Improvement of the convergence property of adaptive feedforward controllers and their application to the active control of ship interior noise

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Abstract: Various structural measures against vibration and noise were taken in a training ship, the Oshima Maru. However, an unpleasant sound persisted in the mess hall, where crews take their breaks. To reduce the noise, active controllers were investigated to satisfy the causality constraint in their update. Some of the controllers were preconditioned using the inverse of the plant because their convergence rates are limited by the dynamics and coupling within the plant response. The overall response from the output of the control filter to the output of the error sensor is thus equal to the all-pass part of the plant response. Because this response has a flat magnitude, the convergence speed of the adaptive algorithm is not affected by resonances in the plant response, as it is for the normal filtered reference LMS algorithm. The algorithms were compared under the same conditions to investigate differences in their properties and also corrected to satisfy the causality of their update processes. A comparison of convergence properties shows that satisfying the causality constraint in their update results in an improvement, and consequently the effect was confirmed by the filtered reference - filtered error LMS algorithm. Simulations are presented for a control system that was introduced using plant responses measured from a loudspeaker to a microphone in the mess hall inside the Oshima Maru. After investigating the convergence speed in various gradient descent adaptation algorithms, the results were integrated with the actual plant response and applied to the active control of ship interior noise. It was also found that although the preconditioned LMS algorithm converges dramatically faster than the ordinary gradient descent adaptation algorithms with an accurate plant model, its convergence rate is still sensitive to the auto-correlation and cross-correlation properties of the reference signals.

Keywords: Feedforward control, Active noise control, Gradient decent adaptation algorithm, Preconditioned LMS algorithm, Ship interior noise

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1. INTRODUCTION

The Oshima Maru is a 226-gross-ton training ship equipped with one 1,300 ps, 370 r.p.m. diesel engine. Various structural countermeasures against vibration and noise have been taken in this ship. However, unpleasant low-frequency sound from the engine room has persisted in the mess hall where crews take their breaks. The main engine room is located adjacent to and partially below the mess hall on the lower deck. Conventional solutions to that noise are subject to considerable structural variations. Low-frequency sound experienced in a ship often cannot be reduced by the use of ordinary passive methods; only active control of noise is capable of achieving a universal solution. Adaptive feedforward systems have found special applications in the active control of sound and vibration [1]. The controller is often a digital FIR filter, and the methods most widely used to adapt to such controllers are based on the LMS algorithm.

We investigated various feedforward controllers in search of the most effective means of reducing the noise. The convergence rate of the algorithms is limited by the correlation properties of each reference signal, their cross-correlation properties, and the dynamics and coupling within the plant response. Various current gradient descent adaptation algorithms — the filtered reference LMS algorithm, the filtered error LMS algorithm [2], the filtered-ε LMS algorithm [3], the filtered reference - filtered error LMS algorithm [4] and the preconditioned LMS algorithm [2] — were corrected to satisfy the causality of their update processes, and then compared to investigate differences in their convergence and bias.

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properties. Both a simulated signal and a measured signal using plant responses measured from a loudspeaker to a microphone inside the ship were applied. A comparison of the adaptive algorithms and adapting them to an actual noise signal have not been performed previously, so these points are accentuated.

2. GRADIENT DESCENT ADAPTATION ALGORITHMS

It is possible to classify the gradient descent adaptation algorithms into two main categories. One type minimizes the mean-square error, while the other minimizes a filtered mean square error and leads to a biased result.

2.1. Non-biased Methods

The adaptation algorithm, which directly minimizes the mean-square value of an error signal, is defined as a non-biased method. Both filtered error and filtered reference LMS algorithms belong to the non-biased method and minimize the mean-square value of the same, output, error [5]. A block diagram for the filtered error LMS algorithm [2] is shown in Fig. 1; the algorithm uses a time-delayed and transposed model of the plant response, \( e(z^{-1}) \), in which \( e(n) \) is the reference signal, \( d(n) \) is the disturbance, \( e(n) \) is the error signal at the sensor which the control system is attempting to minimize, \( W(n) \) is the control filter, \( G(z) \) denotes the response of the physical plant, i.e. the response between the actuator and sensor in the system under control, and \( \hat{G}(z) \) represents the internal model of the plant. The filtered error LMS algorithm can be written as

\[
W_i(n + 1) = W_i(n) - \alpha f(n - J)xT(n - i - J). \tag{1}
\]

