PLASTIC DEFORMATIONAL BEHAVIOR OF AXIALLY LOADED CONCRETE UNDER LOW LATERAL PRESSURE#

—An Evaluation Method for Compressive Toughness of Laterally

Confined Concretes (Part 1)—

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§1. Introduction

For the application of the plastic design method to reinforced concrete (RC) frames, it is the first requisite that constitutive RC members are ductile enough for the redistribution of moment and for the formation of plastic collapse mechanism⁴⁾. It is well known that the preferred failure pattern of RC members is a flexural failure preceding type with adequate warning of collapse, then increase in the toughness (energy absorption capacity) of concrete in the compressive zone of RC members is quite effective to improve their ductility.

It has been pointed out that the mechanism of the toughness improvement of confined concrete and steel fiber reinforced concrete under compression is mainly due to the lateral confining effect of lateral bars and steel fibers caused by the Poisson's effect of concrete after failure⁵⁾. This effect can be easily related to the behavior of plain concrete under multiaxial stress states⁶⁾⁻¹⁰⁾. Thus, it is possible to discuss systematically the toughness improvement of various kinds of laterally confined concretes, based on triaxial compression test data, especially on the post-failure behavior of concrete under a low confining lateral pressure of less than about 20 kgf/cm²¹⁰⁾.

However, information is not adequate to date to develop a reliable constitutive model including such a post-failure behavior of concrete^{7), 11)-13)}. Some of the main reasons are in the difficulties in the control of strain rate and the measurement of lateral strains, resulted from the fact that the dilatation of concrete beyond ultimate stress is remarkable in this range of lateral pressure. The authors, therefore, have developed a triaxial testing method in which a high rigidity compression testing machine and a new type of lateral loading device are combined, and confirmed, using this method, that fundamental information can be obtained for discussing the mechanism of the toughness improvement of laterally confined concretes¹⁴⁾⁻¹⁶⁾.

There are two main purposes in the present study. The first is to propose an evaluation system for the effect of various types of lateral confinements on the compressive toughness of concrete, based on the behavior of concrete under triaxial compression (Part 1). The second is to discuss in terms of the toughness evaluation system the rational combination pattern of concrete and other materials, especially steel, to provide a required ductility to RC members (Part 2).

The present paper (Part 1) describes the result of the investigation related to the first purpose. First, the effect of various factors on the plastic deformational behavior of concrete under triaxial compression was examined in an experiment. Next, a systematic method based on the triaxial compressive test data was proposed for the evaluation of the compressive toughness of various kinds of laterally confined concretes. Further, the stress (σ_1)-strain (ϵ_1) curves in the direction of the maximum principal compressive stress were numerically expressed, the expression being used as a standard in the evaluation system.

This paper is based on the earlier works presented in Refs. 1) through 3).

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§2. Experimental procedures

2.1 Outline of experiment

The outline of experiment is shown in Table 1. The variables in the experiment were the water-cement ratio ($W/C=0.45, 0.55$, and 0.70), height (H)/width (D) ratio of specimen ($H/D=1, 1.3$, and 2), magnitude of lateral pressure ($\sigma_L=0, 1, 3, 6$, and 12 kgf/cm^2 . Here, these values of σ_L gradually increase from about $\epsilon_1=5 \times 10^{-3}$ in the stress descending range due to the rapid dilatation of concrete in the experiment.), spacing of loading point of lateral pressure ($S=0, 5$, and 10 cm), and friction at the specimen-loading platen interfaces (coefficient of static friction $\mu=0$ and 0.4). The term "pressure (σ_L)" is used for the representation of the balanced lateral stresses, i. e. $\sigma_L = \sigma_2 = \sigma_3$, and the subscripts of both stress (σ) and strain (ϵ) represent the co-ordinate axes (see Figs. 1 and 2), compression being positive. The cross-section of specimen for the triaxial test was kept to $10 \times 10 \text{ cm}$. Three specimens were prepared for each variable.

2.2 Fabrication and curing of specimens

Ordinary Portland cement, river sand (maximum size : 5 mm) and river gravel (size range : $5\text{--}15 \text{ mm}$) were prepared for concrete. The slump of concretes was designed to be 5 cm for all the batches in order to minimize the variation of mechanical properties of the specimens resulting from bleeding. Concretes were cast into steel molds and consolidated with a wooden hammer and a rotary type poker vibrator from the outside of the molds. All the specimens were capped with cement paste at the age of 1 day and demolded at the age of 2 days, and then cured in an air conditioned room ($23 \pm 2^\circ\text{C}$, relative humidity $75 \pm 5\%$) until testing, except that some cylindrical specimens were cured in water. The tests were carried out at the age of about 50 days.

