EFFECTS OF MONOLITHIC FLOOR SLABS ON THE MECHANICAL BEHAVIOUR OF FRAMED SHEAR WALLS SUBJECTED TO EARTHQUAKE LATERAL LOADS

by Masahide TOMII*, Tetsuo YAMAKAWA** and Toshiharu NINOMIYA***, Members of A.I.J.

1. Introduction

It is considered that one of the necessary studies for aseismic design is to discuss the effects of floor slabs on the mechanical behaviour of framed shear wall (hereafter referred to as "shear wall") subjected to earthquake lateral loads, since the shear walls arranged in the reinforced concrete frame structures are monolithic with floor slabs. The monolithic floor slabs (hereafter referred to as "floor slabs") transfer the earthquake lateral loads as distributed loads on the beams adjacent to wall panel (hereafter referred to as "beams"). Also the axial, flexural and shearing stiffnesses of beams are enhanced by the floor slabs. In this paper the effects of floor slab on the angular distortion of the columns adjacent to uncracked wall panel (hereafter referred to as "columns") are discussed using analytical solutions (concerning shear walls) expressed in terms of Fourier series.1 One aspect of the discussion concerns with the mechanism for applying the lateral load and the other with the stiffness evaluation of beams. As a consequence the nodal stiffness matrix2 of shear wall, considering the effect of-floor slabs, is calculated (as presented in numerical examples) in order to analyze frame structures stiffened with monolithic wall panels using matrix displacement method.

On the other hand, when the diagonal shear cracks occur in the wall panel, the wall panel becomes as an anisotropic plate having and behaves the diagonal compression field by shear, and beams and columns, together with the floor slab, restrain the dilatation of the cracked wall panel. Thus the cracked shear wall exhibits complicated mechanical behaviour. The elastic analyses of the single-bay single-story and the single-bay two-story or two-bay single-story cracked shear walls (hereafter referred to as "single shear wall" and "2-continued shear wall" respectively) have been reported by Tomii et al.3)-5) assuming the walls to be 45-degree orthotropic plates.

In this paper the elastic analyses of uncracked and cracked reinforced concrete shear walls infinitely continuous in one or two directions (hereafter referred to as "infinitely continued shear wall") as well as those of single and 2-continued shear walls are performed using Airy's stress function of 45-degree orthotropic elastic plates. And the effects of monolithic floor slabs on stress resultants and deformations of the beams are clarified.

2. The Effects of Monolithic Floor Slabs on Angular Distortion of Uncracked Shear Wall

In order to examine the effects of floor slabs, the angular distortions of single-bay single-story and single-story infinitely continued uncracked shear walls subjected to earthquake lateral loads (as illustrated in Fig.1) are discussed in the following four cases.

\[ \text{I} \]: Lateral loads are applied as uniform distributed loads to the axis of the beams, where flexural, shearing and axial stiffnesses of the beams are real.

\[ \text{II} \]: Lateral loads are applied as concentrated loads to the nodal points at four corners of the shear wall, where flexural, shearing and axial stiffnesses of the beams are real.

\[ \text{III} \]: Axial stiffness of the beams is assumed to be rigid, while flexural and shearing stiffnesses of the beams are real,

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Concentrated loads
Single-bay single-story shear walls

Distributed loads

Concentrated loads
Single-story infinitely continued shear walls

Fig.1 Uncracked shear walls subjected to earthquake lateral loads

Table 1 The aspect ratios of single-bay single-story shear wall or unit shear wall adopted in five numerical examples

<table>
<thead>
<tr>
<th>Aspect ratios</th>
<th>λ</th>
<th>αb</th>
<th>βb</th>
<th>αc</th>
<th>βc</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1</td>
<td>1.714</td>
<td>0.171</td>
<td>2.5</td>
<td>0.171</td>
<td>3.333</td>
</tr>
<tr>
<td>No. 2</td>
<td>0.5</td>
<td>0.3</td>
<td>4</td>
<td>0.2</td>
<td>4</td>
</tr>
<tr>
<td>No. 3</td>
<td>1</td>
<td>0.3</td>
<td>3</td>
<td>0.2</td>
<td>4</td>
</tr>
<tr>
<td>No. 4</td>
<td>2</td>
<td>0.2</td>
<td>3</td>
<td>0.2</td>
<td>4</td>
</tr>
<tr>
<td>No. 5</td>
<td>3</td>
<td>0.1</td>
<td>2</td>
<td>0.2</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: The proportion of the shear wall adopted as example No. 1 (see Fig. 2 for definitions of aspect ratios of shear wall) is equal to that of the shear wall adopted as the design example shown in the AJJ Standard for Structural Calculation of Reinforced Concrete Structures (revised in 1982).

