Visualization of sound propagation and scattering in rooms

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Abstract: This paper presents visualization of transient sound propagation in 2-dimensional room sound fields in which the typical shapes of concert halls are modeled by applying the finite difference time domain method. As a basic study on room acoustic design, sound propagation in rooms, scattering effect of acoustic diffusers and reflection characteristics of suspended panel arrays are investigated. Through the investigation, it has been confirmed that this kind of visualization technique is very effective to get intuitive comprehension of complex acoustic phenomena which occur in rooms. The technique can be useful tool for discussion on room and acoustic treatment between acoustic engineers and architects.

Keywords: Visualization, Finite difference time domain method, Room acoustic design, Room shape, Acoustic diffuser, Suspended panel array

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1. INTRODUCTION

Visualization technique of sound propagation in a room is very effective not only for education on room acoustics but also for acoustical design work of various kinds of auditoria. For this purpose, such computer simulation techniques as ray-tracing and image-source methods have been developed and being widely used. This kind of techniques based on geometrical acoustics are effective to some extent when obtaining rough estimate of sound reflection and absorption in a room but it is impossible to exactly deal with such complicated phenomena as sound reflection, diffraction and scattering. To overcome this problem, the authors have been investigating the application of “the finite difference time domain (FDTD) method [1]” to transient acoustic phenomena [2]. In this paper, the outline of the FDTD method for 2-dimensional space is firstly introduced. As the examples of its application to visualization of room acoustics, the comparisons of sound propagation in rooms of different shapes, and reflection/scattering effects of acoustic diffusers and suspended panel arrays are presented.

2. OUTLINE OF CALCULATION BY FDTD METHOD

In a 2-dimensional sound field, sound wave is expressed by the following partial differential equations. Equations (1) and (2) are the momentum equations in x- and y-directions, respectively, and Eq. (3) is the continuity equation.

\[
\frac{\partial p(x, y, t)}{\partial x} + \rho \frac{\partial u_x(x, y, t)}{\partial t} = 0
\]

\[
\frac{\partial p(x, y, t)}{\partial y} + \rho \frac{\partial u_y(x, y, t)}{\partial t} = 0
\]

\[
\frac{\partial p(x, y, t)}{\partial t} + \kappa \left( \frac{\partial u_x(x, y, t)}{\partial x} + \frac{\partial u_y(x, y, t)}{\partial y} \right) = 0
\]

where \(p\) is the sound pressure, \(u_x\) and \(u_y\) are the particle velocities in x- and y-directions, respectively, \(\rho\) is the density of the air and \(\kappa\) is the volume elastic modulus of the air.

The spatial and time derivatives of an arbitrary function \(f\), \(\partial f/\partial x\), \(\partial f/\partial y\) and \(\partial f/\partial t\) can be approximated by the central finite difference forms as \((f(x + \Delta x/2) − f(x − \Delta x/2))/\Delta x\), \((f(y + \Delta y/2) − f(y − \Delta y/2))/\Delta y\) and \((f(t + \Delta t/2) − f(t − \Delta t/2))/\Delta t\), respectively. Here, \(\Delta x\) and \(\Delta y\) are the spatial intervals, and \(\Delta t\) is the discrete time step. When applying the staggered grid system with square-grids (\(\Delta x = \Delta y\)) shown in Fig. 1, the following equations are
obtained for a discrete system.

\[
\begin{align*}
\frac{u_x}{t}^{n+1}(i + 1/2, j) &= u_x^n(i + 1/2, j) - \frac{\Delta t}{\rho \Delta h} \left( p_x^{n+1/2}(i, j) \right) \\
- p_x^{n+1/2}(i, j) \\
\frac{u_y}{t}^{n+1}(i, j + 1/2) &= u_y^n(i, j + 1/2) - \frac{\Delta t}{\rho \Delta h} \left( p_y^{n+1/2}(i, j, j + 1) \right) \\
- p_y^{n+1/2}(i, j, j + 1) \\
p^{n+1/2}(i, j) &= p^n-1/2(i, j) \\
&- \kappa \Delta t \left\{ \frac{u_x^n(i + 1/2, j) - u_x^n(i - 1/2, j)}{\Delta h} \\
&+ \frac{u_y^n(i, j + 1/2) - u_y^n(i, j - 1/2)}{\Delta h} \right\}
\end{align*}
\]

where \( \Delta h \) is the size of the square-grid and indices \( n, n + 1/2, n - 1/2 \) and \( n + 1 \) denote time steps. In the calculations mentioned below, \( \Delta h = 1 \) cm for the spatial grid size and \( \Delta t = 0.02 \) ms for the discrete time step were set.

