DAMAGE MECHANISMS AND MECHANICAL BEHAVIOUR OF CONCRETE UNDER CYCLIC LOADS

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Introduction
Concrete is undoubtedly one of the commonest and most important structural materials. In spite of a huge amount of effort to study its strength properties, the fracture process and the mechanism of nonlinear behaviour under cyclic loads are far from being as fully understood as those for metals and polymers (Yoshimoto, Ogino and Kawakami, 1972; Stroven, 1979; Weigler and Klausen, 1979). Unfortunately it is hardly possible to observe directly the cracking inside of concrete materials under load, so far. Since concrete is a brittle material with a very heterogeneous microstructure, the mechanical properties are quite different from those of metallic materials. Moreover, the mechanical behaviour highly depends on the experimental condition such as the type of loading, the temperature, the humidity and so on.

Although many researchers have studied the dynamic fatigue of concrete materials, most of them use experimental approaches to know the widely scattered relation between the maximum load and the fatigue life. Recently the results of some experimental studies presented the deformation properties of concrete materials under cyclic loads (Sparks and Menzies, 1973; Tokumitsu and Matsushita, 1979; Cornelissen and Timmers, 1981). They pointed out that there was a strong relation between the fatigue life and strain rate. Moreover Cornelissen and Timmers carried out the fatigue test under tension-compression cyclic loads. It might be one of the most important findings in their work that the slope of the relation between \( \ln N \) and \( \ln \eta \) in the case of tension-compression cyclic loads is about twice steeper than that in the case of tension-tension cyclic loads; where \( N \) is the mean value of fatigue life and \( \eta \) is the ratio of the maximum stress to the strength. The micromechanism of fatigue damage and deformation, however, is still ambiguous and there are no generally accepted theoretical models to describe such a comprehensive behaviour. In order to predict the mechanical behaviour and the life time of concrete structure under cyclic loads, some theoretical model based on the fundamental properties of the material should be developed.

Recently, the authors have presented a stochastic theory for the fatigue of concrete (Mihashi and Wittmann, 1980; Mihashi and Izumi, 1980). The non-fracture probability \( P(N) \) and the fatigue life \( N \) were given by eq. (1) and eq. (2) respectively.

\[
P(N) = \exp(-A\sigma^n) \\
N = \frac{1}{A\sigma^n}
\]

These theoretical results were in good agreement with published data. They have also analyzed the experimental results of Cornelissen and Timmers by means of fracture mechanics and discussed the mechanism of fatigue process (Mihashi and Izumi, 1984). The purpose of this paper is to present an theoretical model to link the probable mechanism of fatigue processes of concrete to the macroscopic behaviour under cyclic loads.

Probable Mechanism of Fatigue Process
Since concrete is a brittle material with the extremely heterogeneous structure, the fracture and fatigue properties are dominated by the internal structure. The fatigue process may be subdivided into three stages as follows: 1).

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Crack initiation around larger aggregates, being arrested by the neighbouring aggregates; damage accumulation in the matrix and interfaces; unstable crack extension to cause fracture, the first stage is constituted by the following mechanism. Stress is highly concentrated in the vicinity of the aggregates and there is a porous and weak system on the interface. Accordingly cracks initiate even under low stress. However, these cracks immediately come across other aggregates and are arrested by them because of high toughness of aggregates (Fig. 1).

On the other hand, the matrix system is a comparatively homogeneous solid but a kind of composite materials including micropores. Therefore the damage of the system is successively accumulated under a cyclic load. The mechanism of the damage was studied experimentally by Yoshimoto and co-workers (1972). According to his study, the damage may be due to the accumulation of microcracks (boid-cracks) in the paste.

After a certain amount of the accumulation of damage, the fracture toughness of the matrix may be decreased. As the result of that, the arrested cracks will be extended and mutually connected. In other words, the specimen may be fractured when the damage on the second stage is accumulated enough to allow arrested cracks to propagate in an unstable manner. Therefore the fatigue life may be closely related to the rate of the damage accumulation. In the case of tension-compression cyclic loads, the possibility to transient into the third stage may be increased. Because the vicinity of crack tips are thrust in the Mode II and Mode III under compressive loads and vertical bond cracks are also created linking the horizontal tensile cracks.

