METHOD OF ESTIMATION OF ROTATIONAL RIGIDITY OF THE INTERMEDIATE CONNECTIONS OF FRAMED SHEAR WALLS FOR THEIR EQUIVALENT DIAGONALLY-BRACED FRAMES

by MASAFUMI INOUE* and MASAHIDE TOMII**
Members of A.I.J.

INTRODUCTION

The equivalent strut method\(^1\) is used frequently to analyze frame structures (hereafter referred to as "frames") with framed shear walls, because the equivalent diagonally-braced frames (hereafter referred to as "equivalent frames", see Fig.1) for continuous framed shear walls can be obtained easily by calculation. In this paper, an improved equivalent strut method is proposed to obtain more accurately the stress resultants of the connecting members. The problem of neglecting the nodal rotational rigidity of the conventional equivalent frames whose beams are assumed to be rigid on the stress resultants of the connecting members is made clear.

To consider the nodal rotational rigidity, a method of estimating the flexural rigidity of beams in the equivalent frame corresponding to the intermediate beams of continuous framed shear walls is proposed. A computer program to obtain the interpolated value of the flexural rigidity and a table of 75 exact solutions to be memorized in the computer are made.

A method of estimating flexural rigidity of beams in the equivalent frame, corresponding to the edge beams of the one-bay-one-story framed shear walls, was reported in the previous paper\(^1\).

A method of estimating flexural rigidity of beams in the equivalent frame, corresponding to the edge beams of the multi-bay framed shear walls, is assumed to be the same as the method in a previous study (see reference 2) by dividing the multi-bay framed shear walls into one-bay-one-story framed shear walls.

In this paper, one-bay-two-story and two-bay-one-story framed shear walls, whose exact solutions were obtained by using Airy's stress functions, are adopted as examples of continuous framed shear walls.

1. Analyzed Frames

The configuration of the one-bay-two-story and two-bay-one-story framed shear walls (hereafter referred to as "shear walls") arranged in the analyzed frames is shown in Fig.2. The dimensions and weights of the assumed building in this analysis are shown in Fig.3. The distribution of the weights of the building is determined by assuming that the sum of dead load and live load during an earthquake is 1.0 ton/m\(^2\), and that each outer column and each inner column respectively sustains the loads corresponding to the floor areas 18 m\(^2\) and 36 m\(^2\).

The dimensions of the members of the open frame around the shear walls are assumed to be the same as those of the boundary frames of the shear walls shown in Fig.2. The dimensions of the beams in the first floor are assumed to be 45 cm \times 100 cm.

The analyzed frames in which shear walls are arranged in various patterns are shown in Fig.4.

---

* Associate Professor of Structural Engineering, Department of Architecture, Faculty of Engineering, Oita University, D. Eng.
** Professor of Structural Engineering, Department of Architecture, Faculty of Engineering, Kyushu University, D. Eng.
The number of the stories of the frames is assumed to be 6 or 3 so as to be divided into three parts; the lower, middle and higher stories. The number of the bays of the frames is assumed to be 3 or 6 so as to make outer bays and inner bays.

Young's modulus and Poisson's ratio are assumed to be $2.1 \times 10^5$ kg/cm² and 1/6 respectively. The shape factor for shearing deformation of the beams and columns of the open frame is assumed to be 1.2.

The horizontal external forces adopted in this static analysis (see Fig.6) are determined from the shear coefficients $C_i$ shown in Fig.5.

2. Conventional Equivalent Strut Method for Shear Walls

The equivalent frames for framed shear walls are shown in Fig.1. The cross sectional areas of the columns and braces in the equivalent frames are determined by the method mentioned in the previous paper. 

The exact solutions adopted in this analysis are induced from the elastic analysis of shear walls by using Airy's stress functions, where the axial strain of the beams of the boundary frame of the shear wall is assumed to be zero for the antisymmetrical loads with respect to the longitudinal axis of the shear wall, and the relative displacement between the ends of the beam of the boundary frame is assumed to be zero for the symmetrical loads with respect to the longitudinal axis of the shear wall by attaching a rigid bar with pin joints.
on the connections at both ends of the beam.

The axial rigidity of the beams of the open frame is also assumed to be infinite, and the rigid zones of the connections of the beams and columns are considered.