As the initial condition assuming an impulse source, a smoothly continuous distribution of sound pressure described by the following equation was set (see Fig. 2).

\[
p(r) = \begin{cases} 
1 + \cos \pi \frac{r}{12 \Delta h} & (r < 12 \Delta h) \\
0 & (r > 12 \Delta h)
\end{cases}
\]

where \( r \) is the distance of a grid point from the source position.

Based on the assumption that the boundary is locally reactive, the normal component of the particle velocity on the boundary is expressed as follows.

\[
\nu_{n|\text{boundary}} = \frac{p}{Z_n}
\]

where \( \nu_{n|\text{boundary}} \) is the normal component of the particle velocity on the boundary, \( p \) is the sound pressure and \( Z_n \) is the normal acoustic impedance on the boundary. Here, by assuming that a part of the boundary is approximated by the two sides of a square-grid as shown in Fig. 3 and \( p \) can be represented by the sound pressure at the center point of the square-grid \( p(M, N) \), the following expressions are derived for the \( x \)- and \( y \)-components of the particle velocity on the boundary.

\[
\begin{align*}
\frac{u_x}{t}^{n+1}(M + 1/2, N) &= \frac{p_x^{n+1/2}(M, N)}{Z_n} n_x \\
\frac{u_y}{t}^{n+1}(M, N + 1/2) &= \frac{p_y^{n+1/2}(M, N)}{Z_n} n_y
\end{align*}
\]

where \( n_x \) and \( n_y \) are the \( x \)- and \( y \)-components of the unit vector normal to the boundary under consideration.

To simplify the problem, it was assumed that the normal acoustic impedance on the boundary consists only of real part in this study. In this case, the relationship between the normal acoustic impedance \( Z_n \) and the normal sound absorption coefficient \( \alpha_n \) is expressed as follows.
In the calculation mentioned in sections 3 and 4 in this paper, it is assumed that $\alpha_n = 0.2$ (corresponding to $Z_n = 7.357 \text{ Ns/m}^3$) for over all frequencies to simplify the boundary condition. (The authors have been investigating the method to simulate the boundary condition of arbitrary normal acoustic impedance in the FDTD method [3].)

Under these initial and boundary conditions, the sound pressure and particle velocities at each grid point were calculated successively using Eqs. (4), (5), and (6).

3. VISUALIZATION OF SOUND PROPAGATION IN ROOMS

In architectural design of concert halls and theaters, rectangle (so called “shoe-box style”), fan-shape, round shape and ellipse are often chosen. By the difference of such room shapes, acoustic properties are much varied and therefore the design of fundamental room shape is an essential problem not only from the architectural viewpoint but also from the acoustic viewpoint.

In order to examine the acoustic characteristic determined by such fundamental room shape, sound propagation characteristics in the 2-dimensional rectangular, fan-shaped and elliptic rooms shown in Fig. 4 were calculated by the FDTD method. In this calculation, it was

Fig. 4  Sound propagation in a rectangular room (a), fan-shaped room (b) and elliptic room (c) without diffusing treatment.
assumed that these three 2-dimensional rooms have the same area of 518.4 m$^2$.

Figures 4 shows the typical calculation results in the form of “snap shot” in the time lapse after the emission of the impulse source. (When demonstrating the results by computer animation, the successive propagation of the wave front of the impulse can be clearly visualized.) In each figure, the black circle indicates the source position and the white one indicates the receiving position for the calculation of impulse response mentioned later.

Comparison of these figures reveals that the propagation of the wave front is much different in each hall. In the case of the rectangular room, it is clearly seen that the number of wave front increases with the progress of time, whereas in the cases of the fan-shaped and elliptic rooms, a tendency that the wave front deflects and concentrates is seen. Especially, in the case of the elliptic room, it is clearly seen the wave front focuses at around the source position and its symmetrical point alternately.

Figure 6(a) shows the impulse responses at the receiving point in each room. In these results, it is seen that the reflections are dense and smoothly diminishing in the case of the rectangular room, whereas the reflections are scattered and uneven in the fan-shaped and the elliptic rooms.
4. THE EFFECT OF SOUND SCATTERING BY DIFFUSERS

In concert halls and theaters, wall and ceiling are often made irregular to increase sound diffusivity. To examine the effect of such diffusion treatments, the FDTD calculation was again performed for the three types of rooms by making their walls irregular. As the shape of irregular wall, a zigzag shape (Type-2 in Fig. 7) was assumed. The snap shots of the calculation results are shown in Fig. 5. By comparing the results with those in the case of no diffusion treatment shown in Fig. 4, it is clearly seen that the distinct wave fronts have been much diminished and scattered in all of the three rooms.