**Deformation due to Crack Initiation at the First Stage**

It is supposed that the increase of deformation in stage 1 is due to the accumulation of mesocracking which takes place in succession from the weakest region such as interfaces. According to the elements of Linear Fracture Mechanics, the deformation in the y direction on the point P(x, y) (Fig. 2) is given by eq. (3) in the case of plane strain.

\[ \varepsilon = \frac{K_i}{2G} \sqrt{\frac{r}{2\pi}} \sin \frac{\theta}{2} (x + 1 - 2 \cos \frac{\theta}{2}) \]  

where \( x \) is equal to \((3-4\nu)\) and \( G \) is the shear modulus. The displacement on the center of the crack surface: \( \bar{u} \) is obtained from eq. (3) as follows:

\[ \bar{u} = \frac{\sqrt{2(1-\nu)}}{G} \sigma c \]  

The strain due to a single crack initiation may be described by eq. (5).

\[ \varepsilon_0 = 2\sqrt{2(1-\nu)} \frac{\sigma c}{\bar{d}G} \]  

where \( \bar{d} \) is the distance between marked points.

Since the strain is caused by cracking, the increasing rate of the strain may be proportional to the probability of the crack initiation: \( L\sigma^a \) (Mihashi and Wittmann, 1980); \( L \) is a parameter of the internal structure and environmental conditions. Moreover, the magnitude of the strain may be controled by the number of cracks and

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Fig. 1 Schematic Description of Probable Mechanism of Fatigue Process of Concrete.

Fig. 2 Geometry of Single Crack.
their length. The number of crack initiation may be dependent on the non-fracture probability which decreases as the number of loading cycles \( N \) increases.

From the consideration mentioned above, the following equations are obtained.

\[
\dot{\varepsilon}_i = \varepsilon_i L \sigma^\alpha \exp(-A \sigma^\beta) \exp(-A \sigma^\beta) \tag{6}
\]

\[
\varepsilon_i = \varepsilon_i \frac{L}{A} (1 - \exp(-A \sigma^\beta)) \tag{7}
\]

**Deformation Due to Damage Accumulation at the Second Stage**

After most of weaker regions highly concentrated with stress release the strain energy by cracking, the damage accumulation process may become dominant. The strain rate of the damage accumulation process may be supposed to be given by eq. (8).

\[
\dot{\varepsilon}_i = k_e \dot{\varepsilon} \tag{8}
\]

where \( k_e \) is a material constant, \( \dot{\varepsilon} \) is the mean value of the strain increase per one loading cycle and \( \mu_e \) is the probability to cause the strain increase per one loading cycle. Since the strain increase is due to microcracking (void-crack) (Yoshimoto et al., 1972), the strain-increase-probability may be proportional to the micro-crack-initiation-probability. Provided microcracking is a kind of rate process dependent on stress, eq. (9) is obtained (Yokobori, 1974; Mihashi and Wittmann, 1980).

\[
\mu_e \propto \mu_s = L \sigma^\beta \tag{9}
\]

where \( \mu_e \) means the microcrack-initiation-probability under stress \( \sigma \); \( \beta \) is a material constant affected by the temperature and the humidity, and \( \sigma \) is the maximum stress. Substituting eq. (9) into eq. (8), the following equation are obtained.

\[
\dot{\varepsilon}_i = k_e \dot{\varepsilon} L \sigma^\beta \tag{10}
\]

\[
\varepsilon_i = k_e \dot{\varepsilon} L \sigma^\beta \tag{11}
\]

**Deformation due to Unstable Crack Propagation**

Since the matrix and interfaces are damaged with the accumulation of microcracks, the fracture toughness of the system may be decreased after the second stage. According to fracture mechanics, the catastrophic fracture occurs when the fracture mechanical parameter such as the fracture toughness reaches a certain critical value. Since the stability of crack propagation is proportional to the remained toughness of the system, the probability for the system to reach the critical condition may be in inverse proportion to the survival probability at the second stage. Supposing that the unstable crack length is proportional to the kth power of \( N \), the following equations are obtained.

\[
\varepsilon_i = \frac{2 \sqrt{2}}{\pi} \frac{(1 - \nu^* \Delta N^*)}{2^G} \sigma \varepsilon \exp(A \sigma^\alpha) = k_i \sigma \varepsilon \exp(A \sigma^\alpha) \tag{12}
\]

\[
\dot{\varepsilon}_i = \frac{2 \sqrt{2}}{\pi} \frac{(1 - \nu^* \Delta N^*)}{2^G} \sigma \varepsilon \exp(A \sigma^\alpha) \tag{13}
\]

\[
= k_1 \sigma (A \sigma^\alpha \varepsilon) \exp(A \sigma^\alpha) \tag{14}
\]

**Discussion**

Since the strain rate and strain under cyclic loads are obtained as the summation of those on three stages, the following equations are obtained.

\[
\dot{\varepsilon} = \dot{\varepsilon}_s + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 \tag{14}
\]

\[
\varepsilon = \varepsilon_s + \varepsilon_2 + \varepsilon_3 + \varepsilon_t \tag{15}
\]

where \( \varepsilon_t \) means the elastic strain under the maximum stress.

