ELASTIC-PLASTIC LATERAL-TORSIONAL BUCKLING
OF H-SHAPED STEEL BEAM-COLUMNS UNDER
AXIAL AND HORIZONTAL LOADS

by Chiaki Matsui*, Shosuke Morino** and
Junichi Kimura***, Members of A.I.J

I INTRODUCTION

The lateral-torsional buckling, one of factors reducing strength and rotation-capacity of steel structural members, makes the derivation of a design formula of beam-columns difficult, because of diversity of the condition governing the problem. Consequently, a simple interaction formula based on the lateral-torsional buckling moment of beams and the critical axial force of centrally loaded columns is given for the design of beam-columns in the AIJ Standard for Structural Design of Steel Structures published by Architectural Institute of Japan

Among the previous studies on the lateral-torsional buckling of beam-columns, Van Kuren and Galambos9 carried out experimental investigation on strengths and deformation characteristics of practical wide-flange beam-columns. Galambos, Adams and Fukumoto9, and Miranda and Ojalvo6 analyzed theoretically the lateral-torsional buckling problem of beam-columns by treating it as an eigen-value problem. In Japan, the lateral-torsional buckling of beam-columns subjected to uniform and nonuniform bending has been investigated by Suzuki, Kubodera and Kure9, and Wakabayashi et al.47 The former tested beam-columns with H-shaped cross section in the wide-width flange series, and discussed the relation between the theoretical buckling load and the post-buckling maximum load obtained in the tests. The latter showed the experimental and theoretical results of the post-buckling behavior of beam-columns with H-shaped cross section in wide- and medium-width flange series. All studies mentioned above are, however, concerned with beam-columns prevented from side-sway. As for the study of beam-columns permitted to sway, only Kato and Akiyama9 carried out a series of studies, in which the rotations at both ends of a beam-column are perfectly restrained in plane of the web of H-shaped cross section.

In this paper, governing equilibrium equations of an elastic thin-walled member formulated by others9 are first extended to the elastic-plastic state, and then the elastic-plastic lateral-torsional buckling problem of steel beam-columns permitted to sway is treated as the eigen-value problem. In view of the results of the parametric study on the effect of axial force ratio, relative stiffness of beam to column, slenderness ratio of the column and other parameters, the relations between the lateral-torsional buckling load and the maximum in-plane load carrying capacity are discussed in comparison with the design formula given in the Guide to Plastic Design of Steel Structures of AIJ40.

II ANALYSIS OF LATERAL-TORSIONAL BUCKLING LOAD

Fundamental Consideration

The model proposed in this study is an H-shaped steel column of length l, elastically restrained against rotation at both ends by beams of length l/2, the other ends of beams being simply supported, as shown in Fig. 1. The model is supposed to simulate the behavior of an interior column of a multi-

* Professor, Department of Architecture, Faculty of Engineering, Kyushu University, Dr. Eng.
** Associate Professor, Department of Architecture, Faculty of Engineering, Kyushu University, Ph.D.
*** Research Assistant, Department of Architecture, Faculty of Engineering, Fukuoka University, M. Eng.
story frame permitted to sway. Ends of the column are simply supported with respect to out-of-plane deformation, in other words, the beams do not provide any restraint against out-of-plane deformation of the column. The model is subjected to constant compressive axial force $P$ and varying horizontal load $H$ at the top end of the column, and the relative column displacement occurs, their positive directions being shown in Fig. 1. In addition, the bending moments $M_u$ and $M_t$ are applied at the top and bottom joints of the model, which are regarded as the external moments transmitted from the upper and lower columns adjacent to the column under consideration. The coordinate system is taken such that the positive $z$-axis is along the longitudinal axis of the column with the origin at the bottom end.

The positive $x$- and $y$-axes are taken as shown in Fig. 2, which are the principal axes of H-shaped cross section with depth $d$, width $b$, flange thickness $t$ and web thickness $w$.

Assumption

The following assumptions are made to simplify the problem. The stress-strain relation of the column material is assumed to be elastically perfect plastic. However, the beams are assumed to be entirely elastic. Deflections, slopes and curvatures are small. The residual stresses and initial imperfections are not present. Cross sections do not distort. Moreover, in order to treat an H-shaped beam-column as a thin-walled member, shear strains due to shear stresses in equilibrium with the changes of normal stresses can be neglected, and shear strains in the planes normal to the middle surface of thin wall element can be neglected. These assumptions are the same as made in Ref. 9).