2.3 Methods of loading and measurement

Table 1 Outline of experiment

Notation of Specimen	W/C	H/D	σ_L (kgf/cm ²)	S (cm)	μ
e.g.) T6-5-1-0.4 H/D=1 S=5cm $\sigma_L=6 \text{ kgf/cm}^2$ Triaxial test	0.45	1.0	0	0	0.4*
			1		
			3		
	0.55	1.3	5*	0	0
			10*		
			10*		
	0.70	2.0	6*	5	0
			12*	10	
			12*	5	
			12*	10	

[Notes] W/C: Water-cement ratio, H/D: Height/width of specimen, σ_L : Lateral confining stress, S: Spacing of lateral steel bar, μ : Approximate value of coefficient of static friction, *¹: Only for H/D=1, *²: Only for H/D=1 of W/C=0.55, *³: Only for H/D=1 and 2 of W/C=0.55, *⁴: Only for W/C=0.55.

Fig.1 Triaxial loading device

A triaxial compressive loading device illustrated in Fig. 1 was used, with which arbitrary lateral stresses are applied to the specimen by means of the flexural moment resistance of steel plates. The lateral loads applied are estimated by the strain of steel bolts set between the steel platen and a steel bar. The initial values of lateral stresses are controlled by tightening the steel bars with nuts. All the specimens were set upside down, that is, a capped surface was set on the other side of a tilting platen (see Figs. 1 and 2) in order to alleviate the concentration of the failure in a specimen¹⁷⁾. All the specimens for the uniaxial and triaxial tests were loaded in the longitudinal (first) direction under the constant strain rate of about $2 \times 10^{-3}/\text{min.}$ by using a high rigidity compression testing machine. For the dispersed type lateral confinement, a set of steel bars with $5 \times 5 \text{ mm}$ section were set between a specimen and lateral loading platens (see Fig. 7 (a)).

The complete stress-strain curves were recorded by an X-Y recorder up to a specified strain ($\epsilon_1=30 \times 10^{-3}$). The

Fig.2 Convention for subscripts of stress and strain

friction at the specimen-loading platen interfaces was reduced by placing friction reducing pads. The pad consists of two polypropylene sheets with silicon grease between them, with which sufficient uniformity of stress distribution on the ends of the specimen can be expected¹⁸⁾. It was confirmed by using "pressure sheets"¹⁸⁾ placed at the specimen-platen interfaces that the lateral stress distribution induced by the loading device was adequately uniform. For all the specimens, a couple of differential transformers were set between the tilting platen and the machine base platen to measure the over-all strains. For cubic specimens, lateral strains were measured by two deformation transducers as shown in Fig. 1. For the specimens under uniaxial compression, two couples of wire strain gages (W.S.G.) were glued to the specimen to measure the longitudinal and lateral strains.

§ 3. Test results and discussion

Since the displacement was measured by a couple of differential transformers set between platens, the strain in the stress ascending portion of the curves is over-estimated due to the presence of the friction reducing pads. Therefore, the uniaxial longitudinal stress (σ_1)-longitudinal strain (ϵ_1) curve was used up to the stress at which the tangent modulus of σ_1 - ϵ_1 curve from the triaxial test becomes compatible with that of the curve from the uniaxial test.

3.1 Failure patterns of specimens

Typical ultimate failure patterns of various shapes of specimens are shown in Photo. 1. As seen in Photo. 1, cracks extend throughout the whole height for the specimen of $H/D=1$ and 1.3, while crushing occurs only at the upper end zone which corresponds to the upper zone of specimen at casting, for the specimen of $H/D=2$. This trend is observed regardless of the level of lateral pressure applied in the experiment.

3.2 Effect of magnitude of lateral pressure

Figures 3 (a) and (b) show the effects of uniformly distributed lateral pressure on the deformational behaviors of the specimens of $H/D=1$ and 2, respectively. For both shapes of specimens, large increases in load-carrying capacity and toughness are observed even under a small lateral pressure such as $\sigma_L=3 \text{ kgf/cm}^2$. The increases in peak stress and toughness due to a certain level of lateral pressure are almost constant regardless of the H/D ratio of specimen. Further, lateral strain at a certain longitudinal strain level in the stress descending range decreases almost proportionally to the magnitude of lateral stress.

3.3 Effect of water-cement ratio

Figures 4 (a) and (b) show the effect of water-cement ratio (W/C) on the σ_1 - ϵ_1 curves. A similar trend of the effect of W/C on the σ_1 - ϵ_1 curve of concrete under uniaxial compression is observed for concrete under triaxial compression. Namely, the strain level at the beginning of converging branch, which is usually observed in the descending portion of the σ_1 - ϵ_1 curves of the concrete under uniaxial compression¹⁹⁾, gradually increases with increasing lateral pressure. The concrete specimen at this strain level is considered to suffer a certain critical damage regardless of the strength of concrete.

3.4 Effect of height/width ratio of specimen

Figure 5 shows the effect of H/D ratio of specimen on σ_1 - ϵ_1 curves. The stress and strain at the peak point slightly increase with decrease in the H/D ratio of specimen. The effect of the H/D ratio of specimen on the shape of the

descending portion of σ_1 - ϵ_1 curve is quite remarkable regardless of the magnitude of lateral pressure, i. e. the shape of the curve becomes steep as the H/D ratio of specimen increases. Such change is almost independent of the magnitude of lateral pressure.

