように増加してゆくとも解釈できる。図-11, 12, 13 は帯筋の種類や、せん断耐力の曲げ耐力に対する割合 (Q_s/Q_m) の大小等の影響を、せん断スパン比 2.0 の場合について示したものである。しかしこれらの変数による効果に関しては、結論を導き出すまでには至っていない。なお図-14 の各図に、変形限界値そのものと Q_s/Q_m 値との関係を見るためにプロットを試みた。当然乍ら、あまり関連性はみられない。最後に、現行の規準書にある提案式 (本文の 13) との比較を行い、本論文の提案式 (本文 10~12) がより適切であることをのべた。

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