The impulse responses at the receiving points in the three rooms were calculated in this case, too. The results are shown in Fig. 6(b) in comparison with those without diffusion treatment. In these results, it is obviously seen that the impulse responses have become much denser and smoother than the case of no diffusion treatment shown in the upper figures. When hearing these impulse responses through a loudspeakers or headphones, it can be clearly judged that the reverberation decays have much improved to be natural and smooth by the diffusion treatment, although the early fluttering sounds caused by the sound concentration are still slightly remaining in the cases of the fan-shaped and elliptic rooms. This fact indicates that the general tendency of sound concentration caused by the fundamental room shape can not be prevented by this kind of diffusion treatment on the room boundaries.

In order to examine the effect of sound scattering by diffusers in more detail, a further study was performed on the rectangular room. In this study, four kinds of zigzag shapes shown in Fig. 7 were assumed. Among them, Type-1, Type-2 and Type-3 are similar in shape but the size was varied in three steps. The ratio of the height of the apex to the width of a triangle was set 0.15 according to the results of the experimental study made by Ishii [4]. Type-4 is a “two-way” diffuser composed of Type-3 and Type-1.

Figure 8 shows the calculation results. To compare these results with those in the case of no diffusion treatment shown in Fig. 4, it is clearly seen that the sound is scattered after the first reflection on the diffusive boundaries and the space is filled with sound pressure fluctuation. In the results of Type-1, Type-2 and Type-3, it is seen that the scattering effect is dependent on the size of the diffusers. That is, in the case of Type-1, relatively strong and continuous wave fronts are still remaining, whereas they are much diminished in the case of Type-3. In the result of Type-4, the effectiveness of “two-way” diffuser can be observed.

In the calculation by the FDTD method, instantaneous sound pressure at every mesh point is obtained. By squaring the sound pressure, instantaneous potential energy
distribution in the room can be obtained and consequently the time variation of acoustic diffusivity in the room can be evaluated quantitatively from a viewpoint of the spatial uniformity of sound energy [5].

5. REFLECTION CHARACTERISTICS OF SUSPENDED PANEL ARRAYS

In order to provide early reflections to the stage and audience areas, suspended panel arrays (so called “floating clouds”) are often equipped in concert halls. To investigate the effect of such suspended panel arrays, the reflection characteristics of typical arrangements of panel arrays were examined by the 2-dimensional FDTD calculation. As shown in Fig. 9, three kinds of arrangements were examined in this study: Type-1 is a straight arrangement with straight panels, Type-2 is a terraced arrangement with straight panels and Type-3 is a terraced arrangement with curved panels. The size of each panel is 1.7 m in both cases of the straight panels and the curved ones. These panels are arranged at an interval of 2.5 m in horizontal direction in all of the three cases. In the two cases of the terraced arrangement, the gap between the adjacent panels in vertical direction is 0.44 m. In this calculation, the suspended panels were assumed to be perfectly reflective.

The snap shots of the calculation results are shown in Fig. 9, in which sound reflection on the surface of each panel and sound diffraction and transmission through each gap can be obviously seen. In the case of the straight arrangement (Type-1), it is seen that the reflections from each panel forms a continuous wave front, whereas in the cases of the terraced arrangement (Type-2 and Type-3), each reflections are separated and the wave front is divided in space. In the case of Type-3, the wave front of each reflection is almost semi-circular and crossing each other, whereas the strength (sound pressure) is relatively weak.

Regarding this kind of suspended panel arrays, the frequency characteristic of sound reflection is another essential problem and the authors are investigating this problem based on the Fresnel-Kirchhoff diffraction theory [6].

Fig. 8 Comparison of sound propagation in the rectangular room with four types of diffusing treatments.
6. CONCLUSIONS

As a basic study on visualization of room acoustics using numerical simulation technique, the sound propagation in rooms of different shapes, the scattering effect of acoustic diffusers and the reflection characteristics of suspended panel arrays have been investigated by applying the FDTD method. As a result, it has been found that this kind of visualization technique is very effective to get intuitive comprehension of acoustic phenomena in rooms. It will be a useful tool for acoustic education not only for students and acoustic engineers but also for architects who design concert halls and theaters.

Besides the studies on visualization of acoustic phenomena in room acoustics introduced in this paper, the authors are now developing the technique to auralize the results of numerical simulation [7]. By combining these visual and aural simulation techniques, researches on room acoustics could be further advanced in the future.

REFERENCES