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Sound fields caused by diffuse-type reflectors with periodic profile

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For the purpose of providing highly diffuse reflections, the reflection surface with periodic roughness has been often used. The degree of diffusion of this type can be evaluated by an objective measure *DNSD* proposed by the author. Optimum diffusers with periodic profile can be designed by using this measure, but this type of diffuse reflections might have some aspects of misgivings due to its periodicity, which may be a kind of so-called "tone coloration." In other words, highly diffuse reflections might have a problem of some particular effect on the spectrum caused by interference between the direct and the reflected waves, which might be different from an ordinary case of the plane surface. In this paper, to clarify the physical properties and the mechanism of diffusion caused by periodic-type diffusers, SPL characteristics and impulse response of the reflected field as well as the scattering performances for three types of reflectors are discussed including the measured data from 1/8-scale model experiments. As a result of detailed discussion on the relation between the surface profile and the incident wave, the effect of periodic roughness on the reflection is revealed quantitatively.

Keywords: Diffuse reflections, Scattering, Periodic surface, Coloration

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1. INTRODUCTION

There are many cases of uneven surfaces used as walls and ceilings of halls and theaters to provide diffuse reflections. These are, for example, sinusoidal, triangular and rectangular profiles. As for the shape and the size of this unevenness, discussions have been given exclusively in relation to the wavelength of the incident wave, and an actual use of this type of diffuser has been done in an empirical manner. Regarding this matter, an objective measure named *DNSD* was introduced by the author,¹⁾ which can express the degree of diffusion of a reflector with periodic profile. With the help of this measure, quantitative evaluation of diffuse reflections is possible, and the design of effective diffusers can be attained.

In addition to this property of diffusion, reflection of the surface is needed to have good characteristics of the response, which should not have a serious effect on our subjective experience. A well-known problem is "coloration" caused by interference between the direct and the reflected waves, which might seriously appear in this case due to periodicity of the surface. However, even the physical properties of diffuse reflections have not been known well.

This paper concerns the problems regarding the matter described above, and has two purposes. One is to clarify the physical properties of the reflected sound field caused by a diffuse-type reflector. Another is to identify the critical condition of diffuse reflections with respect to both the roughness size of the reflector and the angle of inci-

dence. Any findings for this critical condition can be served as a good tool for designing a good room-acoustic condition.

As for the former purpose, it would be appropriate to examine the physical properties of diffusion in relation to the phenomenon "coloration," then the discussion will be given exclusively to the frequency characteristics of the sound pressure level, in comparison with the case of an ordinary plane reflector. The impulse response of the reflected field is also presented to clarify the temporal structure of diffuse reflections. This information can be used as an indicator of image sources produced behind the diffuser, which seem to be different from the case of ordinary plane reflector as described in Ref. 6. However, the details, which may give a key to the mechanism of diffuse reflections, have not been known well.

For the discussion about diffusion of the reflector, the degree of diffusion will be an important factor. Even though the roughness size of the diffuser is relatively large, this fact does not always gives sufficient diffusion. The results derived by using reflectors with unknown degree of diffusion may have less significance in investigation of the physical properties caused by the diffuser. From this reason, in Sec. 2 introduction of an objective measure *DNSD* for evaluating diffuse reflections and its application for designing an optimum diffuser are summarized briefly.

On the basis of these results, in Sec. 3 physical properties of the sound fields caused by the diffuser with highest diffusion for wide frequency range are investigated both theoretically and experimentally. To verify the accuracy of the theoretical model for predicting the sound field due to the diffuser is also the purpose of this section.

In Sec. 4, by using exclusively the theoretical model based on the verification discussed in the previous section, the discussion is given to the latter purpose of this paper. As a result of this section, the condition whether the reflection is specular or diffuse is presented as a simple relation having two parameters, the surface period and the angle of incidence.