Equilibrium Equations and Boundary Conditions

The governing differential equations of a thin-walled member under thrust, flexure and torsion have been formulated by Nishino, Kasemset and Lee\(^9\), which are based on an arbitrary coordinate system with a fixed origin and an arbitrary reference point for the displacement on the section. By making use of those formulations, it is possible to avoid the difficulties in defining the centroid and the shear center of elastic-plastic cross sections, and consequently the geometrically clear equations of equilibrium in the elastic-plastic state can be obtained by keeping the origin of the coordinate system and the reference point for the displacements on the centroid of the original elastic cross section. In reference to the literature mentioned above, the differential equations in terms of the displacements of the centroid of the original elastic cross section are derived as follows:

$$
[E\bar{I}_x]\ddot{v}^\alpha + P(v + Y)\dot{v}^\alpha = 0 \quad \text{(1a)}
$$

$$
[E\bar{I}_x](u^\alpha + \beta v^\alpha) + E\bar{I}_w x e \beta^\alpha - (E\bar{I}_x x v^\alpha + PY) \dot{\beta} + Pu^\alpha = 0 \quad \text{(1b)}
$$

$$
[E\bar{I}_w e \beta^\alpha + E\bar{I}_w x e (u^\alpha + \beta v^\alpha)]^\alpha - [(GJ_e + K)\beta']^\alpha - [E\bar{I}_x x v^\alpha + PY]u^\alpha + [E\bar{I}_x x u^\alpha + E\bar{I}_w x e \beta^\alpha]v^\alpha = 0 \quad \text{(1c)}
$$

where $u$, $v$ and $\beta$ are the displacements of the centroid of the original elastic cross section in $x$- and $y$-directions and angle of twist, respectively, as shown in Fig. 2. The internal moment $M_x$ about $x$-axis and stress resultant $V$ in $y$-direction are given by

$$
M_x = -E\bar{I}_x x v^\alpha - PY \quad \text{(2a)}
$$

$$
V = -[E\bar{I}_x x v^\alpha + P(v + Y)]' \quad \text{(2b)}
$$

In Eqs. (1) and (2), the following notations are introduced to simplify the expressions:
\[ I_{xx} = I_{xx} - S_{xx}/A_e \] 
\[ Y = \left[ 1 - \{ (S_{xx} - S_{xx}) A_e/(A_e S_{xx} + (A_e - A_t)/A_t) N_o/P\} S_{xx}/A_e \right] \] 
\[ K = ES_{xx} I_{xx}^2 \nu^2 - Pr^2 \] 
\[ r_x^2 = \frac{1}{e} [ (A_t - A_e)/A_e + (I_{zt} + I_{xt} - I_{xc} - I_{sc})/A_e] N_o/P \] 
\[ i_x = \sqrt{I_{xx}/A_e}, \quad i_{xx} = \sqrt{I_{xx}/A_e} \]

where \( I_x \) and \( J_y \) are moments of inertia about \( x- \) and \( y- \) axes, respectively, \( S_x \) is statical moment of area about \( x- \) axis, \( A_e \) is cross-sectional area, \( I_{xx} \) is warping moment of inertia, and \( I_{xx} \) is warping statical moment about \( y- \) axis, \( J = St. \) Venant's torsion constant, \( N_o \) is yield axial load in tension, \( E = Young's \) modulus, \( G = E/(1 + \nu) \) is shear modulus, \( \nu \) is Poisson's ratio \((\approx 0.3)\); the subscript \( g \) denotes quantities computed for the original elastic cross sections, and \( e, c \) and \( t \) for elastic portion, yielded portions in compression and tension of a partially yielded cross section, respectively; and \( (') \) denotes the derivative with respect to \( z \). In the computation of quantities given by Eq. (3), it is taken into account the extension of yield zone in direction of the normal to the middle surface of the thin wall element.