It is well known that i) the σ_1 - ϵ_1 curve measured in a conventional uniaxial compression test is significantly influenced by the strain measurement region and ii) it is quite important to make clear the equivalence of the failure zones of a tested concrete specimen and a reinforced concrete member to which the σ_1 - ϵ_1 curve is to be applied in the

Photo. 1 Typical ultimate failure patterns

 (a) $H/D=1$

 (b) $H/D=2$

 Fig. 3 Effect of lateral pressure (σ_L) on σ_1 - ϵ_1 , ϵ_2 , and ϵ_3 curves of axially loaded concrete

 Fig. 4 Effect of water-cement ratio (W/C) on σ_1 - ϵ_1 curves

deformation analysis^{17), 20)-22)}. Koyanagi et al²²⁾ reported an analytical model for the load-deformation curves of specimens ranging from $H/D=2$ to 4 in which the deformations in failure and elastic zones are superimposed. In the experiment, the casting direction of concrete was normal to a loading axis and the friction at the specimen-loading platen interface was not reduced. The authors¹⁶⁾ discussed the relation between the length of failure zone and the measured σ_1 - ϵ_1 curve, based on the data from a triaxial test of the specimens of $H/D=1$ to 2 in which the casting direction of concrete was parallel to the loading axis and the friction was reduced in the same manner as in the present experiment; and proposed an idealized failure zone model shown in Fig. 6 (a). Based on the Photo. 1 and previous

Fig. 5 Effect of height/width (H/D) ratio of specimen on σ_1 - ϵ_1 curves

evidences¹⁶⁾, the σ_1 - ϵ_1 relation in the idealized crushed zone is assumed to be equivalent to the σ_1 - ϵ_1 curve from the specimen of $H/D=1$ as well as in the earlier report¹⁶⁾. The dashed lines in Fig. 6 (b) are the predicted σ_1 - ϵ_1 curves for the specimen of $H/D=1.3$ and 2, obtained on the condition of $l_p=D$ (l_p : length of idealized crushed zone, D : width of specimen). The predicted curves are in good agreement with experimental ones. Although the σ_1 - ϵ_1 curves for the crushed and non-crushed zones bifurcate at the peak point in the present model, the internal failure of a specimen progressively develops after the critical stress point. Therefore, different σ_1 - ϵ_1 curves should be provided after the critical stress point e.g. after $0.8 F_c$ (F_c : peak stress) regardless of lateral stress level for more detailed discussion.

(a) Idealized failure zone model

(b) Comparison between measured and predicted curves

(c) Prediction of post-peak σ_1 - ϵ_1 curves

Fig. 6 Idealized failure zone model and its applicability

The applicability of the present model to the prediction of the stress descending portion is discussed. Figure 6 (c) shows the comparison between the predicted and the measured σ_1 - ϵ_1 curves in the stress descending range for the specimens of $H/D=2$ with various compressive strengths and lateral pressures. Fairly good agreement is observed between the two curves regardless of the level of lateral pressure for $W/C=0.45$ and 0.55 , while the predicted curves are steeper than the measured ones, and the difference becomes larger as the level of lateral pressure decreases for $W/C=0.70$. One of the reasons for such a difference in prediction may be due to the fact that the failure of the concrete of $W/C=0.70$ is rather ductile compared to that of $W/C=0.45$ and 0.55 . Namely, due to the relatively small degradation of the load-carrying capacity in the crushed zone, the failure concentration or the strain localization is considered to be alleviated. In order to reflect such a trend in the present model, the length of the crushed zone (l_p) should be slightly increased.

3.5 Effect of dispersed lateral confinement

In the preliminary experiment, it was observed that the σ_1 - ϵ_1 curve is not influenced so much by the arrangement of lateral bars ($\square$ and $\square$) for the pitch (S) of 10 cm. Therefore, in the present study, the former arrangement was adopted. Figure 7 (a) shows the effects of the pitch (S) of lateral confinement on the σ_1 - ϵ_1 curves of concrete. The evident effect of the pitch is not observed on the stress and strain at the peak point as far as the present experiment is concerned, i.e. the pitch (S) being less than the width (D) of specimen and the lateral pressure being not more than 12 kgf/cm^2 . However, the effect of the dispersion of lateral pressure becomes gradually remarkable after the peak point, that is, the decrease in the load-carrying capacity of a specimen becomes larger as the pitch increases. Further, such trend becomes more marked with increase in the lateral pressure. Figure 7 (b) shows equivalent lateral pressures ($\bar{\sigma}_L$) for the specimens of $H/D=1$ and 2 under dispersed lateral confinement, where the σ_1 - ϵ_1 curves of the specimens are evaluated by those of the specimens under uniformly distributed confinement.