IV: Axial stiffness of the beams is assumed to be rigid, while flexural and shearing stiffnesses of the beams are increased together twice and five times.

The numerical calculations of angular distortions for five proportions of shear wall (see Table 1) are carried out for each case mentioned above. (See Fig. 2 for definitions and aspect ratios of each unit shear wall.) The results are presented in Tables 2 and 3. In Table 2, the values shown in the columns with headings $R_s/R_{im}$ and $R_{in}/R_{im}$ are ratios of angular distortions induced in cases I and II, respectively, to those in case III. In this table, $R_{im}$ represents the coefficient $x_{s}$ of the angular distortions of the shear wall in case III, where the angular distortions are computed as $x_{s}Q/(Glt)$. Notice that the angular distortions of columns in case II are much larger than those in case I and dependent on the proportions of shear walls (compare numerical examples No. 1, No. 4 and No. 5 with No. 2 and No. 3). This suggests that the actual angular distortions of columns will be overestimated if the angular distortions in case II are used. Also the discrepancies between angular distortions of columns and those defined at midspan of the shear wall in case II may become remarkable as the span length becomes longer and the axial stiffness of beam smaller. However the angular distortions in case I are almost uniform throughout the shear wall, and these values are nearly equal to those in case III (see Table 2).

In order to discuss the effect of floor slabs on angular distortions of shear wall, the ratios of angular distortions in case IV to those in case III, where flexural and shearing stiffnesses of the beams are increased by enlarging the beam.
3. Numerical Examples on Fundamental Nodal Stiffness Matrix of Single Shear Wall Restrained by Floor Slab

The results discussed in Section 2 suggest that the axial stiffness of the beams can be assumed to be rigid in the formulation of the fundamental nodal stiffness matrix $K^*$ of the shear wall. If each $K^*$ of fundamental type (see Tables 4 and 5) is derived as the analytical solutions, a general stiffness matrix $K$ of shear walls is formulated easily by making use of transformation matrix $T$. Note that Types I, II, III and IV to be mentioned below are the

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Ratios $R_i/R_{ii}$ and $R_i/R_{iii}$ of angular distortions of the shear walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locations</td>
<td>Single-bay single-story shear walls</td>
</tr>
<tr>
<td>Ratios</td>
<td>Column</td>
</tr>
<tr>
<td>No. 1</td>
<td>0.99</td>
</tr>
<tr>
<td>No. 2</td>
<td>0.99</td>
</tr>
<tr>
<td>No. 3</td>
<td>0.99</td>
</tr>
<tr>
<td>No. 4</td>
<td>0.99</td>
</tr>
<tr>
<td>No. 5</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: 1) $R_i$ is the angular distortions of the shear wall subjected to lateral uniform distributed load $Q$ in case I.
2) $R_{ii}$ is the angular distortions of the shear wall subjected to lateral concentrated load $Q$ in case II.
3) $R_{iii}$ is the angular distortions of the shear wall subjected to lateral load $Q$ in case III.
4) The values shown in column with heading $R_{iii}$, are corresponding to the values of the coefficients $k_{ij}$ of the angular distortions of the shear wall in case III, in which the values of $R_{iii}$ are computed as $k_{ij}Q/(Gt)$, where $G$ is modulus of elasticity in shear, $t$ is thickness of wall and $t$ is center-to-center distance of columns of the shear wall.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>The effect of the width of beams on the ratios $R_{i}/R_{ii}$ of angular distortions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical examples</td>
<td>Single-bay single-story shear wall</td>
</tr>
<tr>
<td>$\phi = 2$</td>
<td>$\phi = 5$</td>
</tr>
<tr>
<td>No. 1</td>
<td>0.97</td>
</tr>
<tr>
<td>No. 2</td>
<td>0.93</td>
</tr>
<tr>
<td>No. 3</td>
<td>0.93</td>
</tr>
<tr>
<td>No. 4</td>
<td>0.96</td>
</tr>
<tr>
<td>No. 5</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Note: 1) $R_{iii}$ is the angular distortions of shear walls in case III, where the beam width to wall thickness ratio $\beta_B$ is 2.5.
2) $R_{ii}$ is the angular distortions of shear walls in case II, where the beam width to wall thickness ratios $\beta_B$ are 5.0 and 12.5.
3) $\phi$ is the rate of enlargement for the width of beams.

width, are provided, as shown in Table 3. This table shows that the effect is generally small.