In the derivation of the differential equations of the lateral-torsional bucking of the beam-column, Galambos, Adams and Fukumoto\(^3\) neglected all products of displacements and their derivatives as higher order terms, and as a result their analysis neglected the effect of the prebuckling displacements, while it is taken into account in the present study by substituting the numerical solution for \( v \) of Eq. (1 a) into Eqs. (1 b, c). Miranda and Ojalvo\(^5\) discussed in detail the effect of the prebuckling displacements and pointed out that neglect of this effect leads to serious errors of the lateral-torsional buckling load on the unsafe side. In addition, the term \( EI_{xx}^n \nu^m \), neglected in the analysis by Miranda and Ojalvo, is also included in Eq. (1 c), which is known as the stabilizing effect of deflection in the lateral-torsional buckling problem of beams.\(^9,10\) Consequently, the lateral-torsional buckling load obtained in the present study is expected and assured by numerical computation to be intermediate between the results of analyses by Galambos, Adams and Fukumoto, and Miranda and Ojalvo.

The lateral-torsional buckling load of the model shown in Fig. 1 can be obtained by solving Eqs. (1 a~c) numerically together with the following boundary conditions. In reference to Fig. 1, the boundary conditions in the plane of applied loads are as follows:

At \( z = 0 \); \( v = 0 \) and \( 2 M_{bt} + M_s = M_l \) \hspace{1cm}(4 a, b)
At \( z = l \); \( 2 M_{bu} - M_s = M_u \) and \( V = H \) \hspace{1cm}(4 c, d)

where \( M_{bt} \) and \( M_{bu} \) denote beam-end moments of lower and upper beams, respectively, and clockwise moments are taken as positive. \( M_{bt} \) and \( M_{bu} \) are expressed in terms of \( v \) as

At \( z = 0 \); \( M_{bt} = (6 EI_{bt} / l_b) v' \) \hspace{1cm}(5 a)
At \( z = l \); \( M_{bu} = (6 EI_{bu} / l_b) v' \) \hspace{1cm}(5 b)

where \( I_{bt} \) and \( I_{bu} \) denote moments of inertia of lower and upper beams, respectively. The external moment \( M_u \) is approximated by

At \( z = l \); \( M_u = (H_l + P_v)/2 \) \hspace{1cm}(6 a)

and in this analysis, \( M_l \) is assumed to be related to \( M_u \) with a constant parameter \( \sigma \) as

\[ M_l = \sigma M_u \] \hspace{1cm}(6 b)

Substituting Eqs. (2 a, b), (5 a, b) and (6 a, b) to Eqs. (4 a~d) leads to the boundary conditions expressed in terms of \( v \) as follows:

At \( z = 0 \); \( v = 0 \) \hspace{1cm}(7 a)
\[ (12 EI_{bt} / l_b) v' - E I_{xx} \nu' - P Y = \sigma (H_l + P_v)/2 \] \hspace{1cm}(7 b)
At \( z = l \); \( 12 EI_{bu} / l_b) v' + E I_{xx} \nu' + P Y = (H_l + P_v)/2 \] \hspace{1cm}(7 c)
\[ -[E I_{xx} \nu' + P Y] - P v' = H \] \hspace{1cm}(7 d)

where \( j \) denotes the column top displacement.

For the out-of-plane deformations, both ends of the column are assumed to be simply supported, and thus the boundary conditions are

At \( z = 0, l \); \( u = u^* = \beta = \beta^* = 0 \) \hspace{1cm}(8)
Numerical Solution by Finite Difference Method

When the yielding starts in the column, the explicit solution of Eqs. (1 a～c) can not be obtained because of the cross-sectional properties varying along the column. The numerical analysis by the finite difference method is thus applied to obtain the in-plane behavior of inelastic beam-columns and their lateral-torsional buckling loads.

The column is divided into $s$ segments of equal length, subdivision points 0 and $s$ being at the bottom and top ends of the column, respectively, when Eqs. (1 a～c) are transformed into the finite difference equations by the first order central difference. Since Eq. (1 a) is independent of Eqs. (1 b, c), the solution for the value of $v$ at each subdivision point for a certain horizontal load $H$ can be independently obtained by solving a set of simultaneous equations generated by transforming Eq. (1 a) into the finite difference equations, in view of Eqs. (7 a～d). Substituting this solution for $v$ into the finite difference expression of Eqs. (1 b, c) in view of Eq. (8) leads to the homogeneous simultaneous equations in which unknowns are the values of $u$ and $\beta$ at subdivision points. The critical condition for the lateral-torsional buckling is that the coefficient matrix of the homogeneous simultaneous equations becomes singular.