§ 4. Evaluation and estimation of compressive toughness of laterally confined concretes

4.1 Equivalent lateral pressure

The improvement of compressive toughness of concrete due to lateral confinements can be evaluated in terms of "equivalent lateral pressure ($\bar{\sigma}_L$)" which is defined as a pressure whose effect on the load-carrying capacity of a specimen at a certain strain level is equivalent to that by uniformly distributed lateral pressure. The equivalent lateral pressure is given as a function of strain (ϵ_1) by comparing the σ_1 - ϵ_1 curves of the confined concretes under uniaxial compression and those of plain concrete under a certain triaxial stress state. One of the standard conditions adopted in the present study is as follows:

Shape of specimen : $H/D=1$

Loading path of lateral pressure : active loading

Ratio of two lateral stresses : $\sigma_2/\sigma_3=1$

Loading point of lateral pressure : uniformly distributed over lateral surfaces of specimen

Fig. 7 Effect of pitch (S) of lateral confinement on σ_1 - ϵ_1 curves

Here, active loading means the loading path in which the lateral stresses (σ_2, σ_3) are applied to a specimen before axial loading (σ_1). The effects of various factors on the σ_1 - ϵ_1 curve of concrete under the above standard condition are schematically shown in Fig. 8.

The evaluation of σ_1 - ϵ_1 curve in terms of the equivalent lateral pressure has a significant merit. Since the equivalent lateral pressure ($\bar{\sigma}_L$) is a physical quantity, it is considered possible to estimate to some extent the toughness improvement due to the combination of various types of composites (multi-order composites) just by superimposing the each value of $\bar{\sigma}_L$ and, if necessary, by introducing a reduction coefficient for the combination of $\bar{\sigma}_L$. The equivalent lateral pressure for the multi-order composites is calculated e. g. by using Eq. (2) in Table 2, where the effects of the factors treated in the present study are assumed to be independent.

4.2 Stress-strain model

The stress(σ_1)-strain(ϵ_1) curves in the direction of the maximum principal compressive stress were numerically expressed. Tension is positive here in the calculation of failure criteria. For normalizing stresses and strains, the absolute values of the uniaxial compressive strength (σ_c) and the strain (ϵ_c) at the uniaxial compressive strength were used as standards.

(1) Strength failure criterion

The four parameter function used in Ottosen's criterion^{23),24)} (see Eq. (1) in Table 3), which is rather simple and may be easily calibrated, was used to express a strength failure envelope. As seen in Fig. 9, however, Ottosen's criterion (dashed line) provides slightly lower values than the data obtained by the authors¹⁴⁾⁻¹⁶⁾ and Mills et al.²⁵⁾ in the range of relatively low lateral pressure. Therefore, the parameters in the function were calibrated based on the present experimental data. The obtained values of the parameters and the criterion are shown in Table 4 and Fig. 9 (solid line), respectively. It was also confirmed that the modified criterion is in rather good agreement with the experimental data in the range of high lateral pressure reported by Richart et al., Balmer, etc²⁴⁾.

(2) Strain failure criterion

Strain at failure (ϵ_{1f}) was assumed to be given by a function of the first invariant of stress tensor at failure (I_{1f}) in the form of Eq. (2) in Table 3. The reason why the criterion was expressed only by the value of I_{1f} or hydro-pressure component is as follows: i) considerable scattering is observed in the measured values of the strain at failure, ii) it is quite difficult to precisely measure the strain at failure in a triaxial test, and iii) in actual confined concretes, increase in the deviatoric stress component due to the imbalance of two lateral stresses is rather small. Figure 10 shows the comparison between the proposed strain failure criterion and the experimental data, empirical constant a in Eq. (2) in Table 3 being determined to be 2.2.

(3) Normalized stress-strain relationships

The expression of normalized stress (σ_1/σ_c)-strain (ϵ_1/ϵ_c) relationship for concretes in uniaxial compress-

Fig. 8 Effects of various factors on triaxial σ_1 - ϵ_1 curve of concrete under standard condition

Table 2 Estimation of equivalent lateral pressure for composite concretes

For First-Order Composites

$$\sigma_L(\epsilon_1) = \alpha \cdot \beta(\epsilon_1) \cdot \gamma(\epsilon_1) \cdot \sigma_L \quad \text{----- (1)}$$

where, α : Coefficient concerning ratio of two lateral stresses,
 $\beta(\epsilon_1)$: Coefficient concerning pitch of confined points,
 $\gamma(\epsilon_1)$: Coefficient concerning lateral stress path,
 σ_L : Maximum lateral pressure or maximum average of two lateral stresses expected.