Consequently, the axial stiffness of the beams can be assumed to be rigid, in case the earthquake lateral loads distributed through the floor slabs are treated as concentrated loads on the nodal points at four corners of uncracked shear walls. This result is useful in the formulation of the nodal stiffness matrix of the shear wall.
basic types of the fundamental components of nodal forces and nodal displacements of the shear walls.

The fundamental nodal stiffness matrices \( K_{\text{I}} \) of Type I and \( K_{\text{II}} \) of Type IV, whose components are symmetric with respect to the longitudinal center line of the wall panel, can be given approximately by pin-jointing rigid bars at the nodal points of the shear wall along the beams. These stiffness matrices are treated as approximate solutions pertaining to the stiffness matrix of the shear walls with rigid floor slabs (see Tables 4 and 5).

Table 4 The representative components \( P^* \) and \( P^o \) of the balancing and unbalancing fundamental components of the nodal external forces on single-bay single-story shear wall

<table>
<thead>
<tr>
<th>Fundamental type</th>
<th>Neglecting monolithic floor slab</th>
<th>Correction term for considering monolithic floor slab</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 4X_{\text{I}} ) { \begin{align*} &amp; X_{\text{I}}^* \given &amp; 3.008 \given 1.444 \given u_I^* \given v_I^* \ &amp; Y_{\text{I}}^* \given &amp; 0.910 \given v_I^* \end{align*} }</td>
<td>( 4X_{\text{I}}^* ) { \begin{align*} &amp; X_{\text{I}}^* \given &amp; 0.761 \given 0.535 \given u_I^* \ &amp; Y_{\text{I}}^* \given &amp; 0.377 \given v_I^* \end{align*} }</td>
<td></td>
</tr>
<tr>
<td>Type II</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 4X_{\text{II}} ) { \begin{align*} &amp; X_{\text{II}}^* \given &amp; 3.854 \given 0.453 \given -0.197 \given u_{\text{II}}^* \ &amp; Y_{\text{II}}^* \given &amp; 7.626 \given 0.362 \given v_{\text{II}}^* \end{align*} }</td>
<td>( 4X_{\text{II}}^* ) { \begin{align*} &amp; X_{\text{II}}^* \given &amp; k_{\text{II}}^d 0 0 \given u_{\text{II}}^* \ &amp; Y_{\text{II}}^* \given &amp; 0 0 \given v_{\text{II}}^* \end{align*} }</td>
<td></td>
</tr>
<tr>
<td>Type III</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 4X_{\text{III}} ) { \begin{align*} &amp; X_{\text{III}}^* \given &amp; 7.492 \given 0.378 \given u_{\text{III}}^* \ &amp; Y_{\text{III}}^* \given &amp; 0.170 \given h_{\text{III}}^e \end{align*} }</td>
<td>( 4X_{\text{III}}^* ) { \begin{align*} &amp; X_{\text{III}}^* \given &amp; k_{\text{III}}^d 0 0 \given u_{\text{III}}^* \ &amp; Y_{\text{III}}^* \given &amp; 0 0 \given h_{\text{III}}^e \end{align*} }</td>
<td></td>
</tr>
<tr>
<td>Type IV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 4X_{\text{IV}} ) { \begin{align*} &amp; X_{\text{IV}}^* \given &amp; 3.627 \given -0.410 \given u_{\text{IV}}^* \ &amp; Y_{\text{IV}}^* \given &amp; 0.179 \given h_{\text{IV}}^e \end{align*} }</td>
<td>( 4X_{\text{IV}}^* ) { \begin{align*} &amp; X_{\text{IV}}^* \given &amp; k_{\text{IV}}^d 0 0 \given u_{\text{IV}}^* \ &amp; Y_{\text{IV}}^* \given &amp; 0 0 \given h_{\text{IV}}^e \end{align*} }</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( k_{\text{II}}^d, k_{\text{IV}}^d = \infty \)
Since there is no lateral loads to balance the system in Type III (see Table 4), the effect of floor slabs on $K_s$ may be neglected. The lengths of the beams do not change by the nodal displacements of Type I. Therefore it is meaningful to pin-jointing rigid bars at the nodal points of the shear wall along the beams for considering the effect of floor slabs. As a result, only the analytical solutions of $K^s$ are required, where the axial stiffness of the beams is assumed to be rigid. In this paper the analytical solutions of $K^s$ of the shear walls with floor slabs are obtained, and after that, the correction term to be added to $K^s$ of the shear walls without floor slab is derived.