The shearing forces of the columns and shear walls, and the bending moments of the beams and columns, obtained

values of shearing forces of shear walls, beams and columns

values of bending moments at the ends of beams and columns

unit of external forces and stress resultants: "ton" for external forces and shearing forces

position of bending moments: "ton-m" for bending moments

expression of numerical results:

i) The exact solutions are mentioned in top row, and the bending moment diagram (B.M.D.) is denoted by solid lines.

ii) The solutions of conventional equivalent strut method are mentioned in middle row, and the B.M.D. is denoted by dashed lines.

iii) The solutions of improved equivalent strut method are mentioned in bottom row, and the B.M.D. is denoted by link lines.

Fig. 6 Stress resultants of shear walls, beams and columns

Fig. 7 Maximum errors in bending moments given by conventional equivalent strut method and improved equivalent strut method

Fig. 8 Comparison of fundamental period of the analyzed frames
by exact analysis and by the conventional equivalent strut method, are shown in Fig. 6. The values of the top row are exact solutions, and the values of the middle row are those obtained by the conventional equivalent strut method. The differences between the values of the shearing forces of the shear walls obtained by exact analysis and those obtained by the conventional equivalent strut method are small. The maximum errors of the bending moments of connecting beams and columns for all frames shown in Fig. 4 are shown in Fig. 7. The values of the shearing forces and bending moments of the beams and columns are smaller than those of the shear walls, but the rates of the errors in those values are predominantly large for all frames shown in Fig. 4. Therefore, although the shearing forces of the shear walls can be obtained precisely by the conventional equivalent strut method, the stress resultants, especially the bending moments of the beams and columns, can not be obtained precisely by this method (see Fig. 6 and 7).

The comparisons between the fundamental periods of the frames obtained by exact analysis and those obtained by the conventional equivalent strut method are shown in Fig. 8. The errors of the fundamental periods of the frames vary according to the arranged patterns of the shear walls. But the maximum error of the fundamental period is 7%. Therefore, the fundamental periods of the frames with the shear walls can be obtained with fairly good accuracy by the conventional equivalent strut method.

3. Improvement of Conventional Equivalent Strut Method

The cause of the errors of stress resultants of the connecting beams and connecting columns induced by the conventional equivalent strut method is investigated.

The typical nodal resultants of the shear wall, where a large error occurs in the resultants if the nodal rotational angles in the equivalent strut method are neglected, are shown in Fig. 9a. These typical resultants can be decomposed into four fundamental components (see Figs. 9b, 9c, 9d and 9e). The dashed lines in Fig. 9 indicate the deformations of the shear walls subjected to the four fundamental components respectively.

The nodal rotational angles of Types I, II and IV are neglected since the flexural rigidity of the beams of the equivalent frame is assumed to be infinite in the conventional equivalent strut method. Therefore, a large error occurs in the stress resultants of the connecting beams and connecting columns.

In this section, a method of estimating the flexural rigidity of the beams of the equivalent frame is proposed.


When the rotational angles, \( \theta_i \) and \( \theta'_i \) (see Fig. 10), at the node \( i \) of the shear wall and the equivalent frame, respectively, are calculated, the flexural rigidity, \( EI' \), of the edge beams of the equivalent frame is obtained by equalizing \( \theta_i \) to \( \theta'_i \).

Although \( \theta_i \) in Fig. 9a should be obtained by exact solutions\(^3\),\(^4\), in this paper we use \( \theta_i \) for the beam of one-bay-one-story shear walls mentioned in the previous paper\(^5\) because the maximum error of resultants is about three percent when \( \theta_i \) for the beam of one-bay-one-story shear walls is used. Therefore, the equivalent moment of inertia of the edge beam of the continuous shear wall is obtained by Eq. 1.
\[ I' = \frac{th^4}{24} (k_{xx} + k_{yy}) \]  

Here, the method mentioned in the previous paper\(^2\) can be used for the edge beam of continuous shear walls.

2 Method of estimation of flexural rigidity of intermediate beam

When the rotational angles, \( \theta_k \) and \( \theta_\delta \) (see Fig.11) at node \( k \) of the shear wall and the equivalent frame are calculated, the flexural rigidity, \( EI_\delta \), of the intermediate beams of the equivalent frame is obtained by equalizing \( \theta_k \) to \( \theta_\delta \).

The \( \theta_k \) (see Fig.11a) is given by Eq. 2.

\[ \theta_k = \frac{1}{Et}(k_{15,15} + k_{16,16}) \]  

Here, \( k_{15,15} \) and \( k_{16,16} \) are the elements of stiffness matrix of one-bay-two-story shear wall (see Eq. 3) and are given as follows.

\[ k_{15,15} = (a_{ij} + d_{ij})/4 \]

\[ k_{16,16} = (a_{ij} - d_{ij})/4 \]

\( a_{ij} \) and \( d_{ij} \) are the elements of the fundamental stiffness matrices given by Eq. 4. These elements are asterisked in Eqs. 3 and 4.
The $\theta'_k$ (see Fig. 11b) is given by Eq. 5.

$$\theta'_k = -\frac{l}{6EI'_i}$$

(5)

where, $I'_i$ = moment of inertia of the cross sectional area of the intermediate beams of equivalent frame.