The fatigue life under cyclic loads is related to the maximum stress level as follows from eq. (2).

\[
\ln \bar{N} = -\beta \ln \nu + \text{const.} \tag{15}
\]

On the other hand, eq. (10) is rewritten as follows:

\[
\ln \dot{\varepsilon}_i - \ln \Delta \varepsilon = \beta \ln \bar{N} + \text{const.} \tag{16}
\]

The values of \( \beta \) calculated from the experimental results by Cornelissen and Timmers are as follows (Mihashi and Izumi, 1984):

\[
\beta = 12.9 \text{ for wet condition}
\]

\[
\beta = 11.2 \text{ for dry condition}
\]


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These values are quite close to those obtained from a completely different test (Mihashi and Wittmann, 1980).

Fig. 3, shows, some examples of the theoretically simulated results according to eq. (14), using the above mentioned different values of $\beta$ for the different environmental conditions. The influence of the humidity on the fatigue deformation properties is represented in Fig. 3(a). It is simulated that the shape of the deformation curves seems to be almost same but the wet condition causes a larger deformation than the dry condition for the same stress level: $\eta$. The influence of the stress level on the fatigue deformation properties can be also described by this model as shown in Fig. 3(b). It is well simulated that a larger stress level gives a larger deformation though the whole fatigue deformation properties are not so much changed in this calculated range. These theoretical predictions are in good agreement with the experimental data published by Cornelissen and Timmers (1981) as shown in Fig. 4. It means that the present theoretical model based on the probable mechanism can predict reasonably the mechanical behaviour of concrete under cyclic loads.

The corresponding strain rates calculated by eq. (14) are also shown in Fig. 3. These changing processes of strain rates are very important to monitor the safety of the system because the final fatigue life is dominated by the unstable crack propagation through the system. According to these theoretical predictions, the changing behaviour of the strain rate at the final stage of wet specimen is not so much sensitive as

![Graph showing strain rate vs. normalized cyclic number](image)

Fig. 4 Fatigue Property after Cornelissen and Timmers.

![Graph showing relation between strain rate and fatigue life](image)

Fig. 5 Relation between the Strain Rate at the Second Stage and Fatigue Life [Experimental Results were obtained by Cornelissen and Timmers].
that of dry specimens. The behaviour on a low stress level is also not sensitive in comparison with that on a high stress level. Therefore the target to be controlled should be very carefully fixed when the system is monitored by the unsensitive parameter. Comparing eq. (15) with eq. (16), the following relation is obtained.

\[ \ln N = \ln \dot{\epsilon}_f \]

that is

\[ \dot{\epsilon}_f = \dot{\epsilon}_c \]

where \( \dot{\epsilon}_f \) is a material constant. Therefore it is expected to estimate the fatigue life by monitoring the strain rate at the second stage. The comparison of eq. (17) with the experimental results by Cornelissen and Timmers is shown in Fig. 5 and one can find the good agreement. This tendency was also presented for the compressive fatigue test by Sparks and Menzies (1973).

Concluding Remarks

In order to investigate the mechanism of the nonlinear mechanical behaviour of concrete under cyclic loads, the fatigue process should be subdivided into three stages.

At the first stage, the strain energy accumulated by the locally concentrated stress around fatal material defects such as larger aggregates and shrinkage cracks is easily released by mesocracking. However, these cracks are arrested by other aggregates or with the change of the stress field at the crack tip. The crack initiation process occurs successively throughout the specimen and continues until the saturated stable condition. The strain is widely scattered because the crack initiation and arrest process is highly influenced by the geometric properties and arrangement of aggregates. Undoubtedly it is necessary to study theoretically the cracking process at the first stage by means of computer simulation in a random media with composite structures.

At the second stage, microcracks (void-cracks) initiate in the matrix because of the heterogeneity in the matrix itself. Since these cracks are gradually accumulated in the matrix, the fatigue process at the second stage may be supposed to be a kind of rate process dependent on stress. Therefore the strain rate may be proportional to \( L_c \).

The fatigue process at the third stage may be due to the extension of a critical crack which links mesocracks occurred on the first stage. Since the stress intensity factor increases with the crack extension, the most critical crack extension may dominate the mechanical behaviour.