2. DESIGN FOR AN OPTIMUM DIFFUSER

The geometry of the problem is shown in Fig. 1. The boundary surface is periodic with the period L

Fig. 1 Geometry of a wave scattering model.

having an arbitrary profile with grooves parallel to the y -axis. The incident plane wave at angle θ and ϕ with unit amplitude expressed as the velocity potential is described as $\psi_i(x, y, z) = \exp[i(\alpha_0 x + \beta_0 y - \gamma_0 z)]$. Owing to the periodicity of the surface, the reflected waves can be expressed as^{2,3)}

$$\psi_r(x, y, z) = e^{i\beta_0 y} \sum_{n=-\infty}^{\infty} \Psi_n e^{i(\alpha_n x + \gamma_n z)}, \quad (1)$$

where

$$\begin{cases} \alpha_n = \alpha_0 + 2n\pi/L \\ \gamma_n = [(k^2 - \beta_0^2) - \alpha_n^2]^{1/2}, \end{cases} \quad (2)$$

with $\text{Re}\{\gamma_n\} \geq 0$ and $\text{Im}\{\gamma_n\} \geq 0$ corresponding to the time factor $\exp(-i\omega t)$, and where $\alpha_0 = k \sin \phi \sin \theta$, $\beta_0 = k \cos \phi \sin \theta$, $\gamma_0 = k \cos \theta$ with k as the wave number.

The unknown factor Ψ_n can be obtained by solving the boundary integral equation with a fundamental solution for the problem of wave scattering from a periodic boundary surface under an appropriate boundary condition.^{4,5)} Owing to the property of a complex quantity, the term such that γ_n is imaginary in Eq.(1) does not contribute to the reflected energy, i.e., the reflected waves in the far field can be expressed as

$$\psi_r(x, y, z) = e^{i\beta_0 y} \sum_{n=-N}^N \Psi_n e^{i(\alpha_n x + \gamma_n z)}, \quad (3)$$

in which N is the number of n under the condition γ_n is real. This expression shows that the reflected sound field can be obtained by a superposition of its component plane waves propagating in the direction determined by two angular factors α_n and γ_n .

An objective measure *DNSD* (Doubly Normalized Standard Deviation) is defined for evaluating the performance of a diffuser,¹⁾ which takes the form

$$DNSD = \frac{\sigma}{\sigma_{\max}}, \quad (4)$$

where σ is the standard deviation of the normalized

sound intensity I_n of each component wave defined as $I_n = |\Psi_n|^2 / \sum |\Psi_n|^2$, and $\sigma_{\max}$ is the maximum value of σ . Thus the value of $DNSD$ varies in the range 0 to 1.0 corresponding to the amount of scatter; $DNSD=0$ for a uniform diffusion, and $DNSD=1$ for a specular reflection. In the case of optimum diffusers with intended directional patterns, a similar procedure can be applied, in which the calculation of $DNSD$ is performed for the quantity I_n that is weighted for the angles corresponding to the emphasized intensity directions.⁶⁾

Consider two types of uneven surface, which are sinusoidal and triangular profiles as shown in Fig. 2. The period L and the height H that give wide-band diffuse reflection can be determined by both the condition of holding the plural number of n and the procedure of minimizing $DNSD$. The details of the procedure are written in Ref.1). As a result for the case of normal incidence at a frequency range of 250 Hz to 2 kHz, the period L is given as $L = 1.5$ m, and the height H is 18 cm for the sinusoidal case

Fig. 2 Periodical diffuse-type reflectors with sinusoidal and triangular profile.

Fig. 3 $DNSD$ characteristics of two types of reflector.

and 20 cm for the triangular case. The $DNSD$ characteristics are shown in Fig. 3 and some examples of the scattering pattern calculated using a method developed by Jordan and Lang,⁷⁾ together with the experimental results for the surface of finite dimension $2L$, are shown in Fig. 4. These results show properties of highly diffuse reflections of the present uneven surfaces in comparison with an even surface (PLANE case), as well as close agreement between theory and experiment.

In Fig. 3, however, an anomalous point can be seen, which occurs at a frequency of 453 Hz. The detailed calculation reveals that similar anomaly

Fig. 4 Scattering pattern of three types of reflector.

occurs at other frequencies corresponding to those estimated from a relation $f = (c_0/L)n$ with $n=1, 2, 3, \dots$ and c_0 as the speed of sound. The detailed discussion will be taken up in Sec. 3 including an explanation of the bottom figure labeled "TRI-ANGLE INFINITE CASE."