Note that the in-plane load-displacement relations in the elastic state are given by the analytical solution of Eq. (1 a), while those in the elastic-plastic state are obtained by a conventional iterative procedure since values of cross-sectional properties change in accordance with the in-plane displacement. In the numerical procedure, the horizontal load increment is taken equal to 2% of the elastic limit value. When the load-displacement curve approaches its peak, it becomes difficult to obtain the converged solution for $v$ by the iterative procedure because of the ill-conditioned nature of the simultaneous equations. To avoid this, the simultaneous equations are modified in such a way that the top-end displacement of the column is treated as known and the horizontal load $H$ as unknowns. As to the application of the finite difference method, the column is divided into 50 segments of equal length. In order to check the accuracy of the numerical results, the variation of the lateral-torsional buckling load is investigated a priori, by increasing the number of segments from 10 to 50 by an interval of 10. The maximum difference between two values of the buckling load associated with 40 and 50 segments is less than 2% of the value obtained for 50 segments. Moreover, the present computation has been applied to the elastic buckling problems investigated in Refs. 3) and 4), and the solution seems to be satisfactory.

III RESULTS OF PARAMETRIC STUDY

Idealized Cross Section

For the purpose of reducing the number of parameters involved in the problem, the real H-shaped cross sections are idealized in the following manner. Referring to Fig. 2, $i_{xx}$ and $i_{yy}$, which are radii of gyration about $x$- and $y$-axes of an H-shaped cross section, respectively, are obtained as follows:

$$i_{xx} = d \sqrt{\frac{1 - (1 - w/b)(1 - 2t/d)^2}{12[2t/d + w/b(1 - 2t/d)]}} \quad (9a)$$

$$i_{yy} = b \sqrt{\frac{2t/d + (w/b)^2(1 - 2t/d)^2}{12[2t/d + w/b(1 - 2t/d)]}} \quad (9b)$$

Then, nondimensional constants $\tau$ and $\mu$ are introduced as

$$\tau = i_{xx}/d = \sqrt{\frac{1 - (1 - w/b)(1 - 2t/d)^2}{12[2t/d + w/b(1 - 2t/d)]}} \quad (10a)$$

$$\mu = i_{yy}/i_{xx} = \frac{b}{d} \sqrt{\frac{2t/d + (w/b)^2(1 - 2t/d)^2}{1 - (1 - w/b)(1 - 2t/d)^2}} \quad (10b)$$

where $\tau$ and $\mu$ are expressed in terms of $b/d$, $w/b$ and $t/d$. It is found from the investigation of rolled H-section specified by Japanese Industrial Standard that the values of $\tau$ and $\mu$ are nearly constant and the mean values are as follows; $\tau = 0.427$ and $\mu = 0.580$ for 19 sections in the category of the wide-width flange series $(d/bw = 1.0)$, and $\tau = 0.415$ and $\mu = 0.271$ for 17 sections in the category of the medium-
Fig. 3 $I_{xg}/I_{yg}$ Relations

Fig. 4 $D_T$–$
u$ Relations

width flange series ($d/b=2.0$). The maximum deviation is 4.8% of the mean value of $\nu$ in the medium-width flange series. Substituting the values of $\tau$ and $\omega$ obtained above into Eqs. (10 a, b), in view of $d/b$ equal to 1 or 2, leads to a set of simultaneous equations for $\omega/d$ and $\psi/b$. The solution of $\omega/d$ and $\psi/b$ in view of the value of $d$ equal to 100 mm results in the following idealized cross section adopted here; H-100×100×4.49×5.39 mm for the wide-width flange series and H-100×50×1.93×2.81 mm for the medium-width flange series. The values of shape factor of these cross sections are 1.119 and 1.123, respectively.

Figure 3 shows the relation between $I_{xg}/I_{yg}$ and the nondimensional curvature $\varphi (=\varphi/\varphi_0)$ of the idealized cross section H-100×50×1.93×2.81 mm under the axial thrust and strong-axis bending, where $I_{yg}$ denotes the moment of inertia about $x$–axis of the original cross sections, $\varphi$ the curvature and $\varphi_0=2\varepsilon_0/d$, $\varepsilon_0$ being the yield strain (=0.0012). The results are compared with the real H-shaped cross sections, H-200×100×5.5×8 mm and H-300×150×6.5×9 mm in which the effects of the fillet are neglected. The difference between the idealized and real sections is very small. Although the results are not shown here, small differences are ascertained in case of other nondimensional cross-sectional parameters appearing in the equilibrium equations such as $I_{xg}/I_{yg}$, $I_{w,el}/I_{yg}d$, $I_{w,el}/I_{yg}$, $K/(N_0\sigma_y^2)$ and $Y/d$, and in case of the other idealized section H-100×100×4.49×5.39 mm as well. The remaining cross-sectional parameter is St. Venant's torsion constant $J$ which substantially affects on the lateral-torsional buckling strength of beams. Figure 4 shows the relations between $D_T (=J/Agd^2)$ defined by Galambos 13 and $\varphi$ of the idealized and real cross sections, where it is assumed that only elastic core provides the torsional resistance. In Fig. 4, large difference is observed. The effect of this difference on the lateral-torsional buckling load of beam-columns is discussed later.