Numerical examples for $K^s$ of the single shear wall are presented in Table 5. The proportion of the shear wall is equal to that shown in the design example in the AJJ Standard for Structural Calculation of Reinforced Concrete Structures (revised in 1982) (see Fig. 2 and shear wall No. 1 in Table 1). Poisson ratio is assumed to be 1/6.

4. Elastic Analysis of Cracked Shear Wall Infinitely Continuous in One or Two Directions

The elastic analyses, by which mechanical behaviour of single and 2-continued cracked shear walls was clarified assuming shear cracked wall panels as 45-degree orthotropic plates, have been reported by Tomii et al.\(^{30-41}\). In this paper, the analysis is applied to analyze cracked shear wall infinitely continued in one or two directions and subjected to external forces that are polar symmetric with respect to the centers of the wall and the intermediate member (see Fig. 3). Since the assumptions and methods in the elastic analysis of this paper are the same as those of single and 2-continued shear walls\(^{30-41}\), except for equilibrium equations on horizontal and vertical cross sections of unit shear walls (see Fig. 4), detailed discussions on the analytical methods and the derivations of equations are omitted. The two typical characteristics are as follows.

i) The deflection of intermediate members is expressed by odd functions and a constant. The odd functions represent flexural and shear deformations of the intermediate members. The constant represents the rigid-body displacement of intermediate members due to dilatation of the shear cracked wall panel. Therefore, if the wall panel is an isotropic elastic plate, the deflection of each intermediate member is expressed by odd functions only.

ii) The axial deformation of each intermediate member is also expressed by odd functions and a constant. In this case, the odd functions represent the elongation of each intermediate member due to the dilatation of the cracked wall panel and the constant represent the rigid-body displacement. The odd functions are not necessary to express the axial deformation of each intermediate member whose adjacent wall panels are isotropic elastic plates.

![Diagram](attachment:image.png)

(a) Single-bay infinitely continued shear wall

(b) Infinitely continued shear walls

Note: 1) The unit parts mentioned by the thick broken lines are analyzed.
2) External forces per single-bay are illustrated.

Fig. 3 Infinitely continued shear walls having shear cracked wall panels and their external forces — 106 —
In the numerical examples, the elastic analyses are carried out for the infinitely continued shear wall whose single-bay single-story unit parts have the same proportions as those for the single shear wall adopted in Section 3. The elastic constants of the cracked wall panels are given by Eqs. (1)-(4). 

\[
E_1 = \left[ \mu_1 + n \left( \frac{\mu_c - \mu_1}{2(\mu_c + np_d)} \right) \right] \epsilon E \tag{1}
\]

\[
E_2 = \left[ \mu_e + n \left( \frac{\mu_c - \mu_e}{2(\mu_c + np_d)} \right) \right] \epsilon E \tag{2}
\]

\[
G_{12} = \frac{\mu_1 + np_d}{2} \epsilon E \tag{3}
\]

\[
\nu = \frac{E_2}{E_1} ; n = \frac{1}{6} \tag{4}
\]

where 

- \(E_1, E_2\): Young's modulus for the principal direction 1 of elasticity of the wall panel and that for the principal direction 2 of elasticity of the wall panel 
- \(G_{12}\): shear modulus in the rectangular coordinates 1 and 2 of the wall panel 
- \(n\): modular ratio (\(n = sE/E = 10\)) 
- \(\epsilon E\): initial Young's modulus of concrete 
- \(sE\): Young's modulus of the reinforcement 
- \(p_d\): ratio of diagonal shear reinforcement area to the gross concrete area of a diagonal section (\(p_d = 0\)) 
- \(p_s\): ratio of horizontal or vertical shear reinforcement area to the gross concrete area of a vertical or horizontal section (\(p_s = 0.0025\)) 
- \(\mu_1\): reduction coefficient of the Young's modulus of concrete in the direction perpendicular to the cracks in the wall panel (i.e. in the principal direction 1 of the wall panel) 
- \(\mu_c\): reduction coefficient of the Young's modulus of concrete in the direction of the cracks in the wall panel (i.e. in the principal direction 2 of the wall panel) (\(\mu_c = 1\)) 
- \(\mu_{12}\): reduction coefficient of the shear modulus of concrete in the rectangular coordinates 1 and 2 of the wall panel (\(\mu_{12} = \mu_c\))
The beam-to-column connections are assumed to be a rigid zone. The elastic constants of the beams and columns are given by Eq. (5).