3. SOUND FIELDS DUE TO REFLECTORS WITH HIGH DIFFUSION

It is known that a sound field composed of a direct wave and the reflection from a plane surface shows peaks and dips with a regular interval in the frequency characteristics, so to say the comb-filtered response.¹⁰⁾ Both the interval and the depth of the gaps depend on the delay and magnitude of the reflection. The case, in which change in spectrum is perceived as the difference in timbre, is so-called "coloration." Ando *et al.*⁸⁾ expressed the time dependence of the source using auto-correlation function as a representative of the source variation, and discussed the perception of coloration in relation to this value. While, in the relation between the delay of the first reflection and its level, Barron⁹⁾ points out a region required the necessary care for the acoustical design, and is apprehensive about large plane reflector above a stage. In the actual hall acoustics, however, large reflectors like a stage floor or stage surroundings are inevitable and other reflections may make up for the tonal distortion due to interference effects. Thus the real state of tone coloration may still remain as a problem that needs further investigation. Then the discussion here will be given in comparison with the case of an ordinary plane reflector.

3.1 Experimental Setup and Procedure

Reflectors treated here are (i) a plane surface, (ii) a periodic surface with sinusoidal profile, and (iii) triangular profile which is often used in actual halls. For the case having periodic roughness, case (ii) and case (iii), the surface was chosen as a reflector with highest diffusion in the frequency range of 250 Hz to 2 kHz under a normal incidence, which was designed by the procedure described in the previous section. The period is 1.5 m and the height of the profile is 18 cm for the sinusoidal type and 20 cm for the triangular type. The experiment was carried out using 1/8-scale models of the reflector set in an anechoic chamber with a sound source of 10 cm-diameter loudspeaker emitting an impulsive signal.

Fig. 5 Experimental setup for measuring reflected sound fields.

The dimension of the reflector was 75 cm in width (x -direction) and 1.8 m in length (y -direction), then in the case (ii) and the case (iii), the width was 4 times as large as the surface period L .

The experimental setup is shown in Fig. 5. The response of the total sound field at a receiving point was obtained by deconvolution of a source signal. An identified impulsive signal was emitted from a loudspeaker in two types of the setup, with and without the reflector. The frequency-response function of the reflected field was calculated in the frequency domain from the ratio of Fourier transformed two signals. This gives sound pressure level (SPL) characteristics of the reflected sound field normalized by the direct sound. Also, the impulse response of the sound field can be calculated by taking the inverse Fourier transform of the response. It is confirmed that stable and reliable results can be obtained by using the following: an identified impulsive signal, a digital low-pass filter and an appropriate time window. DAT recorded spark pulse was used as a source signal. For the recorded digital data that were converted into the data in full scale, a digital filter with a cutoff frequency of 2 kHz and Hamming window of time length 50 ms were adopted in the signal processing treatment, which was processed collectively by a personal computer.

Although the impulse response has the higher rank information about an input-output linear system than that of the frequency characteristics, in general, it seems difficult to extract the particular features from the impulse response data and to evaluate them accurately. While the frequency response of SPL has a disadvantage of a lack of temporal information. However, this has an

advantage of making it easy to evaluate physical features of the sound field as tonal effects on our auditory system regarding the problem of "coloration." Then in the following section, the results of SPL characteristics are mainly presented, and the impulse response data are appropriately used to illustrate the particular features for the case that can illuminate diffuse reflections.

Fig. 6 SPL characteristics of the reflected sound field caused by a plane reflector. The receiving point is (a) 0.4 m, (b) 0.8 m and (c) 1.6 m away from the surface. —, Theory; ----, experiment.