Parametric Study

Nondimensional parameters involved in the present study other than those related to the cross section and slenderness ratio $l/i_{yg}$ of the beam-column are as follows;

$$ p = P/N_0, \quad k_1 = 12I_{yg}/I_{yg}a, \quad k_2 = I_{yg}/I_{yg}, \quad \alpha = M_{y}/M_{yg} \quad \text{..........................(11 a, b, c, d)} $$

It is found from the plastically designed multi-story frames by Lu et al. 13 that the value of $k_1$ ranges approximately from 6 to 18, and the value of $k_2$ is nearly constant and approximately equal to 1.2. In case of a multi-story frame subjected to constant horizontal load at each floor level, the value of $\alpha$ is nearly equal to 2 in the vicinity of the top story, and it approaches to 1 in the lower story. In view of these considerations, the following values are taken for each parameter in the present study of
Fig. 5 Load Deflection Relations

Fig. 6 Load Carrying Capacities (Effect of $\alpha$, Wide-Width Flange Section)

Fig. 7 Load Carrying Capacities (Effect of $k_i$, Wide-Width Flange Section)

Fig. 8 Load Carrying Capacities (Effect of $\alpha$, Medium-Width Flange Section)

Fig. 9 Load Carrying Capacities (Effect of $k_i$, Medium-Width Flange Section)

Fig. 10 Comparison of Load Carrying Capacities
the strength of beam-columns: $p=0.2, 0.4, 0.6$; $k_1=6, 12, 18$; $k_2=1.2$; $\alpha=1.0, 2.0$. As mentioned before, the H-shaped cross section of the beam-column is idealized to be $H-100 \times 100 \times 4.49 \times 5.39$ mm for the wide-width flange series and $H-100 \times 50 \times 1.93 \times 2.81$ mm for the medium-width flange series.

Figure 5 shows examples of in-plane load-displacement curves of beam-columns made of the idealized cross section of the medium-width flange series, where $h=Hl/M_d$, $M_d$ being yield moment, and $d$ is the column top displacement $d$ divided by $d$. Dash-dotted lines are the simple plastic collapse mechanism lines. Figures 6 through 9 show the variation of the maximum in-plane load carrying capacity (dashed lines) and the lateral-torsional buckling load (solid lines) plotted against slenderness ratio for various values of parameters, where dash-dotted lines denote the elastic limit. In Figs. 10 and 11, the maximum load carrying capacities determined by the present analysis (solid lines) are compared with those given by the design formula recommended in the Guide to Plastic Design of Steel Structures published by AIJ (dash-dotted lines)\(^{[10]}\). The maximum load carrying capacities of beam-columns of $l/i_{yg}$ equal to 40 and 80 are shown in the form of interaction curves between the axial load and the column-end moment in Fig. 11.

**IV DISCUSSION AND CONCLUSIONS**

**Maximum Load Carrying Capacity**

From Figs. 6 through 11, the following observations are made.

a) Effect of cross-sectional shapes: In case of beam-columns of H-shaped cross section in the wide-width flange series, it seems that the lateral-torsional buckling load is nearly equal to the maximum in-plane load regardless of the values of slenderness ratio. On the other hand, in case of the medium-width flange series, the difference between two loads becomes substantial. The range of the slenderness ratio where the elastic lateral-torsional buckling occurs is very narrow regardless of the cross-sectional shapes.