\[ E = E_s, \quad G = \frac{2}{7} E \]  

(5)

Fig. 5 The deformations at the boundary between wall and frame [SHEAR WALL 1]

Fig. 6 The deformations at the boundary between wall and frame [SHEAR WALL 2]

Fig. 7 The shearing stress \( \tau_{xy} \) in the wall and the shearing forces \( Q_b \) and \( Q_c \) on the cross section of the frame [SHEAR WALL 1]

Fig. 8 The shearing stress \( \tau_{xy} \) in the wall and the shearing forces \( Q_b \) and \( Q_c \) on the cross section of the frame [SHEAR WALL 2]

Fig. 9 The normal stresses \( \sigma_x \) and \( \sigma_y \) in the wall and the axial forces \( N_b \) and \( N_c \) on the cross section of the frame [SHEAR WALL 1]

Fig. 10 The normal stresses \( \sigma_x \) and \( \sigma_y \) in the wall and the axial forces \( N_b \) and \( N_c \) on the cross section of the frame [SHEAR WALL 2]
Here it is shown how the stress distribution and deformation of the frame and wall panel change according to the propagation of the diagonal shear cracks in the wall panel. This propagation makes the diagonal tension of the concrete unreliable i.e., $\mu_l=0$ (see Figs. 5-14). When $\mu_l=1$, it means that the uncracked infilled wall panel behaves as an isotropic elastic plate.

From the numerical results (shown in Figs. 5-10) concerning infinitely continued framed shear wall subjected to lateral load $Q$, the following observations are obtained.

1) As $\mu_l$ decreases, the compressive normal stresses $\sigma_n$ and $\sigma_p$ on the cross sections at the midspan and midheight of the shear wall remarkably increase (see Figs. 9 and 10). However the shearing stresses $\tau_{xy}$ on the sections are hardly affected by $\mu_l$ (see Figs. 7 and 8).

2) Axial forces $N_n$ and $N_p$ of beams and columns become large tensions and the wall panel dilates as the diagonal shear cracks in the wall panel develop (see Figs. 9 and 10). This fact is also observed in Figs. 5 and 6 which show the deformations of cracked wall panel.

On the other hand, the following observations are obtained from the numerical results concerning infinitely continued framed shear wall subjected to vertical load $N$. Since shearing stresses in wall panel and columns are so small that they can be disregarded, the stress diagrams are omitted in this paper.

1) Before the shear cracks occur in the wall panel, the story drift of the shear wall is zero. When the diagonal shear cracks occur in the wall panel the story drift yields in the direction opposite to that of the lateral load (see Figs. 11 and 12).

2) As $\mu_l$ decreases, the compressive normal stress $\sigma_n$ on the horizontal cross section at the midheight of the wall...
panel decreases. As a result the compression $N_c$ on the cross section of the column increases (see Figs. 13 and 14).

5. Effects of Monolithic Floor Slab on the Mechanical Behaviour of Beams of Single-bay Infinitely Continued Cracked Shear Wall

When the diagonal shear cracks occur in the wall panel, the wall panel becomes as an anisotropic plate, behaves the diagonal compression field and dilates. The monolithic floor slabs together with the boundary frame of the wall panel restrain the dilatation of the cracked wall panel. Subsequently, the role of the monolithic floor slab in restraining the dilatation of the cracked shear wall is discussed based on axial force and deformation of intermediate beam of single-bay infinitely continued framed shear wall. The floor slab is assumed to be an infinitely spreading plate. This plate is a homogeneous one with thickness $t_s$ and is assumed to have plain stress. The dilating forces due to elongation of the beams are assumed to be a pair of concentrated loads $P$ as illustrated in Fig. 15. In this figure, the relative displacements due to a pair of symmetric loads $P$ conform to the elongation of beams due to dilatation of the cracked shear wall subjected to constant shear forces $Q$ and restraint forces $X$ simultaneously. The restraint forces $X$ are equal to the pair of concentrated loads $P$ (see Figs. 15 and 16). The horizontal displacements $u$ at points subjected to the pair of loads $P$ (see Fig. 15) are given by Eq. (6).