The $I'_i$ is given by Eq. 6 when $\theta_k$ and $\theta'_k$ respectively given by Eqs. 2 and 5 are made equal.

$$I'_i = \frac{th^2(k_{1515} + k_{1516})}{6} = \frac{th^2la_{14}}{12}$$

(6)

The sum of $k_{1515}$ and $k_{1516}$ in Eq. 6 is obtained from the fundamental flexibility matrix of the shear wall. In this paper, a practical calculation method of the premium factor, $\phi$, to calculate $I'_i = \phi l_a$ where $l_a$ is the moment of inertia of the cross sectional area of the intermediate beams of the shear walls, is proposed since a complex calculation is needed to get the sum of $k_{1515}$ and $k_{1516}$.

The variations of $\phi$ according to the aspect ratios, $\lambda(=l/h)$ or $a_o(=D_o/h)$ or $a_o(=D_o/h)$ or $\beta_o(=B_o/t)$ or $\beta_b(=B_b/t)$, of the shear wall are respectively shown in Figs. 12-16, where $B_o$ = width of beams, $B_b$ = width of columns, $D_o$ = depth of beams and $D_b$ = depth of columns. The values of the aspect ratios in Cases 1-5 in Figs. 12-16 are shown in Table 1.

The solid lines in these figures represent the variations of $\phi$ when an aspect ratio of each case is variable. The interpolated values of $\phi$ are given by Eq. 7 which is a parabolic function when the values of $\lambda$, $a_o$ and $\beta_o$ are different from the fundamental values mentioned in Table 1 (see Figs. 12, 14 and 16).

$$f(x) = ax^2 + bx + c$$

(7)

where

$$x = \lambda \text{ or } a_o \text{ or } \beta_o$$

$a$, $b$, $c$ = the coefficients corresponding to each case shown in Table 1 (see Figs. 12, 14 and 16)

The interpolated values of $\phi$ are given by Eq. 8 which is a hyperbolic function when the values of $a_o$ and $\beta_o$ are different from the fundamental values mentioned in Table 1 (see Figs. 13 and 15).

$$f(x) = \frac{ax + b}{cx + 1}$$

(8)
Fig. 15 Variation of $\phi$ according to $\beta_a$

![Image of Fig. 15](image)

Fig. 16 Variation of $\phi$ according to $\beta_b$

![Image of Fig. 16](image)

Table 1 The aspect ratios of Cases 1–5

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda$</th>
<th>$\alpha_a$</th>
<th>$\alpha_b$</th>
<th>$\beta_a$</th>
<th>$\beta_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Case 2</td>
<td>1.5</td>
<td>0.15</td>
<td>0.15</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Case 3</td>
<td>2</td>
<td>0.2</td>
<td>0.2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Case 4</td>
<td>2.5</td>
<td>0.25</td>
<td>0.25</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Case 5</td>
<td>3</td>
<td>0.3</td>
<td>0.3</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

$\lambda = \ell/h$, $\alpha_a = a/a_h$, $\alpha_b = b/b_h$, $\beta_a = \beta_a/\ell$, $\beta_b = \beta_b/\ell$

where $x=\alpha_a$ or $\beta_a$

$a, b, c$ are the coefficients corresponding to each case shown in Table 1 (see Figs. 13 and 15).

The computer program to obtain $\phi$ is shown in the previous paper\(^1\), and the values of the fundamental premium factor $\phi_0$, which are the exact solution of $\phi$ corresponding to each case mentioned in Table 1 to be memorized in computer, are shown in Table 2. The number of $\phi_0$ is 75.

