Theoretical development and experimental validation of modal sensors

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1. Introduction

The design and implementation of modal sensors [1] has been the topic of much research applied to active vibration control as well as active structural acoustic control for many years. In general, modal sensors are constructed from piezoelectric materials. A variety of alternative approaches for modal sensors exist, and many have been investigated in previous studies. A type of sensor is a shaped modal sensor constructed from polyvinylidene fluoride (PVDF) film. Design modal sensors using shaped PVDF film can be traced back to Lee and Moon [1]. Depending on the PVDF film shape, the sensor can be made sensitive to selected modal coordinates and filter out other modal coordinates [2–5].

However, the shaped PVDF sensor implementation is often difficult because the design requires one layer of piezoelectric film for each mode [6]. Moreover, for higher order mode the shape of PVDF sensor becomes very complex which makes it difficult to fabricate.

Another alternative approach has been proposed in [7–10], it is to locate a set of independent piezoelectric patches on the surface of the structure by means of optimal design theory to yield a sensor whose output evaluates the modal coordinates. This approach is meant to overcome the difficulties by using shaped PVDF sensors. The segmented design of modal sensors does not require spatially continuous shaping of the film and does not require a separate layer of sensor film for each mode. The use of an array of sensors seems to be a more practical way than shaped sensors.

It appears that no paper has been published concerning the experimental work of sensing modal coordinates from measurement with an array of piezoelectric strain sensors. With a beam example, this paper presents an experimental demonstration of the modal sensors that builds upon results from the above papers [7–10]. In this paper, segments of piezoelectric film made of polyvinylidene fluoride (PVDF) is chosen because it gives little loading on light structures, and is easy to cut into desired segments.

2. Estimation of modal coordinate with PVDF sensors

Consider a beam with N rectangular PVDF film segments equally attached on the top surface, shown in Fig. 1. As referred to Lee and Moon's work [1], The output charge Q_n of every PVDF sensor can be expressed as follows,

$$Q_n = -\frac{h + h_f}{2} \int_0^{L_x} F_n(x) \cdot \left(e_{31} \frac{\partial^2 w(x)}{\partial x^2} \right) dx \quad (1)$$

where h_f is the PVDF sensor thickness, e_{31} is the PVDF sensor stress/charge coefficient, $w(x)$ is the displacement of the beam, $F_n(x)$ is the n th PVDF film shape function.

The PVDF film shape function of the n th segment gives

$$F_n(x) = \begin{cases} P_y & x_n - P_x \leq x \leq x_n \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Substituting Eq. (2) into Eq. (1),

$$Q_n = -e_{31} P_y \frac{h + h_f}{2} \int_{x_n - P_x}^{x_n} \frac{\partial^2 w(x)}{\partial x^2} dx \quad (3)$$

The displacement distribution of the simply supported beam can be represented by a series of expansion.

$$w(x) = \sum_{m=1}^M A_m \sin\left(\frac{m\pi}{L_x} x\right) \quad (4)$$

where A_m represents the m th modal coordinate coefficients. M is the index for the highest order structural mode.

Substituting Eq. (4) into Eq. (3),

$$Q_n = \frac{h + h_f}{2} P_y \sum_{m=1}^M A_m e_{31} \left(\frac{m\pi}{L_x} \right)^2 \times \int_{x_n - P_x}^{x_n} \sin\left(\frac{m\pi x}{L_x}\right) dx = \sum_{m=1}^M K_{m,n} A_m \quad (5)$$

where

$$K_{m,n} = \frac{h + h_f}{2} P_y \cdot e_{31} \left(\frac{m\pi}{L_x} \right)^2 \int_{x_n - P_x}^{x_n} \sin\left(\frac{m\pi}{L_x} x\right) dx.$$

The idea of creating modal sensor using an array of PVDF sensors is illustrated in Fig. 2. For the PVDF segment array to behave as modal sensor. The output signals of an array of PVDF films must be multiplied by appropriate weights vectors to produce the modal coordinates of the beam. The weighted sum of the PVDF segment output is

$$y = \sum_{n=1}^N W_n Q_n \quad (6)$$

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Fig. 1 Beam with an array PVDF segments.

Fig. 2 Principle of the analytical modal sensors.

where W_n are the unknown weight coefficients and n denotes the segment number to which the weight is connected.

Substituting Eq. (5) into Eq. (6), written in the matrix form, that is

$$y = \sum_{n=1}^N W_n \sum_{m=1}^M K_{m,n} A_m = \mathbf{W}^T \mathbf{K} \mathbf{A} \quad (7)$$

where $\mathbf{K}$ is an $M \times N$ matrix with the element $K_{m,n}$. $\mathbf{A}$ is $M \times 1$ vector.