$$u = F_s P/(EI_s)$$

where

$$F_s = \frac{(1+\nu)}{4\pi} (3-\nu) \left[ 1 - \frac{b_c}{2(l+D_c)} \right] (1+\nu)$$

On the other hand, the horizontal nodal displacement of the cracked shear wall is decomposed into two components: antisymmetric component $u_t$ due to rigid-body displacement of the beam and symmetric component $u_s$ due to the elongation of the beam. These displacements $u_t$ and $u_s$ are expressed by the flexibility matrix in terms of shear forces $Q$ and restraint forces $X$ as Eq. (7).

$$\begin{bmatrix} u_t \\ u_s \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} f_{11} & f_{1S} \\ f_{S1} & f_{SS} \end{bmatrix} \begin{bmatrix} Q \\ X \end{bmatrix}$$

The restraint forces $X$ are obtained by making $u$ and $P$ in Eq. (6) equivalent to $u_s$ and $X$ in Eq. (7) respectively as Eq. (8).

$$X = \frac{f_{1S}}{f_{SS}} Q$$

In Fig. 17, the relation between $u_s$ and $\mu_s$ is given by the ratio of the thickness of wall, $t$, to the thickness of slab, $t_s$, that is, $t/t_s$. The ordinate in Fig. 17 is $u_s/t_s = \mu_s - 1$, in which $u_s|_{t=t_s}$ represents the elongation of the beam of cracked shear wall without floor slab when $\mu_s$ is zero.
Fig. 17 Elongation ratio $\frac{\varepsilon_h}{\varepsilon_h |_{t=0}}$ of each intermediate beam

Fig. 17 suggests that the floor slab can not perfectly restrain the dilatation of the shear cracked wall panel. On the other hand, the relation between axial force $N_e$ at the end of the beam of cracked shear wall and $\mu_0$ is shown by parameter $t/t_e$ in Fig. 18. It is observed that the axial tension at the end of the beam is decreased by the restraint due to the floor slab. Considering these facts, it can be deduced that the intermediate shear failure can hardly occur at the end of the beam of shear wall with floor slab.

6. Conclusions

1) The earthquake lateral loads distributed through the floor slabs are treated as concentrated loads on the nodal points at four corners of uncracked shear walls. In this case, if the axial stiffness of the beams is assumed to be rigid, the approximate solutions pertaining to the nodal displacements of the shear walls subjected to earthquake lateral loads can be obtained with accuracy. This result is useful in the formulation of the nodal stiffness matrix of the shear wall.

2) When the diagonal shear cracks occur in the wall panel, the wall panel dilates and the axial tensions yields in the boundary frame of the wall panel. However, the axial tension at the end of beam does not become large if the restraint action of monolithic floor slabs is expected.

References


地震荷重を受ける耐震壁の力学挙動に及ぼす床スラブの影響に関する解析的研究（梗概）

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正会員 山川 哲雄**
正会員 二宮 利治***

限連スパン1層耐震壁の層間変形角を、次の4つの場合について比較検討する。
i. 耐震壁の側ばりの材軸上に水平力を等分布荷重として作用させ
ii. 耐震壁の節点に水平力を集中荷重として作用させる。
iii. 耐震壁の側ばりの曲げおよびせん断剛性をそれぞれ
道にし、側ばりの軸方向のみを無限大にする。
iv. 耐震壁の側ばりの曲げおよびせん断剛性を2および
さらに、側ばりの軸方向を無限大にする。

層間変形角の計算法は耐震壁の5種類の形状について
に行う。表-2より2の場合における柱の部材側は、耐震壁の形状によっては1の場合それよりもかなり大きくなる。したがって、柱の実際の部材側は2の場合の解を使うと過大評価される。2の場合における柱の部材側とスパン中央位置における層間変形角の相異、耐震壁のスパンが大きさ、側ばりの軸方向が小さくなるほど顕著になる。しかしながら、1の場合の層間変形角は耐震壁全体にわたってほとんど一致であり、かつこれらの値は11の場合の層間変形角には等しい。