For Multi-Order Composites

$$\begin{aligned} \sigma_L(\epsilon_1) &= \sum \sigma_{Li}(\epsilon_1) \\ &= \sum (\alpha_i \cdot \beta_i(\epsilon_1) \cdot \gamma_i(\epsilon_1) \cdot \sigma_{Li}) \quad \text{----- (2)} \end{aligned}$$

Table 3 Stress (σ_1)-strain(ϵ_1)-model**Strength Failure Criterion (σ_1, ϵ_1)**

$$f(I_1, J_2, \cos 3\theta) = A \frac{J_2}{\sigma_c^2} + \lambda \frac{J_3}{\sigma_c} + B \frac{I_1}{\sigma_c} - 1 = 0 \quad (1)$$

where, I_1 : First invariant of stress tensor, J_2, J_3 : Second and third invariants of stress deviator tensor, respectively, σ_c : Uniaxial compressive strength (positive value), $\lambda = \lambda(K_1, K_2, \cos 3\theta)$, $\cos 3\theta = 1.5\sqrt{3} J_3 / J_2^{3/2}$, A, B, K_1, K_2 : Empirical parameters.

Strain Failure Criterion (ϵ_1, ϵ_c)

$$\epsilon_{1f}/\epsilon_c = -|I_{1f}/\sigma_c|^a \quad (2)$$

where, ϵ_c : Strain at uniaxial compressive strength (positive value), I_{1f} : I_1 at failure.

Relative Stress (σ_1/σ_{1f}) - Relative Strain (ϵ_1/ϵ_{1f}) Relation

$$\text{Stress ascending portion: } \frac{\sigma_1}{\sigma_{1f}} = \frac{n_a \cdot \epsilon_1 / \epsilon_{1f}}{n_a - 1 + (\epsilon_1 / \epsilon_{1f})^{n_a}} \quad (3a)$$

$$\text{Stress descending portion: } \frac{\sigma_1}{\sigma_{1f}} = \frac{1}{n_d} + \frac{(n_d - 1) \cdot X}{n_d - 1 + X^{n_d}} \quad (3b)$$

$$n_d = \bar{n} n_{d1} \quad (3c)$$

where, $\sigma_{1f}, \epsilon_{1f}$: Stress and strain at failure in the direction of maximum principal compressive stress, $X = (\epsilon_1 / \epsilon_{1f})^m$, $n_a = E_1 / \{E_1 - (\sigma_{1f} / \epsilon_{1f})\}$, $n_d = \bar{n} n_{d1}$, n_{d1} : n_d for uniaxial compressive stress state, E_1 : Initial modulus of elasticity, $m, \bar{n}$: Empirical constants.

Fig.9 Strength failure criterion

Fig.10 Strain failure criterion

(a) $H/D=1$ (b) $H/D=2$ Fig.11 Determination of value of $\bar{\alpha}$ relating to shape of descending portion of σ_1 - ϵ_1 curve

Fig.12 Examples of analytical σ_1 - ϵ_1 curves calculated from proposed model

Table 4 Empirical constants for strength failure criterion

	A	B	K ₁	K ₂
Proposed	1.256	4.030	14.63	0.9870
Ottosen's	1.275	3.196	11.74	0.9801

Table 5 Empirical constants for strain failure criterion and relative stress-relative strain relationship

H/D	Empirical Constants			
	m	n _{d1}	$\bar{\alpha}$	
			$1.0 \leq S \leq 1.15$	$S \geq 1.15$
1	0.2	$1+6.0(\sigma_c/100)^{0.6}$	3-2S	0.7
2	0.5	$1+2.3(\sigma_c/100)^{1.0}$	4-3S	0.55

[Note] $S = -(\sigma_{1f}/\sigma_c)$ Fig.13 Prediction of σ_1 - ϵ_1 curves of concrete subjected to imbalance lateral stresses

ive stress state was extended to express that for concretes in triaxial compressive stress state. For uniaxial compressive stress state, Popovics' formula (Eq. 3 (a) in Table 3) and the formula proposed earlier by the authors²¹⁾ (Eq. 3 (b) in Table 3) were used to express the behaviors in the stress ascending range and the stress descending range, respectively. Figures 11 (a) and (b) show the comparisons between experimental curves (solid line) and analytical curves (dashed and broken lines) obtained by using the above expressions for failure criteria. Note that all the experimental curves of concrete under gradually increasing lateral stresses (see Fig. 3) were modified into those under constant lateral stresses on the assumption that the load-carrying capacity of concrete specimen is proportional to the magnitude of lateral stresses at a same strain level. The broken lines in the figures were obtained on condition that the shape of the stress descending portion of the normalized curves of concretes under triaxial compression is similar to that under uniaxial compression ($\bar{\alpha}=1$), the slope of the broken lines being considerably steeper than that of experimental curves. Therefore, the coefficient $\bar{\alpha}$ in Eq. (3 c) in Table 3 was introduced in the expression to reflect the ductility improvement due to the existence of lateral stresses. Table 5 shows the values of $\bar{\alpha}$ obtained for all the experimental data.

(4) Comparison between analytical and experimental curves

Figures 12 (a) and (b) show the examples of analytical σ_1 - ϵ_1 curves for the specimens of $H/D=1$ and 2, respectively, calculated from the proposed stress-strain model.