The experimental results in comparison with the theory are shown in Figs. 6–8 and 11. Figures 6–8 show the effect of receiver position, in which the receiving point was along the line normal to the surface top, and Fig. 10 shows the case of surface bottom. The results for the case of oblique incidence 45° are shown in Fig. 11. Results of the impulse response, as a few examples of each case, are shown in Figs. 9 and 10 having the direct impulse as a reference signal, in which the vertical scale corresponds to the sound pressure amplitude of the same scale throughout the whole figures. All of these results are converted into full-scale reflector with a parameter of the full-sized distance from the plane or the neutral plane in the corrugated case.

3.2 General Discussion about Diffusion

In all results of Figs. 7, 8, 10 and 11 calculated for SPL characteristics, as compared to specular reflection (Fig. 6), the following some unique properties due to diffuse reflections can be seen. First, sound fields with diffuse reflections has peaks and dips at irregular intervals in the frequency domain, while in the case of specular reflection the interval is constant in the linear frequency scale. Second, for specular reflection from a rigid surface, the peak values are fixed at a constant of 6 dB, while diffuse reflections show the envelope of peaks having fluctuations in the frequency characteristics. Third, as can be seen in Figs. 7 and 8, the results for both types of the diffuser show almost the same characteristics. Then, for the present case of diffusers having the same structural period and different profile designed for providing highest diffusion over a wide frequency range, the characteristics can be determined regardless of the surface details.

Another particular feature shown in these SPL characteristics for both types of the diffuser is some unique sharp peaks, which can be seen particularly in the theoretical results, but cannot be seen clearly in the experimental data. The existence of such peaks is the largest cause of disagreement between theory and experiment, and may be closely related to the anomalous effect shown in Fig. 3. The details about this matter are discussed as follows:

The theory applied to the present case is different from the measurement in such a way that the perfect periodic condition (infinite periodicity) is assumed for the theoretical treatment, while in the experiment it is impossible to fully satisfy this condition. In

the present experiment, the width of the reflector is only four times as large as one period of the surface corrugation. Although the periodicity originally implies the sense of infinity, the case of some repetitions within a finite length is also commonly expressed using the same word "periodicity." Accordingly it is necessary to draw a clear distinction between both cases. Then in the following

discussion, a word "true-periodicity" is used for the infinite case, and "quasi-periodicity" is for the finite case.

Disagreement between theory and experiment would be caused by two kinds of difference between infinite and finite conditions. One is the difference in dimension, *i.e.*, infinite dimension is assumed in the theory, while in the experiment the dimension is

Fig. 7 SPL characteristics of the reflected sound field caused by a sinusoidal-type diffuser. Otherwise as in Fig. 6.

Fig. 8 SPL characteristics of the reflected sound field caused by a triangular-type diffuser. Otherwise as in Fig. 6.

necessarily finite. This cause has a general tendency of making an error increase as the frequency is decreased. In the frequency range of interest, however, even at low frequencies the "plane case" shows rather good agreement between theory and experiment. Then, this cause seems to have less effect than another kind of difference, which is the periodic condition. For both types of diffuser, the true-periodic condition is assumed in the theory, while in the experiment quasi-periodicity is necessarily needed. Such inefficiency of the true-periodic condition in the experiment would be a main cause of disappearance of sharp peaks. In particular at low frequencies, where the effects are more serious, this tendency becomes more remarkable.

According to a scattering theory described in Sec. 2, the analytical formulation can be regarded as being originated from a theory using Helmholtz-Kirchhoff's integral formula for general sound fields. The fundamental solution adopted in the present case can be written as⁴⁾

$$G(\mathbf{r}_p|\mathbf{r}) = \sum_{n=-\infty}^{\infty} \frac{i}{2\gamma_n L} e^{i[a_n(x_p-x) + \gamma_n(z_p-z)]} \quad (5)$$