床スラブによる側ばりの曲げおよびせん断剛性の増大
が耐震壁の層間変形角に及ぼす影響をみるために、2の
場合の層間変形角に対する1の場合の層間変形角の比を
表-3に示す。表-3に示れば、側ばりの曲げおよびせん
断剛性の増大が、耐震壁の層間変形角に及ぼす影響は
はの近くにある。したがって、壁板にせん断ひび割れが発生していない耐震壁に床スラブから分算して用する大震時の水平荷重を、節点水平荷重と取り扱う場合には、側ばりの軸方向を無限大と仮定することができるこ
とを示している。このとき、耐震壁の節点外力と節点
変位の関係である節点剛性マトリックスを定式化する上
で有用である。

3. 床スラブの影響を考慮した単独耐震壁の基本節点
剛性マトリックスに関する数値計算例
2項での検討結果、耐震壁の基本節点剛性マトリック
スK*を定式化する場合には、側ばりの軸方向を無限大
と仮定することができる。各基本型ごとのK*が解析解

1. 序

鉄筋コンクリートラーメン構造に多用されている耐震
壁は、床スラブと一体になっているので、床スラブが地
震荷重を受ける耐震構造の力学挙動に及ぼす影響を検討す
ることは、耐震設計上必要な研究課題の一つと考えられ
る。床スラブは地震時の水平荷重を耐震壁の側ばりに分
布荷重として作用させ、かつ床スラブと一体になった側
ばりの軸剛性や曲げ剛性などを増大させる。そこで、壁
板にせん断ひび割れが発生する前の耐震壁の層間変形角
を及ぼす床スラブの影響に関して、水平荷重の作用形態
と側ばりの剛性評価の観点から耐震壁のフーリエ変数
を利用して検討する。この検討結果をふまえて、マト
リックス変位法による有体系ラーメンの弾性解析におよべ
耐震壁の節点剛性マトリックスに、床スラブの影響を
考慮した数値計算例を示す。

一方、せん断ひび割れが壁板に発生すると、異方性化
した壁板は圧縮域を形成し、床スラブは帯状ラーメンと
ともにその壁板の広がりを拘束する。このように、せん
断ひび割れ後の耐震壁は複雑な力学的挙動をする。せん
断ひび割れ前の単純2.1層耐震壁の壁板を、45°直交異
方性弾性板と仮定したこれらの弾性解析を富井らによっ
て発表された31-35。本論では、富井らが発表した45°直
交異方性弾性板の応力関数36を用いて、せん
断ひび割れが生ずる無限問題耐震壁の弾性解析を行う。次で、この耐震壁に床スラブが取付けられた場合の
帯状ラーメンの応力や変形に関して床スラブが及ぼす力
学的影響を明らかにする。

2. ひび割れが発生していない耐震壁の層間変形角に
及ぼす床スラブの影響

床スラブが耐震壁の層間変形角に及ぼす影響を明らか
にするために、ひび割れが発生していない単独および無

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として求められると、一般的な耐震壁の節点剛性マトリックス $K^v$ が、変換マトリックスを利用し容易に誘導される。壁面の中心を通る縦軸に沿って、対称的な節点群からなる II 型の $K^v$ および IV 型の $K^v$ は、剛性に偏りある両端の節点間にかけた力によって、剛体が取り付いた場合の変形が求める。II 型においては、力のつもり合いを満足する外力を存在しないので、$K^v$ に対しては床スペルの影響を考慮しない。耐震壁が IV 型の節点を構成する場合には、側ばりは材長変化を起こさないので、剛性を付与することによっても、床スペルが取り付く場合の影響を考慮することができない。したがって、側ばりの剛性を無限大と仮定した I 型の解析解 $K^v$ を求める。本論では、その解を求めた後、すでに発表している「床スペルが取り付け条件のスペル版」を併用することが可能である。したがって、側ばりの節点剛性を無限大と仮定した I 型の解析解 $K^v$ を求める。本論では、その解を求めた後、すでに発表している「床スペルが取り付け条件のスペル版」を併用することが可能である。

4. せん断ひび割れが壁面上に発生した等材無限連続耐震壁の壁面剛性解析

壁面にせん断ひび割れが発生した後の単独および 2 連耐震壁について、それぞれ壁面が 45° 直交異方性弾性板と仮定した解析方法をと、それらの力学挙動をより詳細に考察する。これらの解析方法を応用発展させて、一方における二方向等材無限連続耐震壁では、壁面および中間部材の特性に関して、それぞれ対称性外力を受ける場合を解析する。本解析の特徴として次の 2 点がある。