It is already shown in Figs. 11 (a) and (b) that fairly good agreement can be obtained between the experimental and analytical σ_1 - ϵ_1 curves in case of balanced lateral stresses ($\sigma_2 = \sigma_3 = \sigma_L$). Further, Fig. 13 indicates the validity of the proposed model to predict the σ_1 - ϵ_1 curve¹⁵⁾ of concrete subjected to imbalance lateral stresses ($\sigma_2 \neq \sigma_3$). Note that the effect of the imbalance can be reflected also in the equivalent lateral pressure by determining the value of α in Table 2 for, if any, available experimental data.

§ 5. Conclusion

The following statements can be made from the study.

- 1) The increase in the load-carrying capacity beyond ultimate stress of axially loaded concrete due to uniformly distributed lateral pressure (σ_L) is quite marked. The improvement of the compressive toughness of concrete by conventional lateral steel bars and steel fibers may be evaluated from the complete stress-strain curves of axially loaded plain concrete subjected to the lateral pressure of less than about 20 kgf/cm².
- 2) The effect of the height/width (H/D) ratio of specimen on the inelastic deformational behavior of concretes is quite significant even when the friction at specimen-loading platen interface is sufficiently reduced.
- 3) It is possible to relate each other the σ_1 - ϵ_1 curves of different shapes of concrete specimens under triaxial compression as well as those under uniaxial compression by taking into account the failure localization or applying the idealized failure zone model proposed.
- 4) The effect of dispersion of lateral pressure becomes gradually remarkable after the peak point of σ_1 - ϵ_1 curves. The rate of decrease in load-carrying capacity of a specimen becomes larger with increase in the magnitude of lateral pressure and the pitch of the loading point of lateral pressure.
- 5) It is considered to be possible to estimate approximately the toughness improvement due to the combination of various types of confinements just by superimposing each effect, using the proposed physical quantity "equivalent lateral pressure" as a compressive toughness index for laterally confined concretes.
- 6) The equivalent lateral pressures for various kinds of concretes are given by comparing the σ_1 - ϵ_1 curves of the confined concretes and those of plain concrete in a standard triaxial stress condition proposed in Table 3.

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低側圧三軸圧縮下のコンクリートの塑性変形挙動（梗概）

——各種横拘束コンクリートの圧縮靱性の統一評価（第1報）——

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§1. はじめに

鉄筋コンクリート（以下、RC と略記）構造物の塑性設計の際には、構成部材の塑性回転能力が十分に確保されていて、構造物が崩壊機構を形成するに足るモーメントの再配分が行われることが前提となる⁴⁾。そのためには、構成部材の破壊形式が曲げ先行型で、かつ、部材断面の曲げ圧縮側コンクリートの靱性（エネルギー吸収能）が十分確保されねばならない。

コンクリートの圧縮靱性改善のため、すでに多くの試みがなされてきたが、その一つに、帯筋で拘束したコンクリートや鋼繊維補強コンクリートの適用がある。この場合の圧縮靱性改善効果は、主として、内部クラック発生以後のコンクリートの体積膨張が帯筋や鋼繊維によって拘束されることによるものであり⁵⁾、先に筆者らは、これらの横拘束効果はコンクリートに対する三軸圧縮効果⁶⁾⁻¹⁰⁾と等価であるとして解析的に説明できることを示した¹⁰⁾。しかし、この場合の横拘束力は側圧に換算しておよそ 20 kgf/cm² 以下であるため、最大軸圧縮応力到達後はコンクリートの体積膨張が著しく、通常の油圧式の三軸圧縮実験法では高ひずみ領域に至るまでの応力 (σ)-ひずみ (ϵ) 関係の計測はほとんど不可能であった^{7), 11)-13)}。そのため筆者らは、高剛性試験機と試作の側圧載荷装置とを組み合わせた独自の低側圧三軸圧縮実験法を考案・適用し、各種横拘束コンクリートの圧縮靱性改善機構を論ずる上で、基礎的かつ重要な資料が得られることを示した¹⁴⁾⁻¹⁶⁾。

本研究の目的は次の二点である。すなわち、第一に、各種の横拘束方法によるコンクリートの圧縮靱性の改善効果を、コンクリートに対する三軸圧縮効果の観点から統一的に評価すること、第二に、得られた統一評価指標を用いて、RC 部材に所定の靱性を付与するためのコンクリートと鉄筋の合理的複合形式を明らかにすることである。

本報（第1報）では、まず、一連の低側圧三軸圧縮実験を行い、コンクリートの圧縮塑性変形挙動に及ぼす各種要因の影響を調べた。次いで、低側圧三軸圧縮実験結果に基づく各種横拘束コンクリートの圧縮靱性の統一的評価法について検討するとともに、評価の基準となる低側圧三軸圧縮下の最大主圧縮軸方向の応力 (σ_1)-ひずみ (ϵ_1) 曲線のモデル化を行った。