The Eqs. 7 and 8 can be applied to interpolation for $\phi$ if the aspect ratios of the shear wall satisfy the following

Table 2 The values of fundamental premium factor $\phi_0$ to be memorized in computer

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>18.7</td>
<td>34.2</td>
<td>65.8</td>
<td>10.6</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 2</td>
<td>15.1</td>
<td>29.5</td>
<td>58.8</td>
<td>11.6</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 3</td>
<td>11.8</td>
<td>24.4</td>
<td>53.7</td>
<td>19.0</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 4</td>
<td>8.36</td>
<td>20.3</td>
<td>47.5</td>
<td>28.9</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 5</td>
<td>5.32</td>
<td>15.7</td>
<td>43.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\alpha_a = 0.1, 0.2, 0.3$

<table>
<thead>
<tr>
<th>$\alpha_b$</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>18.7</td>
<td>32.6</td>
<td>56.1</td>
<td>25.7</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 2</td>
<td>16.5</td>
<td>28.7</td>
<td>52.3</td>
<td>17.8</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 3</td>
<td>14.0</td>
<td>24.4</td>
<td>49.3</td>
<td>21.5</td>
<td>46.2</td>
</tr>
<tr>
<td>Case 4</td>
<td>12.4</td>
<td>21.9</td>
<td>46.2</td>
<td>18.8</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 5</td>
<td>10.6</td>
<td>18.8</td>
<td>43.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\beta_a = 2, 4, 6$

<table>
<thead>
<tr>
<th>$\beta_b$</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>18.7</td>
<td>30.8</td>
<td>51.1</td>
<td>45.0</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 2</td>
<td>16.4</td>
<td>27.5</td>
<td>48.6</td>
<td>47.0</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 3</td>
<td>14.2</td>
<td>24.4</td>
<td>47.0</td>
<td>45.0</td>
<td>43.9</td>
</tr>
<tr>
<td>Case 4</td>
<td>12.3</td>
<td>21.9</td>
<td>45.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 5</td>
<td>10.4</td>
<td>19.7</td>
<td>43.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
conditions.

\[ \frac{1}{3} \leq \lambda \left(= \frac{l}{h}\right) \leq 3 \]  
\[ 0.1 \leq \sigma_{a} \left(= \frac{D_{a}}{h}\right) \leq 0.3 \]  
\[ 2 \leq \beta_{a} \left(= \frac{B_{a}}{l}\right) \leq 6 \]

The maximum error of the interpolated values of \( \phi \) is 8%.

The computer program and values of \( \phi \) can be checked by confirming the output data \( \phi = 31.031 \) when \( \lambda = 2.75, \) \( \sigma_{a} = 0.15, \) \( \alpha_{a} = 0.1, \) \( \beta_{a} = 2.5 \) are put into the program.

To indicate the validity of the proposed method, some examples of stress resultants of the open frames and arranged shear walls (see Fig. 2 and 4) are shown in Fig. 6. The value of \( \phi \) of the shear wall in the analyzed frames (see Fig. 2) is 21.2. The stiffness matrix of the one-bay-two-story shear wall in the analyzed frames is shown in Eq. 14.

There is improvement in the accuracy of the stress resultants of the connecting beams and connecting columns. The maximum errors of the resultants calculated by the proposed method of all frames shown in Fig. 4 are shown in Fig. 7. They are relatively small. The error of the resultants of the beams and columns seems to be large when shear walls are arranged in all stories. On the contrary, the error seems to be small when there is a story without a shear wall.

The comparisons between the fundamental periods, of the frames with the shear walls (see Figs. 2 and 4), obtained by exact analysis and the improved equivalent strut method are shown in Fig. 8 to indicate the validity of the proposed method. The maximum error of the fundamental periods is not reduced by this method but the error is small.

4. Conclusions

The following conclusions are made clear from the results of the structural analysis to obtain the fundamental periods and stress resultants of the frames with one-bay-two-story and two-bay-one-story framed shear walls.

1. The shearing forces of the shear walls are obtained with sufficient accuracy for practical structural design by the conventional equivalent strut method where the beams of the equivalent frames are assumed to be rigid.

2. The maximum error of stress resultants of the beams and columns of the open frames is several hundred percent of the corresponding exact solution if the conventional equivalent strut method is used. The main cause of the error is due to the nodal rotational angle of the conventional equivalent frames being neglected.

3. In order to consider the equivalent nodal rotational rigidity of the equivalent frames corresponding to the rotational rigidity of the rigid connections of the shear walls, a practical method of estimating the flexural rigidity of the beams assumed to be rigid in the conventional equivalent strut method is proposed. To calculate the flexural rigidity, \( EI_{n} \), of the beams of the equivalent frame, only 75 exact solutions, \( \phi_{n} \), of premium factor \( \phi (= I_{f}/I_{a}) \) of the flexural rigidity, \( EI_{n} \), of the intermediate beam of the shear wall are necessary.

4. The maximum error of the stress resultants of the beams and columns of the frames decreases remarkably if the proposed method is applied.
REFERENCES