Numerical aspects of edge diffraction impulse responses

U. Peter Svensson

Acoustics group, Department of electronics and telecommunications,
Norwegian University of Science and Technology, NO-7491 Trondheim, Norway
(Received 6 September 2004, Accepted for publication 7 October 2004)

1. Introduction

In computational acoustics, methods based on geometrical acoustics (GA) can not take edge diffraction (ED) effects into account. However, the GA solution can be complemented by ED components and for rigid surfaces this addition gives a correct total field. The topic of this paper is numerical aspects of such impulse responses (IRs). ED components give discontinuous sound fields which compensate exactly for the discontinuities in GA sound fields and these discontinuities deserve particular attention. A numerical integration is required for the computation of discrete-time impulse responses, and an analytical approximation is presented which eases the numerical integration around the discontinuities.

2. Edge diffraction impulse responses

A formulation by Medwin et al. [1], the so-called BTM formulation, is based on Biot-Tolstoy’s IR solution for an infinite wedge [2]. The impulse response \( h_{\text{BTM}}(t) \) is given by Eq. (1) whereas an alternative formulation, [3], using directional edge sources (ES), \( h_{\text{ES}}(t) \) is given by Eq. (2).

\[
\begin{align*}
  h_{\text{BTM}}(t) &= -\frac{cv}{2\pi rs_R \sinh(\eta)} H(t - t_0), \\
  h_{\text{ES}}(t) &= -\frac{v}{4\pi} \int \delta \left( t - \frac{m + 1}{c} \right) \frac{\beta}{ml} dz.
\end{align*}
\]

In Eqs. (1)–(2), the function \( \beta \) is a sum of four terms,

\[
\beta = \sum_{i=1}^{4} \beta_i = \sum_{i=1}^{4} \frac{\sin(\psi_i)}{\cosh(\eta_i) - \cos(\psi_i)},
\]

where the angles \( \psi_i \) are \( \psi_1 = \pi + \theta_0 + \theta_R, \psi_2 = \pi + \theta_0 - \theta_R, \psi_3 = \pi - \theta_0 + \theta_R, \psi_4 = \pi - \theta_0 - \theta_R \). The auxiliary variable \( \eta \) takes two different forms,

\[
\eta_{\text{BTM}} = \cosh^{-1} \left( \frac{r^2 + r^2_R + (z_R - z)^2}{2rs_R} \right),
\eta_{\text{ES}} = \cosh^{-1} \left( \frac{1 + \sin \alpha \sin \gamma}{\cos \alpha \cos \gamma} \right).
\]

The geometrical parameters in Eqs. (1)–(5) are illustrated in Fig. 1. In addition, \( c \) is the speed of sound, \( v \) is the so-called wedge index, \( \nu = \pi/2\psi_0 \), \( H(t) \) is Heaviside’s unit step function, \( \delta(t) \) is a Dirac function, and \( t_0 \) is the onset time for the diffraction wave from the edge. The idea of the formulation in Eqs. (2) and (5) is that the \( \beta \) function can be interpreted as a directivity function for edge sources along the edge, [3]. The contributions from all the edge sources are then summed by a line integral along the edge. Whether the expression in Eq. (1) or Eq. (2) is used, a numerical integration is needed to get a sampled/discrete-time IR, \( h(n) \). Most common is to integrate the continuous-time function \( \pm 0.5 \) time samples:

\[
h(n) = \int_{n-0.5/\gamma_c}^{n+0.5/\gamma_c} h(t)dt.
\]

For the form in Eqs. (2) and (5), this integral can also be written as an integral along a short portion of the edge,

\[
h(n) = -\frac{v}{4\pi} \int_{z_1}^{z_2} \frac{\beta}{ml} dz,
\]

where the integration limits, \( z_1 \) and \( z_2 \), are the points along the edge that correspond to the travel times \( (n - 0.5)/\gamma_c \) and \( (n + 0.5)/\gamma_c \), respectively. The integral in Eq. (7) implies that each sample value of the impulse response is given by an integral along a short portion of the edge. The numerical integration of Eq. (7), using Eqs. (3) and (5), will be more well-behaved than the BTM-formulation in Eq. (6), based on Eqs. (1), (3) and (4). Both formulations share one singularity: the \( \beta \) function is singular when the receiver position is placed exactly at a zone boundary, which is where a GA component disappears. The expression in Eq. (1) has a second singularity since \( \sinh(\eta) \) is zero at the time \( t_0 \), that is, at the onset of the diffraction wave, [4]. Interestingly, the integrand has no such singularity when the ES formulation is used in Eq. (7).

However, the first type of singularity needs special attention for the numerical integration of the ES formulation, Eqs. (3), (5) and (7). The singularity is caused by the integrand of Eq. (7), \( \beta/\gamma \), when the receiver is at a zone boundary. The singularity occurs for the apex point of the edge, i.e., the point along the edge which gives the shortest path from the source to the receiver via the edge. The apex point, \( z_a \), is given by

\[
z_a = \frac{z_R s + z_S r_R}{s + r_R}.
\]

By shifting the \( z \)-coordinate so that \( z_a = 0 \), an analytic approximation around \( z = 0 \) is straightforward:

\[
\frac{\beta_i}{ml_{\text{ES}}} \approx B_0 \frac{1}{z^2 + B_1 z + B_2 z + B_3},
\]

where \( B_0, B_1, B_2 \) and \( B_3 \) are constants. For symmetrical cases, i.e., when \( z_R = z_S \), the constant \( B_2 = 0 \). This analytical

Keywords: Diffraction, Computational methods

approximation can be integrated explicitly so that the first value of the sampled IR, \( h(n_0) \), which contains the singularity, can be computed explicitly rather than via a numerical integration. In the next section, some examples of edge diffraction IRs will be shown.

### 3. Results

To illustrate the effect of the singularity at zone boundaries, a single edge like in Fig. 2 was studied in a first example. The receiver position was moved along an arc and the two first values of the IR were calculated, and plotted in Fig. 3. It is clear that there is a discontinuity as the receiver passes a zone boundary, but only for the onset of the edge diffraction IR, i.e., the first sample value of the IR.

In a second example for the edge in Fig. 2, the receiver was moved along a straight line and IRs were calculated, including both the GA and the ED components. Figure 4 shows low-pass filtered IRs. In Fig. 4(a), where only GA components are included, the truncated wavefronts show evidence that GA is non-physical. The addition of the ED components lead to perfectly continuous wavefronts. It can be noted in Fig. 4(a) that exactly at the zone boundary, GA components are assigned half their ordinary values. It can also be noted, in Fig. 4(b), that the amplitude of the wavefront decreases as the receiver is coming closer to the zone boundary.
Acknowledgements

Part of this research was financed by the Research Council of Norway.

References