where $\mathbf{r}$ represents a point (x, z) on the surface and $\mathbf{r}_p$ is for an arbitrary point (x_p, z_p) in the sound field. As can be seen from this equation, reflection caused by a periodic surface has singularities at frequencies calculated from the condition $\gamma_n=0$. On consideration of γ_n depending on the angle of incidence, surface period L and the frequency, one can easily expect the existence of frequency that satisfies this condition at any value of integer n . At these frequencies, it's impossible to calculate such type of equations numerically. The solution, however, can be obtained as a converged value by putting the frequency close to the singularity. All results for the corrugated surfaces are calculated by means of this method. Originally, such a singularity is originated from the assumption of true-periodic condition. Owing to this assumption the reflection can be expressed as a superposition of discrete elemental plane waves as shown in Eq.(1). Interference between such reflection and the direct wave forms constructive superposition similar to that in a source-receiver system set in parallel walls,¹⁰⁾ as shown in the later section. However in a quasi-periodic case like measurements in the laboratory, the effect of constructive interference is weakened. This fact may weaken the sharpness of the peaks.

Such matters as described above may have also a

relation to a matter regarding the difference between theory and experiment with respect to the reflection directivity. The reflection directivity obtained from a theory with true-periodic conditions has some directional intensity components at discrete angles, which differ from the experimental results characterized by a smooth directivity with side lobes. One example is shown in Fig. 4 (bottom), in which the results were calculated directly from a theory for a triangular-type diffuser with true-periodic corrugation. Comparing this with the finite case labeled "500 Hz" and "TRIANGLE," one can find a difference originated from a similar cause to that can be seen in the SPL characteristics.

Singularities in SPL characteristics obtained from theoretical calculations, which are for example sharp peaks occurred at 229 Hz, 459 Hz, 688 Hz and so on as shown in Figs. 7 and 8, correspond to the frequencies determined as $f=(c_0/L)n$, which is derived from the condition $\gamma_n=0$, and the same form as in Sec. 2. Thus the difference at around these frequencies between theory and experiment can be supposed as a result caused by an assumption of true-periodic condition. As for the anomaly occurred at 459 Hz in *DNSD* characteristics, one can consider the same reason. The effect of such singularities is clearly seen also in some cases of the impulse response of Fig. 9. Discussions on this matter are given in the next section.

3.3 Impulse Response

Figs. 9(a), (b) and (c) show the impulse response corresponding to the case (b) of Figs. 6, 7, and 8, respectively, in which the time history after the arrival of the direct sound is presented. The receiver point is 0.8 m away from the neutral plane of each reflector. The first pulse is the direct sound, which is used as a reference value of vertical scale of the pressure amplitude. All figures of the impulse response are expressed relative to this value. Then it is possible to compare all pulses in the same scale.

Some pronounced characteristics inherent in diffusion of the surface can be seen in these results. One is a series of pulses being attenuated with a regular interval, which is seen in the theoretical results at more than 10-ms delay after the arrival of the direct sound, and corresponds to sharp peaks at frequencies of 229 Hz, 459 Hz, 688 Hz and so on in the theoretical results of Figs. 7 and 8. The frequency interval of 229 Hz between these frequencies

corresponds to the time interval of 4.4 ms ($=1/229$ s) between each pulse. Such phenomenon would be caused by the true-periodic condition as discussed in the previous section, and seems to be similar to multi-reflection phenomenon of a source-receiver system in parallel walls, which is so-called "flutter echo." From these discussions, one important conclusion can be drawn as follows: When the repetition of the surface period increases, there is a possibility of appearance of the phenomenon predicted by a scattering theory based on the true-periodic condition. In other words, a situation similar to flutter echo may become reality in sound fields near a surface of large dimensions with quasi-periodic corrugation. However, the details still have been unknown, and the effect of such phenomenon on our auditory perception is interesting, then further study is needed to investigate the physical phenomenon and to determine to what extent the phenomenon contributes to the subjective quality of reflected

sound fields.

Another feature is scattered pulses at around 5-ms delay. The accurate delay times of the reflection referred to the direct sound are 4.64 ms for the plane case, 4.16 ms and 6.24 ms for the first and second scattered pulses for both types of diffuser. In the light of delay times due to geometrical path length between receiver and surface bottom, which are 5.18 ms and 5.24 ms for both types, it is found that the second pulse arrives with an excess delay of about 1 ms, which corresponds to 34 cm longer than expected from the geometrical relation.