The task now is to determine the weight vector $\mathbf{W}$ so that the weighted sum of the PVDF segment output y emulate the k th modal coordinate coefficients A_k , where $1 \leq k \leq M$. Assume that $M = N$, i.e., the number of the modes included in the analysis is equal to the number of the PVDF sensors. So the $\mathbf{K}$ matrix is square.

From Eq. (8), we see that if we select $\mathbf{W}$ according to

$$\mathbf{W}^T = \mathbf{B}^T \mathbf{K}^{-1} \quad \text{with} \quad B_i = \begin{cases} 1 & i = k \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

we get

$$y = A_k \quad (9)$$

It follows that, by selecting the weight coefficients according to Eq. (8), the weighted sum of output y defined by Eq. (6)

transform an array of PVDF segment outputs into the modal coordinate A_k . It is important to note that weight vector $\mathbf{W}$ given by Eq. (8) is independent of the type and position of the excitation.

3. Experimental validation

To verify the above conclusions about sensor development, an steel beam 310 mm long $\times$ 25 mm wide $\times$ 3.5 mm thick was equally covered with $N = 9$ segments of PVDF film. Thin metal shim was used to connect each end of the beam to the flame. This shim gave almost no resistance to bending, but was very stiff in the beam's transverse direction. Therefore, the shim was assumed to be a simply supported boundary condition. The physical properties of the PVDF film in the experiments are listed in Table 1. The first nine natural frequencies of beam are list in Table 2. The weights of every PVDF film for sensing the first four mode coordinates are shown in Fig. 3. These weights have been normalized so that the maximum absolute value is unity.

A B&K4810 shaker located at $x_d = 125$ mm on the back of the beam, was used to provide the primary driving force. And a B&K8200 force transducer installed between the shaker's stinger and the beam was used to measure the input force. HP35665A dynamic signal analyzer was used to perform the sweep-sine measurement. Swept-sine excitation

Table 1 The physical properties of PVDF film.

Parameter	PVDF film
Length P_x (mm)	30
Width P_y (mm)	12
Density (kg/m ³)	1.78×10^3
Poisson's ratio	0.28
Young's modulus (N/m ²)	$2-4 \times 10^9$
Piezo strain constant d_{31} (m/V)	23×10^{-12}
Maximum operating voltage (V/ μ m)	750
Relative permittivity ϵ/ϵ_0	12

Table 2 The first nine natural frequencies of beam (Hz).

1	2	3	4	5	6	7	8	9
81.4	325.6	732.6	1,302.4	2,023.0	2,930.4	3,998.6	5,209.6	6,593.4

Fig. 3 Sensor weight W_n for transforming 9 segment outputs into the first four modal sensors.

was used to ensure clean transfer functions and prevent aliasing. Each PVDF sensor output signal was first passed through a charge amplifier and then measured with HP35665A dynamic signal analyzer. All measurements were limited to the frequency range below 2,000 Hz.

At first, the transfer functions between force and the segments were measured. All these experimental transfer functions were scaled to equivalent to the corresponding theoretical ones at the first natural frequency to facilitate the comparison. Then these transfer functions multiplied by the corresponding PVDF segment weights of the k th mode results in single k th mode responses. The magnitudes of the transfer functions between force and multiplier outputs are shown in Fig. 4. Each of these outputs is intended to emulate a modal amplitude. As illustrated, For the most part, the transfer functions show reasonable modal filtering effects. The highest peaks in the transfer functions are always at the natural frequencies for which the modal filters were intended, although some undesired small peaks near the other natural frequencies were detected experimentally. According to reference [1,2], those undesired peaks are likely due to the inability to attach the segments in perfect positions.

It must be note that the modal sensors are designed using

the first nine structure modes since only nine segments PVDF film covered on beam. So the contribution of higher order structure modes is not considered in the weight of the sensors. This modal truncation may result in residual mode contribution in the PVDF segment outputs. The off-resonance differences between the experimental results and theoretical ones are presumably due to the higher order modes contributions. If the number of PVDF segment is added, accuracy of the sensors can be increased because more modes are taken into account. However, in general the experiment showed reasonable modal filtering effects. The computed weight vector $\mathbf{W}$ (see Eq. (9) and Fig. 3) can transform an array of PVDF sensor outputs into modal coordinates.

4. Conclusions

This paper presents a technique to measure the modal coordinates from a one-dimensional structure using an array of PVDF segments with very simple shape. The outputs of the PVDF segments can be transformed into the modal coordinates by multiplying them by appropriate weights. The weight of the sensors is independent of the type and position of the excitation. One advantage of the method is that when boundary condition of structure is changed, weights can be

Fig. 4 Magnitude response from force to weighted combinations of sensor outputs. Solid lines: experimental; dashed lined: analytical.

adjusted to compensate for the sensor imperfection in stead of changing the shape and location of PVDF sensors. The other advantage is that this method can be easily extended in the case of other boundary condition and 2D structures. The experimental results show the feasibility of this new type of modal sensor.

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