The vector of the delayed filtered error signals can be written as

\[
f(n - J) = \sum_{j=1}^{J} \hat{G}_f^T(n - f). \tag{2}
\]

2.2. Biased Methods

The filtered-\( \epsilon \) LMS algorithm [3] minimizes the mean-square value of a filtered error signal. It should be noted that the filtered-\( \epsilon \) LMS algorithm in [3] uses a delayed model of the inverse of the plant response to filter the error, rather than a delayed model of the time-reversed plant. The filtered reference - filtered error LMS algorithm [4] uses a similar scheme. They both converge to a biased version of the optimal least-squares filter because of the use of the inverse of the plant response. A block diagram for the filtered-\( \epsilon \) LMS algorithm is obtained by substituting \( \hat{G}_f^{-1}(z^{-1}) \) for \( \hat{G}_f(z^{-1}) \) in Fig. 1. The filtered-\( \epsilon \) LMS algorithm can be written as

\[
W_i(n + 1) = W_i(n) - \alpha f(n - J)xT(n - i - J). \tag{3}
\]

\[
f(n - J) = \sum_{j=1}^{J} \hat{G}_f^{-1}e(n - j). \tag{4}
\]

The filtered reference - filtered error LMS algorithm [4] tries to solve the problem caused by the dynamics of \( |G(z)| \). An FIR filter is applied that filters both the error and the reference signal. This filter is designed so that the resultant magnitude response is approximately unity. Figure 2 shows the algorithm. If \( |\hat{G}(z)\hat{G}_0^{-1}(z) = 1| \), it is equal to the filtered-\( \epsilon \) algorithm with a minimum-phase plant. If \( G(z) \) is a minimum-phase plant, a non-delayed inverse of \( \hat{G}(z), \hat{G}_0^{-1}, \) would suffice for the adaptation algorithm. In general, the algorithm can be written as

\[
W_i(n + 1) = W_i(n) - \alpha f(n - J)rT(n - i). \tag{5}
\]

\[
f(n) = \sum_{j=1}^{J} \hat{G}_0^{-1}e(n - j). \tag{6}
\]

\[
r(n - i) = \sum_{j=1}^{J} \hat{G}\hat{G}_0^{-1}x(n - i - j). \tag{7}
\]

When the plant is at a non-minimum phase, a delayed inverse, \( \hat{G}_f^{-1} \), must be used, as in Fig. 3. In this case, the algorithm can be written as

\[
W_i(n + 1) = W_i(n) - \alpha f(n - J)rT(n - i - J). \tag{8}
\]

\[
f(n - J) = \sum_{j=1}^{J} \hat{G}_f^{-1}e(n - j). \tag{9}
\]

\[
r(n - i - J) = \sum_{j=1}^{J} \hat{G}\hat{G}_f^{-1}x(n - i - j). \tag{10}
\]
As of the estimated plant response, preconditioned by the inverse of the minimum phase part shown in Fig. 4, the output of the control filter, \( W \) response by decoupling the channels of the system [1,6].

A way to overcome these problems is to use a spectral decomposition to precondition the plant and uncorrelated reference signals, and an all-pass/minimum phase decomposition to precondition the plant response by decoupling the channels of the system [1,6].