b) Effect of external moment ratio $\alpha$ and stiffness ratio $k_1$: It is observed from Figs. 6 and 8 that the effects of $\alpha$ on both in-plane load carrying capacity and lateral-torsional buckling load are negligible regardless of the cross-sectional shapes and the value of $k_1$. It appears in Figs. 7 and 9 that the value of $k_1$ more substantially affects on the carrying capacity of beam-columns in the wide-width flange series than in the medium-width flange series, because of the carrying capacity plotted against the value of $l/i_{yg}$. When two beam-columns, the one in the wide-width flange series and the other in the medium-width flange series, with an identical value of $l/i_{yg}$ are compared, the strength reduction due to the decrease in the value of $k_1$ appears to be nearly in the same order.

c) AIJ design formula: From Figs. 10 and 11, it is observed that the strength determined by the design formula in the Guide to Plastic Design of Steel Structures of AIJ\(^{[10]}\) is quite lower than those by the present analysis, and the difference is pronounced with the increase in the slenderness radio. But however, a more detailed study is needed to draw conclusion on the conservativeness of the design formula, since the factor of safety is involved in the formula to take into account the effects of imperfections such as initial crookedness and residual stresses, which are not considered in this study. Note in Fig. 11 that the critical loads of centrally loaded columns of $l/i_{yg}$ equal to 40 and 80 are both equal to the yield axial load $N_y$, since elastic-perfectly plastic material is assumed in the present analysis.
Comparison Between Idealized and Real Sections

The values of $D_T$ of the idealized cross sections adopted here are $0.89 \times 10^{-3}$ for the wide-width flange series and $0.21 \times 10^{-3}$ for the medium flange series. It is revealed from the examination of H-shaped cross sections listed in JIS that the value of $D_T$ ranges $0.50 \times 10^{-3}$ to $1.22 \times 10^{-3}$ for the wide-width series and $0.18 \times 10^{-3}$ to $1.35 \times 10^{-3}$ for the medium-width series. Therefore, the idealized sections adopted in the present analysis represent closely the weakest cross section against torsion.

In order to examine the effect of $D_T$ on the lateral-torsional buckling load, the result of a beam-column of the idealized section H-100×50×1.93×2.81 mm is compared with those of the real sections H-200×100×5.5×8 mm and H-150×75×5×7 mm, as shown in Fig. 12. These two real sections are categorized in the medium-width flange series, and the value of $D_T$ of the former is equal to $0.43 \times 10^{-3}$ and the latter $0.50 \times 10^{-3}$. The difference of the value of $D_T$ affects the lateral-torsional buckling load. However, note that the values of $D_T$ of 13 sections among 17 sections in the category of the medium-width flange series of JIS are below $0.43 \times 10^{-3}$. The difference in the maximum in-plane load carrying capacity observed in Fig. 12 is negligible, and this is already expected since $\tilde{I}_{xu}/I_{xg}$ and $Y/d$ relations, which affects the in-plane load carrying capacity, are nearly identical for the idealized and real cross sections as indicated before.

Conclusions

The elastic-plastic lateral-torsional buckling load and the maximum in-plane load carrying capacity of H-shaped beam-columns with side-sway are numerically analyzed, and the results of the parametric study are discussed. Parameters selected here are the slenderness ratio $l/\bar{r}_p$, the axial load ratio $p$, the beam-to-column stiffness ratio $k$, the external moment ratio $\alpha$ and the cross-sectional shapes idealizing the real H-shaped sections in the wide-width flange series and the medium-width flange series. Summarizing the results of analysis, it is concluded:

(1) The idealized cross sections adopted here are nearly the weakest cross sections against torsion, and thus the results give conservative prediction to the lateral-torsional buckling loads of beam-columns with the real sections. But however, as to the in-plane load-displacement behavior, prediction based on the idealized sections is very well.

(2) As to the beam-columns of sections in the wide-width flange series, the lateral-torsional buckling load is nearly equal to the maximum in-plane carrying capacity regardless of the values of slenderness ratio.

(3) As to the medium-width flange series, the difference between two loads becomes substantial.

(4) It appears that the carrying capacity determined by the design formula in the Guide to Plastic Design of Steel Structures of AIJ is quite conservative. However, it should be noted that the factor of safety to take care of initial imperfections is not considered in the analysis.