From these findings as well as that in the far field, as can be seen from Eq.(1), the reflection is mainly generated as a superposition of plane waves with respect to n under the condition γ_n is real, one can estimate the following: The fundamental mechanism of diffusion of periodic rough surfaces can be considered as a kind of multiple reflections composed of discrete waves with various delays and directions. In the case of quasiperiodicity, as a practical case, the discrete superposition will be changed into smeared one. As a result of this section, a possible interpretation about the mechanism of diffuse reflections can be drawn as described in the discussion regarding Fig. 18.2 of Ref. 6).

3.4 Effects of Receiver Position and Angle of Incidence

In Fig. 10, where a receiver is located at 0.4 m away from the neutral plane along the line normal to the plane through the surface bottom, both results of the SPL characteristics and the impulse response show rather unique response. As for the impulse response, both types of diffuser showed almost the same results. Then only the case of triangular type is presented here. As can be seen from the comparison with the results (a) of Figs. 7 and 8, a shift of half a period of the receiving point along parallel to the surface has serious effects on the response, which are decreased fluctuations and gradual increase of the total level with increasing frequency.

In the impulse response also, a reflected pulse of about two times as large pressure amplitude as the direct sound can be seen. These seem to be caused by a dent formed by one structural period of the surface corrugation, which may be an effective reflector in the near field, *i.e.*, the receiving point is near a focal point of the dent. From these observations, it is found that in such a near field with a

Fig. 9 Impulse response at a receiving point 0.8 m of the reflected sound field caused by (a) a plane reflector, (b) a sinusoidal-type diffuser and (c) a triangular-type diffuser, corresponding to the case (b) of Figs. 6, 7 and 8, respectively.

certain geometrical uniqueness, the response is affected by such geometrical characteristics of the surface rather than its diffusion. It is, however, confirmed that the effect is decreased as the distance

from surface to receiver increases.

In the case of oblique incidence shown in Fig. 11, a peak at 134 Hz in the theoretical results of both diffuser-types is the same singularity as discussed in Sec. 3.2, which occurs at a frequency corresponding to the condition $\gamma_n=0$ with $\theta=45^\circ$ of an order $n=-1$. In such an oblique incidence, for all sur-

Fig. 10 Effect of the receiver position on the response of the reflected sound field caused by (a) a sinusoidal-type diffuser and (b) a triangular-type diffuser, and (c) the impulse response for a triangular case. The receiving point is shifted in half-period of the surface with reference to the case of Figs. 7, 8 (a).

Fig. 11 SPL characteristics of the reflected sound field under 45° incidence caused by (a) a plane reflector, (b) a sinusoidal-type diffuser and (c) a triangular-type diffuser, with the receiving point 0.8 m away from the neutral surface. —, Theory; ----, experiment.

face types, there comes out a discrepancy of the path length between theory (infinite source-distance) and experiment (finite source-distance), which has an effect causing disagreement of the peak to peak interval or the dip to dip interval between them. One method to compensate this discrepancy is to change the surface-receiver distance depending on the difference of the path length. There is no other effect of this change on the response of the plane wave case. In the case of corrugated surface, however, changing the distance has an influence on the response somewhat, because of the receiver distance from the surface being an important factor for this case. Such effects can be seen in Figs. 11(b) and (c). In any case, although any small particular characteristics at low and mid frequencies due to its incidence condition can be seen, the general tendency of the reflection seems to be similar to the case of normal incidence.

On the whole, it is seen that the surface with sinusoidal profile and the surface with triangular

profile have almost the same reflection characteristics. This result shows that both the surface period and the macroscopic profile, regardless of the details, exclusively affect reflection from the periodical uneven surface. Then in the subsequent section the effects of surface roughness on the reflection characteristics are investigated for the case of triangular-type diffusers.