Since the fatigue life may be in inverse proportion to the strain rate of the second stage, it will be possible to predict the fatigue life by monitoring the strain rate at the second stage.

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Appendix
The following notations are used in the present paper.

- $A$: Parameter to describe the environmental condition and also the frequency of the cyclic load (see Mihashi and Wittmann, 1980).
- $A_i, A_{ii}, A_{iii}$: the value of $A$ at the $i$th stage.
- $2c$: crack length.
- $2\theta$: the initial value of the equivalent crack length in the critical condition.
- $G$: shearing modulus.
- $K_i$: stress intensity factor.
- $k, k_m, k_i$: material constants.
- $L$: parameter of the internal structure of the specimen and the environmental condition.
- $L_i, L_{ii}$: the value of $L$ at the $i$th stage.
- $N$: number of loading cycles.
- $N$: the expected fatigue life, i.e. the mean value of the number of loading cycles to collapse the specimen.
- $\Delta N$: increment of the cyclic number of the load.
- $P(N)$: non-fracture probability, i.e. the probability that the specimen still survives after $N$ cycles of the load.
- $\nu$: displacement in the $y$ direction on a certain point $(x, y)$ around a crack.
- $\overline{\nu}$: displacement on the center of the crack surface.
- $\beta$: material constant as a function of the environmental temperature and humidity.
- $\bar{e}$: the mean value of the strain increment per one loading cycle at the second stage (damage accumulation process).
- $e_i$: strain due to a single crack initiation.
- $e_i, e_{ii}, e_{iii}$: strain due to the mechanism at the $i$th stage.
- $\dot{e}_i, \dot{e}_{ii}, \dot{e}_{iii}$: strain rate due to the mechanism at the $i$th stage.
- $e_m$: elastic strain under the maximum stress.
- $\Delta e$: strain increment.
- $\eta$: ratio of the maximum stress to the strength.
- $x$: material constant ($=3-4v$)
- $\mu_e$: probability of microcrack initiation.
- $\mu_s$: probability to cause the strain increase per one loading cycle.
- $\nu$: Poisson's ratio.
- $\nu^*$: apparent Poisson's ratio of the damaged system.
- $\sigma$: the maximum stress of the cyclic load.
- $\phi$: material constant.
繰返し荷重を受けるコンクリートの損傷機構と力学的挙動に関する基礎的研究所（梗概）

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コンクリートは最も重要な構造材料の一つであり、繰返し荷重の作用を受けるのが通例である。最大荷重と疲労寿命の関係については数多くの実験的研究によって長年研究されているが、繰返し荷重による損傷機構あるいは損傷過程については未解明な部分がほとんどと言えるのが現状である。

本論文は、疲労損傷のメカニズムを考察し、その損傷過程を表現する理論モデルを構築するための一つの考え方を示そうとするものである。疲労損傷過程を3つの段階に分類して、各段階の損傷機構を破壊力学的視点から考察する。さらに損傷モデルに基づいた変形理論曲線を求め、実験結果と比較検討する。

3つに分類された損傷過程の第1段階は、クラック発生過程であり、大きな粗骨材や乾燥収縮クラックなどに生じた局所集中応力によって蓄積されたひずみエネルギーは、ポンドクラックやモルタルクラック等のメソレベルのクラック発生によって容易に解放される。しかしながら、これらのクラックは周囲の骨材や応力場の変化によって伝播を阻止される。このクラック発生過程は、

供試体内の至る所で次々と起こり局所的に集中した高ひずみエネルギーが解放される必要のなくなるまで続く。この段階のひずみは、その原因となるクラックの発生および伝播阻止過程が粗骨材の幾何学的形状や配置に強く依存するために大きなバラツキを示す。

第2段階においては、マトリックス部分自身の非均質性のためにセメントベースト中に微細クラック（ポンドクラック）が生じる。これらの微細クラックはマトリックス内にゆっくりと蓄積され、第2段階における疲労損傷過程は一種の応力依存型過程と考えられる。したがって第2段階のひずみ速度はL1σを比例するものと考えられる。

第3段階における疲労損傷過程は、第1段階で発生したポンドクラックやモルタルクラックを連結するクラックの伸展によるものと考えられる。クラック先端における応力拡大係数は、クラックの伸展とともに増大するので主なクラックの伸展が第3段階の力学的挙動を支配する。

疲労寿命は、第2段階におけるひずみ速度に反比例するものと思われるので、第2段階におけるひずみ速度をモニターする事によって疲労寿命を予測する事が可能と考えられる。

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