In the case of the preconditioned LMS algorithm shown in Fig. 4, the output of the control filter, \( W(z) \), is preconditioned by the inverse of the minimum phase part of the estimated plant response, \( \hat{G}^{-1}_{\text{min}}(z) \), before driving the physical plant, \( G(z) \). Thus, the overall response from the control filter output to the error signal is equal to the all-pass part of the plant’s response, \( G_{\text{all}}(z) \). So if

\[
G(z) = G_{\text{all}}(z)G_{\text{min}}(z) \tag{11}
\]

then

\[
G_{\text{all}}(z) = G(z)G_{\text{min}}^{-1}(z) \tag{12}
\]

\[
G_{\text{all}}^{-1}(z^{-1}) = G_{\text{min}}^{-1}(z^{-1})G(z^{-1}) \tag{13}
\]

This algorithm can be written as

\[
W_i(n + 1) = W_i(n) - \alpha a(n - J)u^T(n - i - J). \tag{14}
\]

\[
a(z) = G_{\text{all}}^{-1}(z^{-1})e(z). \tag{15}
\]

\[
u(z) = F^{-1}(z)x(z). \tag{16}
\]

### 3. PRECONDITIONED LMS ALGORITHM

The convergence rates of such steepest descent algorithms in multichannel systems are known to be limited by two distinct effects: first, the correlation of the reference signals, and second, the coupling of the plant response. One way to overcome these problems is to use a spectral factorization of the reference signal’s spectral density matrix to define a preconditioning filter \( F^{-1}(z) \), which operates on the reference signals to produce a set of white and uncorrelated reference signals, and an all-pass/minimum phase decomposition to precondition the plant response by decoupling the channels of the system [1,6].

In the case of the preconditioned LMS algorithm shown in Fig. 4, the output of the control filter, \( W(z) \), is preconditioned by the inverse of the minimum phase part of the estimated plant response, \( \hat{G}^{-1}_{\text{min}}(z) \), before driving the physical plant, \( G(z) \). Thus, the overall response from the control filter output to the error signal is equal to the all-pass part of the plant’s response, \( G_{\text{all}}(z) \). So if

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This algorithm can be written as

\[
W_i(n + 1) = W_i(n) - \alpha a(n - J)u^T(n - i - J). \tag{14}
\]

\[
a(z) = G_{\text{all}}^{-1}(z^{-1})e(z). \tag{15}
\]

\[
u(z) = F^{-1}(z)x(z). \tag{16}
\]

### 4. SIMULATIONS

#### 4.1. Training Ship Oshima Maru

In simulations of active noise control of a ship interior, the signals used, including the reference signals, the desired signals, and the plant response, were measured in the mess room of the *Oshima maru*, a 226-gross-ton training ship equipped with one 1,300 ps, 370 r.p.m. diesel engine. In this ship, the main engine room, located on the lower deck, is adjacent to and partially below the mess room. The space just above the main engine allows clearance to the second deck. Compared to the sounds in the mess hall, those in the engine control room, separated from the mess hall only by a corridor, were not unpleasant at all.

In the mess room, the noise is primarily due to the panel vibrations of walls and floors, as confirmed by results using the multi-coherence function and wavelet method [7]. Figure 5 shows the arrangement of sensors on the upper deck of the *Oshima Maru*. In the simulations, the plant response measured in Fig. 5 is used as the nominal plant response, \( G(z) \). The measurements of \( G(z) \) were made using a white noise excitation signal under low noise conditions.

#### 4.2. Simulations with Plant

In the simulations, the plant response measured in Fig. 5 is used as the nominal plant response, \( G(z) \). Its magnitude and phase response is shown in Fig. 6. It is a simple transfer function because the position of the error sensor is close to the actuator.

For the preconditioned LMS algorithm, a minimum-phase and an all-pass decomposition can be made using the complex cepstrum [8]. The negative time components of the complex cepstrum come from the all-pass cepstrum, since no negative time components are included in the minimum-phase part. The all-pass cepstrum has only the phase cepstrum of an odd function. If we subtract the odd function of the all-pass phase cepstrum from the complex cepstrum, we then get the minimum-phase cepstrum. The all-pass components can be obtained from Eq. (13). The impulse response of \( G_{\text{min}}(z) \), \( G_{\text{all}}(z) \) and \( G_{\text{all}}^{-1}(z) \) for the plant used in the simulations are shown in Fig. 7. Figure 8 shows both \( G_{\text{all}}^{-1}(z) \) and \( G_{\text{all}}^{-1}(z) \) which are used as the filtered reference - filtered error LMS algorithm and the
filtered error LMS algorithm, respectively.