§2. 実験方法

実験要因として水セメント比 (W/C)、側圧の大きさ ($\sigma_2, \sigma_3 = \sigma_3$)、供試体の高さ一辺長比 (H/D)、側圧載荷点のピッチ (S)、および供試体上下端面一載荷板間の摩擦（静摩擦係数： μ ）の有無を取り上げた。表—1 に実験の概要を示す。供試体の製作には、普通ポルトランドセメント、川砂（5 mm 未満）、および川砂利（5～15 mm）を用い、設計スランプを 5 cm、調合強度を 250 kgf/cm² とした。コンクリートの練混ぜには可傾式ミキサーを使用し、棒状バイブレータによって振動締固めを行った。供試体は断面 10×10 cm、高さ 10 cm、13 cm、および 20 cm の 3 種類の角柱体、 $\phi 10 \times 20$ cm の円柱体とし、コンクリートは供試体の高さ方向に打ち込んだ。打設後、材令 1 日目にキャッピング、2 日目に脱型した後、実験室内（温度 23±2°C、相対湿度 75±5 %）で養生した。試験材令は約 50 日、供試体の個数は各要因ごとに 3 個とした。

三軸圧縮載荷装置の模式図を図—1 に示す。本装置は、鋼材の曲げ抵抗モーメントを利用して側圧を加える方式を採っており、鋼材の両端にセットした鋼棒のひずみをワイヤーストレインゲージ (W.S.G.) で測定することにより、供試体に加わる側圧の検出が可能である。また、本装置を高剛性試験機に組み入れることにより、最大主圧縮応力方向のひずみ速度が制御される。主圧縮載荷方向のひずみ速度は、毎分 2×10^{-3} に保ち、原則として、ひずみが載荷板間で 30×10^{-3} に至るまで載荷を継続した。載荷面の減摩材としては、載荷端面において比較的均等応力分布が期待できるポリプロピレンにシリコングリスを塗布した減摩パッドを使用した¹⁸⁾。なお、離散型側圧載荷は 5×5 mm 断面の角鋼棒を供試体と側圧載荷

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用鋼板との間に挟んで行った。

§3. 実験結果とその考察

本実験により、低側圧三軸圧縮下のコンクリートの応力-ひずみ曲線に関する以下の事項が明らかとなった。

1) 側圧の大きさの影響：図-3(a) および (b) は、 $H/D=1$ および 2 の供試体の縦応力 (σ_1)-縦ひずみ (ϵ_1) 曲線、および縦応力 (σ_1)-横ひずみ ($\epsilon_2=\epsilon_3$) 曲線に及ぼす側圧（この場合は全面一様側圧）の大きさの影響を示したもので、図中には側圧の実測値 (σ_L) が併示してある。これらの図によれば、側圧の増加とともに靱性は著しく増加し、横ひずみは比例的に減少する。

2) 水セメント比の影響：一軸圧縮载荷を受けた強度の異なるコンクリートの σ_1 - ϵ_1 曲線群の応力下降域に見られる収束領域は、連続したクラックがモルタルマトリックス中に進展し、その結果、コンクリート供試体の耐荷能力がモルタルマトリックスの強度に依存しなくなるひずみレベルであるといえる¹⁹⁾。したがって、収束領域の開始点は供試体の損傷がある一定のレベルに達した時点と考えることができる。図-4(a) および (b) によれば、収束領域は側圧の増加に伴い高ひずみ高応力域に移行している。

3) 供試体の高さ-辺長比の影響：図-5 によれば、曲線のピーク点の応力およびひずみは供試体の H/D の減少に伴い若干増加する。下降域の曲線形状に及ぼす H/D の影響は側圧の大きさにかかわらず顕著であり、 H/D の増加とともに曲線形状は脆性的となるが、靱性の変化量は側圧の大きさにはそれほど依存しない。筆者らは既報¹⁶⁾において、図-6(a) に示すような理想破壊域モデルを提示した。ここでは既報¹⁶⁾と同様、破壊域 (l_p ：理想化された破壊域の長さ) の σ_1 - ϵ_1 関係は $H/D=1$ の供試体の σ_1 - ϵ_1 曲線と一致するものと仮定し、応力下降域における本モデルの適用性を調べた。図-6(c) に、圧縮強度および側圧レベルの異なる $H/D=2$ の供試体の応力下降域の σ_1 - ϵ_1 曲線の実測結果と理想破壊域モデルを用いて求めた推定曲線との比較例を示す。

4) 側圧の離散载荷の影響：図-7 によれば、本実験の範囲、すなわち側圧加力用鋼棒のピッチ (S) が供試体の辺長 ($D=10$ cm) 以下の範囲では、 σ_1 - ϵ_1 曲線のピーク時の応力およびひずみに及ぼす側圧の離散度すなわちピッチの影響はほとんど認められない。しかしながら、ピーク点以後は離散度の影響が徐々に現れ、ピッチが大きいほどひずみの増加に伴う応力の低下が顕著になる。また、ピッチの増加による下降域の σ_1 - ϵ_1 曲線の脆性化は側圧のレベルに依存し、側圧が大きくなるほどその影響が著しくなる。