REFERENCES

軸力と水平力を受けるII形鋼柱の弾塑性曲げ破れ座屈荷重†（梗概）

正会員 松井 千秋*
正会員 森 野 捷
正会員 木村 潤

I. 序
構造部材の強度及び変形能力を減少させる曲げ破れ座屈は、それを支配する条件の多様性のために、合理的な柱材設計式の導出を困難なものにしている。従って、日本建築学会鋼構造設計規準においては、柱材の設計式として便宜的に、梁の横座屈モーメントと中心圧縮柱の座屈荷重に基づいた相関式が提案されている。

柱材の曲げ破れ座屈問題に関して既往の研究は、柱の水平移動が拘束されている場合とそうでない場合の2つに大別できる。前者の場合には比較的数多くの実験的あるいは解析的研究が報告されているが、後者の場合には文献8）にみられるような、柱両端で構内壁の内圧が拘束されたH形鋼柱に関する一連の実験的・解析的研究があるのではある。

本論文の目的は、まず文献9）の薄肉開断面部材の釣合微分方程式を弾性性体を仮定し、次に一定軸力と水平力を受けけるH形鋼柱に関して、柱曲げ比、梁曲げ及び柱細長比等を種々変化させた数値解析により、曲げ破れ座屈荷重と断面内最大耐荷力との関係を考察することにある。また、これらの結果を文献10）の柱設計式による耐力と比較検討した。

II. 曲げ破れ座屈解析
解析モデル 図1に示す解析モデルは、水平移動を生ずる多層スパン骨組の中柱部材を想定したものである。長さlのH形鋼柱はその柱頭に一定軸力Pと水平力Hを受ける。さらに柱両端は軸力を仮定する上下柱の材枠モーメントマトリックスM_wとM_tが作用する。また、柱材は両端で長さl/2の梁による面内弾性回転拘束を受け、面外曲げに対しては曲げと破れに対して単純支持とした。梁は完全弾性とし、面外変形は考慮していない。

解析仮定 解析にあたって次の仮定を設けている。
(1) 鋼材の応力-歪関係は完全弾塑性。
(2) 変形は微少。
(3) 断面は変形しない。
(4) 薄板要素の板厚中心面内で、直応力に釣合せ断応力によるせん断帯は小さく無視できる。
(5) 3次板要素の板厚中心面に垂直で、部材軸線に平行な面内でのせん断帯は小さく無視できる。
(6) 部材の応力と変形を必要とする。

釣合微分方程式と境界条件 文献9）に、軸力と曲げ及び破れを受ける薄肉開断面部材の釣合微分方程式が、変位の基準点を任意の点として導かれる。この方法に従えば、断面の幅長比に伴って生ずる断面内力やせん断中心の移動を考慮する必要があるが、弹性時に断面全面に関与する釣合を考えることによって幾何学的にも明瞭な釣合微分方程式を求める。上記文献を参考にした。

数値計算 周知のように、柱材に降伏荷重が生じる(1a)式の解は直接的には得られない。ここでは、非弾性柱の面内変形及び曲げ破れ座屈荷重を次のような差分法を用いた数値計算によって求めた。

(1b)式は導入した(1a)式を、境界条件(7a)式を考慮して差分表示する。その結果、vに関する連立一次方程式が得られ、これを解くことによって面内変形vが求まる。(1b, c)式は境界条件(8)式を考慮して差分表示し、それを解いたvを代入するとuとβに関する連立一次方程式が得られ、この係数行列の行列式の値が0になるときの水平力Hの値を求めれば、これが曲げ破れ座屈荷重となる。

III. 数値計算結果
仮想断面 本解析に用いた断面は図2に示すH形断面で、幅材系及び中幅断面をそれぞれH-100×100×4.49×5.39 mm, H-100×50×1.93×2.81 mmなる仮想断面で代表させた。これらは、JIS 規格 H形断面においては、(10a, b)式で定義されるγとνの値が幅材系(d/b=1)と中幅系(d/b≈2)の各断面においてほぼ一定となることをを利用して求めたものである。すなわち、各断面系について、γとνにはJIS 規格H形断
面の平圧値を代入し、$d/b$ の値を与えて $d = 100 \text{mm}$ と仮定すると (10a, b) 式は $w$ と $t$ に関する連立方程式になるから、これを解き上記の仮想断面を定めた。

軸力と曲げを受ける仮想断面の断面性能と曲率の関係を図-3, 4 に示し、実在断面のそれと比較した。解析方程式中の断面性能が、幅断面及び中幅断面を問わず図-3 に示す例のように、仮想断面が実在断面を充分に近似得ることが確かめられた。ただし、図-4 から、曲げ振れ座屈荷重に顕著な影響を与える $D_T$ については、仮想断面と実在断面の間に差があることがわかる。これは仮想断面が実在断面のうち、かなり低い挙げ剛性をもつ断面に対しては、柱の曲げ振れ座屈荷重にどのように影響するかについては後述する。