(1) 中間部材のあたりは、奇関数と定数で表される。奇関数は中間部材の曲げ変形とせん断変形を表す。一方、定数はせん断ひび割れにともなう壁面のはみだしによる中間部材の剛体変位を意味する。したがって、壁面が等方性弾性板であれば等材無限連続耐震壁の中間部材のたわみは、奇関数のみで表される。

(2) 中間部材の輸送機も (1) と同様に奇関数で表される。奇関数はせん断ひび割れにともなう壁面の面積拡張による伸び変形を表し、定数は剛体変位を表す。壁面が等方性弾性板であれば奇関数を必要とする。

付帯ラーメンおよび壁面の各応力分布や各変形がせん断ひび割れの進展とともに、コンクリートの斜め引張応力が低下し、すなわち、せん断ひび割れが生じない耐震壁を意味する。水平外力 $Q^v$ を受ける等材無限連続耐震壁の数値計算結果から、次の諸点が明らかになった。

1) $\mu_v$ が低下するにつれて、壁面中央における水平断面上の圧縮応力 $\sigma_v$ および $\tau_v$ は顕著に増大するが、同じ断面上のせん断応力 $\tau_v$ は $\mu_v$ ほとんど影響されない。

2) 付帯ラーメンの軸力 $N_w$、$N_v$ は異方性化が進展するにつれて、大きな引張応力となっている。これは、せん断ひび割れの進展にともない壁面が膨張していることを示している。このことは、図 5, 6 の壁面の変形図からも明らかである。

一方、鉛直外力 $N^v$ を受ける等材無限連続耐震壁の数値計算結果から、次の諸点が明らかになった。

1) 壁面にせん断ひび割れが発生する前は耐震壁の歪変位が零であったが、壁面にせん断ひび割れが発生すると、歪変位が水平外力の向きを反対方向に生じる。

2) $\mu_v$ の低下にともなって壁面の中央水平断面上の圧縮応力度 $\sigma_v$ が小さくなる。その結果、柱の軸圧縮力 $N_v$ が大きくなる。

5. せん断ひび割れが壁面上に発生した 1 スパン無限連続耐震壁におけるひずみの力学挙動に及ぼす床スペルの影響

せん断ひび割れが壁面上発生すると、壁面が異方化し圧力場を形成して膨張しようとする。壁面の面積膨張に対して、耐震壁の付帯ラーメンとともに床スペルの拘束作用が生じることになる。そこで、床スペルの拘束作用を 1 スパン無限連続耐震壁の中心部分の伸び率と軸力の変化に注目して解析する。床スペルは厚さ $t_v$ を有する無限に広がる平板として取り扱う。ひびの伸びによる軸力変化を図 15 に示したような 1 種の集中荷重 $P$ と考える。$P$ による相対変位を、せん断ひび割れが生じた耐震壁に一定せん断力 $Q$ と拘束力 $K$ が、作用することによって生じたひびの伸び量を基盤とする。

図 17 に $\mu_v$ と $u_v$ の関係を、壁面の厚さ $t_v$ と床スペルの厚さ $t_k$ の比例を $l_t/l_v$ のパラメータで示す。図 17 に示すように床スペルは壁面のせん断ひび割れにともなう耐震壁の面積膨張を、完全に拘束できないことを示している。一方、図 18 に $\mu_v$ と耐震壁の中心より端材の軸力 $N_v$ の関係を $l_t/l_v$ のパラメータで示す。図 18 によれば、端材の引張応力が床スペルの拘束作用によりかなり減少していることがわかる。このことは、床スペルがひびに取り付けた耐震壁では、ひびのせん断破壊が起こりにくいことを示唆している。

6. 結論

1) 壁面にせん断ひび割れが発生していない耐震壁に床スペルから発生して作用する地震時の水平荷重を、節点の水平荷重として取扱う場合には、側ばりの軸剛性は無限大と仮定すると、節点変位に基準、良い近似解が得られる。このことは、耐震壁の節点外力を節点変位の関係である節点剛性マトリックスを定式化する上で有用である。

2) せん断ひび割れが壁面上に発生すると壁面が膨張し、付帯ラーメンに引張応力が生じるが、床スペルの拘束作用が期待できる場合には、より材端の引張応力はそれほど大きくならない。