§4. 各種横拘束コンクリートの圧縮靱性の統一評価

4.1 等価側圧

帯筋や鋼繊維などによって横拘束を受けたコンクリー

トの圧縮塑性変形挙動を、別に定めた基準状態（たとえば、供試体形状： $H/D=1$ 、側圧载荷経路：能動载荷、二方向の側圧の比： $\sigma_2/\sigma_3=1$ 、側圧载荷点：全側面一様载荷）の三軸圧縮下のプレーンコンクリートの塑性変形挙動と対比させ、各ひずみレベルの耐荷力に応じて、これらの横拘束効果を基準状態時の側圧（ここでは、等価側圧と呼ぶ）に換算することができる。この基準状態に対する各種要因の影響を模式的に示せば図-8 のようになる。

このような等価側圧を用いる σ_1 - ϵ_1 曲線の評価方法には、等価側圧という物理的意味の明確な量を用いて各種の横拘束コンクリートの σ_1 - ϵ_1 関係を統一的に表示し得る点に大きな特長があり、種々の力学的複合法の組み合わせによる圧縮靱性の改善効果を等価側圧の累加によって推定することもある程度可能になるものと考えられる（この場合、横拘束の組み合わせに対する新たな低減係数を導入する必要がある）。また、等価側圧の概念を用いることにより、構造設計時に部材に所定の靱性を付与するためのコンクリートの複合形式を各種の複合法の中から選択することも可能になるなど、いくつかの有効な適用も考えられる。

4.2 応力-ひずみ曲線のモデル化

最大主圧縮軸方向の応力 (σ_1)-ひずみ (ϵ_1) 曲線をモデル化した。破壊条件式の定量化に際しては、応力およびひずみの符号は引張を正とし、これらの値の無次元化の基準値として、一軸圧縮強度 (σ_c) および一軸圧縮強度時のひずみ (ϵ_c) の絶対値を用いた。

1) 応力の破壊条件：破壊包絡線の表示性能が比較的良く、かつ適用が簡便な破壊条件式としては Ottosen の提案式（表-3 中の (1) 式）がある^{23), 24)}。ここでは、図-9 中の筆者らの実験結果¹⁴⁾⁻¹⁶⁾に基づいて (1) 式中のパラメータを求め、これを適用した Ottosen 式を用いた。得られたパラメータの値を表-4 に、破壊条件を図-9 中に実線で示す。

2) ひずみの破壊条件：ひずみの破壊条件は破壊時の応力の 1 次の不変量の関数で与えられるものとして、ここでは表-3 に示した (2) 式を用いた。図-10 は、提案した破壊条件と筆者らの実験結果を比較したものであり、この場合、実験定数 a の値として、2.2 が得られた。

3) 相対応力-相対ひずみ関係：ここでは、一軸圧縮下のコンクリートの圧縮軸方向の相対応力 (σ_1/σ_c)-相対ひずみ (ϵ_1/ϵ_c) 曲線の表示式を三軸場に拡張することにした。まず、一軸圧縮時の応力上昇域の表示式として、Popovics 式（表-3 中の (3a) 式）を、応力下降域の表示式として、筆者らの提案式（表-3 中の (3b) 式）を用いた²¹⁾。表示式と実測による三軸圧縮下の相対応力 (σ_1/σ_{1f})-相対ひずみ (ϵ_1/ϵ_{1f}) 曲線の比較例を図-11 に示す。ここで、相対応力-相対ひずみ曲線の実測

値としては、本実験によって得られた σ_1 - ε_1 曲線を側圧 (σ_L) 一定時の曲線に補正し、ピーク点の応力 (σ_{1p}) およびひずみ (ε_{1p}) で無次元化したものを用いた。補正は、ひずみレベル (ε_1) が等しい時点では、供試体の耐荷応力 (σ_1) が側圧の大きさ (σ_L) と比例するものと仮定して行った。図によれば、三軸圧縮下の相対応力-相対ひずみ曲線の下降域の形状は、側圧の存在によって延性的となる。そのため、この傾向を表す指標として (3c) 式中の係数 $\bar{\alpha}$ を導入した。能働側圧時のすべての実験結果に対して得られた係数 $\bar{\alpha}$ の表示式を表—5 に

示す。

4) 解析曲線と実測曲線との比較：以上によって低側圧三軸圧縮下のコンクリートの最大主圧縮軸方向の応力-ひずみモデルが定量化できる。本提案モデルによって得られる σ_1 - ε_1 曲線の一例を図—12(a) および (b) に示す。また、このモデルを用いた解析曲線と筆者らによる本実験および既往の実測曲線との比較例は図—11 および図—13 に示すようであって、両者は比較的良く一致するといえる。