Initially, to focus on the improved performance obtained using each method, the single reference signal was assumed to be Gaussian white noise.

The averaged convergence curves for each of the algorithms in the single input/single output system are shown in Fig. 9. The conditions were an FIR control filter, \( W(\z) \), having 128 coefficients and a convergence coefficient set for each algorithm at half the value, which just makes the algorithm unstable. A Gaussian white noise reference signal was used to generate the disturbance signal via a 64-tap delay. The sampling rate was 2 kHz, and the plant response \( G(\z) \) was the signal measured in the mess room of the Oshima Maru. The average error was calculated by a 32-point length window, and 100 experiments were run using different types of white noise.

The difference in convergence speeds between the filtered-\(\epsilon\) LMS algorithm and the revised filtered reference - filtered error LMS algorithm is due to the delay of \( |G(\z)G^{-1}(\z)| \). Not only does the revised filtered reference - filtered error LMS algorithm have the same convergence speed as the filtered-\(\epsilon\) LMS algorithm, but its residual error is much smaller. The original filtered reference - filtered error LMS algorithm converges faster than the revised one because the delay in the adaptation path is longer for the revised algorithm and so the convergence coefficient must be smaller. The residual mean-square error of the revised algorithm is smaller than the original one. Thus, the revised filtered reference - filtered error LMS algorithm, which satisfies the causality condition, can be said to possess an improved convergence property.

The convergence speed of the filtered-\(\epsilon\) LMS algorithm is close to the preconditioned algorithm. However, the residual mean-square error is much larger than the simple filtered error algorithm, as the filtered-\(\epsilon\) LMS algorithm minimizes the mean-square value of a filtered error signal. In this comparison, it was confirmed that the preconditioned LMS algorithm surpasses the other algorithms with regard to the convergence property. Its superiority is due to the effect of the plant response on the correlation structure in the filtered reference signal, which normally limits the convergence speed of the filtered reference LMS algorithm, having been removed, and the path from the control filter output to the error signal having a frequency response with the flat magnitude response of the all-pass part of the plant, \( G_{\text{all}}(\z) \). The filtered-error algorithm eventually converges to give the same error reduction as the preconditioned algorithm (\( -43 \) dB), which in this case is the least-squares solution.

The averaged convergence curves in the case of a multi-reference system, case 5-1-1, are shown in Fig. 10. Case 5-1-1 means that the adaptive controller has five reference signal inputs, one actuator, and one error sensor. The disturbance signal is defined as the sum of reference signals via a 64-tap delay. The preconditioned LMS algorithm converged more quickly than the other algorithms in the simulation. The revised filtered reference - filtered error LMS adaptive algorithm shows rapid convergence, but is still biased in the steady state. The main reason for the differences in convergence speed is the loop gain of the adaptation, \( \hat{G}(\z)\hat{G}_j^{-1}(\z) \). If \( \hat{G}(\z)\hat{G}_j^{-1}(\z) = z^{-j} \), the two methods correspond. In the loop gain, the difference between them is the error of estimating the inverse filter.

4.3. Simulations with the Ship Interior Noise Signal

Figure 11 shows the averaged convergence curves for the filtered error LMS algorithm and the filtered-\(\epsilon\) LMS algorithm. Vibration signals from selected accelerometer positions (Fig. 5) were used as reference signals for the adaptation algorithms. The convergence property improved using the biased method with the filtered-\(\epsilon\) LMS algorithm (Fig. 11). Regarding the other algorithms, there was no difference in the improved property between the filtered-\(\epsilon\) LMS algorithm and the revised filtered - reference
filtered error LMS algorithm. Because the reference signals have been colored and correlated, the effects of plant dynamics on solving the problem have been reduced.

A single-channel single-point delay prediction error filter (PEF) was introduced in each channel to whiten the reference signals. Figure 12 [9] is a generalization of the single-channel PEF discussed in the example of Haykin [10]. The PEF depends on $P(z)$, which minimizes the sum of the squared error contained in vector $e(z)$. Each $P(z)$ calculated using the LMS algorithm and the Levinson

Fig. 7 The impulse response for the plant used in the simulations.