解析パラメータ 解析にあたっては前記 2 種類の仮想断面及び柱の外断面、中幅比 $I_{rad}$ の他に、(11 a～d) 式で与えられる $p$, $k_1$, $k_2$ 及び $a$ の 4 つの諸量を解析パラメータにとった。これらの値を文献 13) を参考に、$p = 0.2$, $0.4$, $0.6$; $k_1 = 6, 12, 18$; $k_2 = 1.2$; $a = 1.0$, $2.0$ と変化させた場合の、構面内最大荷重及び曲げ振れ座屈荷重を細長比を関係を図-6, 9 に示す。図-5 は中幅断面仮想断面の面内荷重変形曲線の 1 例であり、図-10, 11 は日本建築学会鋼構造鉄造性設計指針の柱設計式による耐力と本解析結果とを、水平力と細長比の関係及び軸力と柱最大モーメントの相関関係で比較したものである。

IV. 考察及び結論

最大耐力 図-6～11 に示す柱耐力についての解析結果をまとめると次のとおりである。

a) 断面系の影響：幅断面仮想断面の場合には、細長比にはかかわらず曲げ振れ座屈荷重は面内最大耐力にはほぼ等しい。これに対して、中幅断面仮想断面の場合には両耐力の差は大きい。また、弾性曲げ振れ座屈が生ずる細長比の範囲は両断面系ともに非常に狭い。

b) 外力モーメント比 $\alpha$ と梁剛性 $k_1$ の影響：図-6, 8 に示すように、$\alpha$ の値の変化が面内最大耐力及び曲げ振れ座屈荷重に与える影響は、柱の断面系及び $k_1$ の値にかかわらず小さい。一方、図-7, 9 にみられるように、$k_1$ の値を小さくすると両耐力は低下するが、低下の度合は細長比 $I_{rad}$ の増加とともに大きくなる。このことは中幅よりも幅断仮想断面に顕著に現れているが、これは両図の横軸を $I_{rad}$ に示しているための現象であり、両図の横軸を $I_{rad}$ に置換して $k_1$ の値を小さくしたことによる両耐力の低下の場合を比較すると、それはほぼ同程度となっている。

c) 鋼構造鉄造性設計指針柱設計式：図-10, 11 に示すように、この設計式による柱耐力は本解析結果と比較してほぼ全般的に安全側であり、細長比が大きくなる程傾向が著しい。しかしながら、本解析は部材の元たわみや残留応力などの不正確性を不完全性を考慮していないので、安全率を含む柱設計式を評価するためには、今後、より詳細な研究が必要である。

仮想断面と実在断面の比較 JIS 規格 H 形断面の $D_T$ 値の分布は、幅断面は $0.50 \times 10^{-3}$～1.22$\times 10^{-3}$、中幅断面は $0.18 \times 10^{-3}$～1.38$\times 10^{-3}$ である。これに対して、本解析に用いた幅断仮想断面の $D_T$ 値は $0.89 \times 10^{-3}$、中幅仮想断面は $0.21 \times 10^{-3}$ であり、従って本論文は比較的挙げ剛性の小さい断面についての解析結果を示すことになる。この $D_T$ 値の差が曲げ振れ座屈荷重に与える影響の度合を検討するために、$H=200 \times 100 \times 5.5 \times 8 \text{mm}$ ($D_T=0.43 \times 10^{-3}$) と $H=150 \times 75 \times 5 \times 7 \text{mm}$ ($D_T=0.59 \times 10^{-4}$) の実在断面についても解析し、仮想断面柱耐力の解析結果を併せて図-12 に示した。図からわかるように、$D_T$ 値の差は断面形状が同じであるにかかわらず、JIS 規格の幅断 H 形断面 17 個中、13 個の $D_T$ 値が $H=200 \times 100 \times 5.5 \times 8 \text{mm}$ に応じる $0.43 \times 10^{-3}$ 以下であることに留意されたい。なお、III. 数値計算結果において比較された仮想断面と実在断面の断面性能と曲率の関係からも推測できるが、図-12 に示すように、仮想断面と実在断面の間の面内最大耐力の差は小さい。