4. QUANTITATIVE DESCRIPTION FOR SURFACE DIFFUSION

As can be seen in Eq.(3), an elemental plane wave of order n propagates towards the direction depending on the factor γ_n with the condition that γ_n is real. Reflection from a surface with properties of diffusion means that the value of N is greater than 2. From Eq.(2), a reflection corresponding to the condition $n=0$ always occurs, which is the specular element. A state of $n=\pm 1$, in which at least one scattering element exists, yields the condition

Fig. 12 Effect of the surface period L on the SPL characteristics, (a) $L = 1.0$ m, (b) $L = 0.5$ m, (c) $L = 0.25$ m and (d) $L = 0.15$ m, for the case of triangular-type diffuser with the roughness height of 20 cm. The receiver point is 0.8 m away from the surface under a normal incidence.

$$\sqrt{1 - \cos^2 \phi \sin^2 \theta} \geq \left| \sin \phi \sin \theta \pm \frac{\lambda}{L} \right|, \quad (6)$$

where λ is the wavelength. Then the lowest frequency that causes scattering is written in the form

$$f_{\min} = \frac{c_0}{L(|\sin \phi| \sin \theta + \sqrt{1 - \cos^2 \phi \sin^2 \theta})}. \quad (7)$$

In examples shown in previous section, $f_{\min}$ is 229 Hz for a normal incidence, and 134 Hz for an oblique incidence of 45° . Some irregular characteristics at frequencies above this value can be seen in both theory and experiment of Figs. 7, 8 and 10. However, it seems difficult to find the critical point in the frequency characteristics of measured data, from the reason owing to the quasi-periodicity as discussed in Sec. 3.2. Then, to identify this property, the following discussion will be given exclu-

sively from a theoretical point of view, and the measured data will be used supplementary to confirm the findings.

The calculated results for the triangular case with a fixed value of 20 cm as the roughness height and a parameter of the period L , under a normal incidence, are shown in Figs. 12 and 13, in which the receiving point was located 0.8 m away from the neutral surface. For the extreme cases, which are the case of $L=1$ m and $L=0.15$ m, the results are presented as compared to the experimental data. The values of $f_{\min}$ calculated from Eq.(7) are 344 Hz, 688 Hz, 1376 Hz and 2293 Hz for the case of $L=1$ m, 0.5 m, 0.25 m and 0.15 m, respectively, and each value is indicated by an arrow in each figure.

Particular features due to diffuse reflections, as discussed in the beginning of Sec. 3.2, appear at frequencies above the critical frequency $f_{\min}$ in the theoretical results. A similar disturbance can be seen also in the measured data of Fig. 12(a) in comparison with Fig. 6(b). At frequencies below this critical frequency, a surface with any corrugation behaves like a flat mirror. In this region sound waves are reflected in such a way as neglecting the surface roughness. Figure 12(d) shows the most extreme case, in which the critical frequency is out of the frequency range of interest. Even though the surface has the roughness height of 20 cm, the reflection from this corrugated surface seems to be almost specular as compared to Fig. 6(b). It is seen from this fact that, for the case where the surface period is too short to have an influence in the frequency range of interest, there is no effect of the roughness height of a periodic-type diffuser on the response.

The impulse response also clearly shows the variation of diffuse reflections depending on the change in the surface period. It is seen, in particular in the theory, that a series of pulses with a regular interval caused by true-periodicity appearing after the delay time of 8 ms is gradually attenuated with decreasing the surface period. Although this phenomenon is not so remarkable in the experimental results because of a difficulty in realizing the true-periodic condition, a pronounced difference between diffuse and specular reflections well appears in the measured data.

According to Eq.(7), features described above depend on the angle of incidence also. The results for the case of $\phi=90^\circ$ and $\theta=45^\circ$ are shown in Fig.

Fig. 13 Effect of the surface period L on the impulse response. Otherwise as in Fig. 12.

Fig. 14 Effect of the angle of incidence on (a) the SPL characteristics and (b) the impulse response, with reference to the case (d) of Figs. 12 and 13.

14, in which the critical frequency $f_{\min}$ ($=1343$ Hz) is indicated by an arrow. There is a disagreement between theory and experiment similar to that discussed in Sec. 3.4 regarding Fig. 11. However, particular features due to diffusion can be seen at frequencies above $f_{\min}$, which are closely related to the disorder of the impulse response at the delay time after 5 ms.