Fig. 8 $\hat{G}_0^{-1}(z)$ and $\hat{G}_J^{-1}(z)$ with $J$ = 32 samples.

Fig. 9 The convergence curves for case 1-1-1 with a white noise reference signal.
method [10] was compared. The signal used was one of the reference signals, the floor’s vibration signal obtained from the *Oshima Maru*. The resulting spectrum, which is shown in Fig. 13, indicates that the Levinson method has effectively whitened the spectrum. But the adaptation method using the LMS algorithm could not sufficiently estimate $P(z)$ because the adaptation speed could not follow the sudden changes in the ship interior vibrations.

To illustrate the procedure derived above, the whitening method was introduced into the case of the 5-1-1 system using signals obtained from the ship as reference signals. It was confirmed that the whitened version converges faster than original, as shown in Fig. 14. To obtain better results, a multichannel generalization of this method is required to overcome the correlation between each channel.

5. CONCLUSION

In the present study, to reduce the ship interior noise with active control, various adaptive active controllers were investigated in terms of their convergence speed and residual error. These controllers included the filtered reference LMS algorithm, the filtered error LMS algorithm, the filtered-$\epsilon$ LMS algorithm, the filtered reference - filtered error LMS algorithm, and the preconditioned LMS algorithm. A comparison of convergence properties showed that satisfying the causality constraint in their update gives rise to their improvement, and consequently the effect was confirmed by the filtered reference - filtered error LMS algorithm. In the simulations, the convergence speed of the filtered-$\epsilon$ LMS algorithm was close to those of the preconditioned LMS algorithm and the revised filtered reference - filtered error algorithm; however, its residual mean-square error was larger than that of the simple
filtered reference algorithm because the filtered-$\epsilon$ algorithm minimized the mean-square value of a filtered error signal. The preconditioned LMS algorithm surpassed the other algorithms in the convergence property. These algorithms were then applied to the actual adaptive controllers of ship interior noise for multi-channel feedforward control and their theoretical effects were confirmed in practice. Notably, a multichannel prediction error filter had not been introduced into the preconditioned LMS algorithm, so it did not function effectively. However, according to the simulation results, the preconditioned LMS algorithm appears to be the most effective method. A multichannel prediction error filter can in the future be introduced into the algorithm, and the effect can be confirmed. It was confirmed in the present study that the whitened version converges faster than the original. Whitening is the part of the preconditioned LMS algorithm. However, it whitens each channel, the cross-correlation between each channel remains. As such, a multichannel prediction error filter would be desirable. In future studies, a multichannel prediction error filter will be introduced into these methods.

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REFERENCES


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Stephen J. Elliott graduated in physics and electronics from the University of London, England, in 1976, and received the PhD degree from the University of Surrey in 1979 for a dissertation on musical acoustics. After a short period as a Research Fellow at the ISVR working on acoustic intensity measurement and as a temporary Lecturer at the University of Surrey, he was appointed Lecturer at the Institute of Sound and Vibration Research (ISVR), University of Southampton, in 1982. He was made Senior Lecturer at ISVR in 1988 and Professor in 1994. His research interests have been mainly concerned with the connections between the physical world and digital signal processing, originally in relation to the modeling and synthesis of speech and, more recently, in relation to the active control of sound and vibration. This work has resulted in the practical demonstration of active control in propeller aircraft, cars and helicopters. His current research interests include the active control of structural waves, active isolation, adaptive algorithms for feedforward and feedback control, the control of nonlinear systems and biomedical signal processing and control. Professor Elliott is co-author of Active Control of Sound (with P. A. Nelson), Active Control of Vibration (with C. R. Fuller and P. A. Nelson) and author of Signal Processing for Active Control. He is a member of the Acoustical Society of America and the UK Institute of Acoustics, from whom he was jointly awarded the Tyndall Medal in 1992.