Up to now the horizontal angle of incidence is fixed at 90° . However, as can be seen from Eq.(7), diffusion performances of a surface depend on also this horizontal angle ϕ . Considering the effect of change in angle of incidence on the reflection, it can be seen that $f_{\min} = c_0/L(1 + \sin \theta)$ for the case of $\phi = 90^\circ$, and $f_{\min} = c_0/L \cos \theta$ for $\phi = 0^\circ$. The former is the case under an incidence normal to the grooves, and the later for the parallel case. It is seen from this result that the tendency of θ -dependence is quite different between two cases, i.e., as the vertical angle θ increases, the critical frequency is decreased in the former case, and increased in the latter case. Then

Fig. 15 Effect of the horizontal angle of incidence on (a) the SPL characteristics and (b) its impulse response for the case of $\phi = 90^\circ$, and (c), (d) for the case of $\phi = 0^\circ$, under an grazing incidence of $\theta = 80^\circ$. The receiving point is 1.6 m away from the neutral plane of a triangular-type diffuser with $L = 1.0$ m.

as the angle approaches near grazing, the effect of horizontal angle ϕ becomes remarkable, especially in the case of $\phi=0^\circ$ the critical frequency shifts to the region being independent of the surface roughness. As an example of the comparison between $\phi=90^\circ$ and $\phi=0^\circ$, the calculated results for the case $L=1$ m and $\theta=80^\circ$ are shown in Fig. 15 with a distance 0.8 m from the neutral plane and $f_{\min}=173$ Hz and 1981 Hz, respectively, which are marked by an arrow in the figure. From this finding, as a practical situation, when a diffuser with one-directional corrugation is exposed by a source at large angle of incidence, it should be noted that the degree of diffuse reflections strongly depends on the direction of the corrugation.

5. CONCLUSIONS

The effect of periodic roughness of a diffuse-type reflector on the sound field, as compared to specular reflection, was investigated both theoretically and experimentally. This investigation has two purposes. One is to clarify the physical properties of diffuse reflections in relation to the clarification of the mechanism. Another is to identify the critical condition of diffuse reflections.

Some particular features depending on the periodicity and roughness of the diffuser were specified as follows:

- (i) deviations from the regularity of the frequency response, *i.e.*, irregular peaks and dips with irregular interval in the frequency characteristics;
- (ii) scattered pulses of reflection in the impulse response at around an arrival time estimated from the geometrical path length and disorder of the temporal structure following them;
- (iii) sharp peaks appearing in the frequency characteristics, which correspond to a series of attenuating pulses with a regular interval in the impulse response.

Features (i) and (ii) are closely related to the period and roughness of the reflector. Even a few repetition of the corrugation (quasi-periodic roughness) can cause these effects. Feature (iii) can be interpreted as a similar phenomenon as occurred in the case of a source-receiver system in parallel walls (flutter echo). This phenomenon is caused by an infinite repetition of the corrugation (true-periodic roughness) of the diffuser. It should be noted that, however, in practical situations as the dimension of

the diffuser increases there is a possibility of appearance of this phenomenon. These findings would confirm an interpretation about the mechanism of diffuse reflections described in the previous paper.⁶⁾

The condition whether the reflection is specular or diffuse depends on the surface period, wavelength and the angle of incidence, which can be quantitatively estimated by a simple relation between them. A frequency estimated from this relation gives a critical value, which determines a turning point of reflection characteristics. At below this critical frequency, a surface with any corrugation behaves like a flat mirror regardless of its roughness size. Then this relation will be served as a useful tool for determining whether the reflection is specular or diffuse at any design frequencies. While in the specified frequency range, a surface with one-directional periodic corrugation has double-faced character, which is diffuse or specular depending on the direction of the incidence.

Appropriate application of the findings obtained in this work and another unsolved problems regarding the effect of diffuse reflections on our subjective experience, which is now under investigation, will provide a good tool for designing good room-acoustic condition.

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