A preprocessing method for multichannel feedforward active noise control

Guoyue Chen^{1,*}, Huigang Wang², Kean Chen² and Kenji Muto³

¹Faculty of System Science and Technology, Akita Prefectural University

²College of Marine, Northwestern Polytechnical University

³Tokyo Metropolitan College of Aeronautical Engineering

(Received 22 September 2004, Accepted for publication 17 November 2004)

Keywords: Adaptive whitening, Spatio-temporal decorrelation, Multichannel ANC

PACS number: 43.50.Ki, 43.60.Lq [DOI: 10.1250/ast.26.292]

1. Introduction

Multichannel active noise control (ANC) has many potential applications and many correlative algorithms have been reported in different kinds of literatures. Some popular algorithms, like multichannel filtered-x LMS (MFxLMS) algorithm and multichannel filtered-e LMS (MFeLMS) algorithm, have been analyzed detailedly [1]. The convergence properties of these algorithms, however, are affected seriously by the correlation of reference signals and the interactions among secondary paths. A preconditioned LMS (PLMS) [2,3] algorithm combined with prior information is used to improve the algorithm convergence speed. In PLMS, the spectral density matrix for the reference signals is decomposed into the product of two spectral factors, and the secondary path model response can also be decomposed into the products of an all-pass component and a minimum-phase component. All these information is combined into the MFxLMS algorithm and a faster convergence speed is obtained.

In practical application, however, the prior information is too difficult to be obtained. If the information has collected by other way in advance, the time-varying environment and noises can degrade the PLMS algorithm performance. So an adaptive algorithm is expected to learn from the environment and preserve the PLMS performance in time-varying condition. On the other hand, correlation reduction filter can also be used to reduce the correlations among reference signals to improve the performance of the MFxLMS algorithm [4].

In this letter, a robust adaptive whitening algorithm is applied to preprocess the reference signals before the MFxLMS algorithm. There are two commonly used linear models from the original noise to the reference signal, the instantaneous mixture and the convolutive mixture. These will be discussed respectively. A simple simulation is used to test the efficiency of the proposed algorithm.

2. An adaptive whitening algorithm

The framework of a multichannel adaptive feedforward ANC system is shown in Fig. 1. It is assumed that the j th reference signal $x_j(n)$ originates from the superposition of multiple noises, and the channel transfer function from the white noise $s_i(n)$ to $x_j(n)$ is modeled as a linear time-invariant (LTI) system polynomial $B_{ji}(k)$, so the reference signal can be

written as

$$x_j(n) = \sum_{i=1}^I \sum_{k=0}^K B_{ji}(k) s_i(n-k), \quad (1)$$

where K is the length of the filter $B_{ji}(k)$. For J reference sensors and I white noises, the system can be described in matrix form

$$\mathbf{x}(n) = \sum_{k=0}^K \mathbf{B}(k) \mathbf{s}(n-k), \quad (2)$$

where the reference signal vector $\mathbf{x}(n)$ is defined as $\mathbf{x}(n) = [x_1(n), \dots, x_J(n)]^T$, the white noise vector $\mathbf{s}(n) = [s_1(n), \dots, s_I(n)]^T$ is an uncorrelated column vector, so $E\{\mathbf{s}(n)\mathbf{s}^T(n)\} = \mathbf{I}$.

The covariance matrix of $\mathbf{x}(n)$ is

$$\mathbf{R}_{xx}(m) = E\{\mathbf{x}(n+m)\mathbf{x}^T(n)\}. \quad (3)$$

If $\mathbf{B}(k)$ is a diagonal polynomial matrix, which means that there isn't cross-path between the reference sensors and the white noises, the MFxLMS algorithm has fast convergence speed. In practical environment, each reference sensor can receive all noises and $\mathbf{x}(n)$ is the superposition of $\mathbf{s}(n)$ with different combinations. There are two basic ways: the instantaneous mixture and the convolutive mixture, which will affect MFxLMS algorithm convergence speed. In this letter, a robust whitening algorithm is provided to remove the effect of the mixing channel to MFxLMS algorithm.

2.1. The instantaneous mixture

The instantaneous mixture means that $\mathbf{x}(n)$ is the sum of $\mathbf{s}(n)$ with different scales, where no delayed component of $\mathbf{s}(n)$ is included. In other words, $\mathbf{x}(n)$ at time n is only determined by the value of $\mathbf{s}(n)$ at the same time,

$$x_j(n) = \sum_{i=1}^I B_{ji} s_i(n), \quad j = 1, \dots, J \quad (4)$$

where B_{ji} is the scale from the i th noise to the j th sensor. Eq. (4) can be written in matrix form,

$$\mathbf{x}(n) = \mathbf{B} \mathbf{s}(n). \quad (5)$$

So the covariance matrix between reference sensors can be written as

$$\mathbf{R}_{xx}(m) = E\{\mathbf{x}(n+m)\mathbf{x}^T(n)\} = \mathbf{B} \mathbf{R}_{ss}(m) \mathbf{B}^T. \quad (6)$$

*e-mail: chen@akita-pu.ac.jp

Fig. 1 The framework for multichannel adaptive ANC.

For uncorrelated white noise $s(n)$, $\mathbf{R}_{ss}(m)$ is a constant diagonal matrix and $\mathbf{R}_{xx}$ is only determined by the mixing matrix,

$$\mathbf{R}_{xx} = \mathbf{B}\mathbf{B}^T. \quad (7)$$

So the eigenvalue spread of the covariance matrix $\mathbf{R}_{xx}$, which limits the convergence speed of MFxLMS algorithm, is only decided by the singular value of $\mathbf{B}$.

2.2. A robust adaptive whitening algorithm

The purpose of whitening is to find a linear transform matrix $\mathbf{V}$ to the reference signals and make the output covariance matrix be a unit matrix. So the whitened output is

$$\tilde{\mathbf{x}}(n) = \mathbf{V}\mathbf{x}(n), \quad (8)$$

and

$$\mathbf{R}_{\tilde{x}\tilde{x}} = \mathbf{I}. \quad (9)$$

If we know the mixing matrix or the covariance matrix $\mathbf{R}_{xx}$, a simple preprocessing will eliminate the effect of different mixing matrix. If $\mathbf{B}$ is known and its inverse exists, the optimal whitening matrix is $\mathbf{V}_{\text{opt}} = \mathbf{B}^{-1}$, and if we know $\mathbf{R}_{xx}$, an eigendecomposition of $\mathbf{R}_{xx}$ is

$$\mathbf{R}_{xx} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H, \quad (10)$$

where $\mathbf{Q}$ is a unitary matrix and $\mathbf{Q}\mathbf{Q}^H = \mathbf{I}$, $\mathbf{\Lambda}$ is a diagonal matrix which include all eigenvalues. The optimal whitening matrix is

$$\mathbf{V}_{\text{opt}} = \mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{Q}^H. \quad (11)$$

However, the mixing matrix $\mathbf{B}$ is variable and unknown to the whitening algorithm under a time-varying environment, so the estimated covariance $\hat{\mathbf{R}}_{xx}$ from the observed reference signals is used to find the optimal whitening matrix. Finite samples and time-varying environment will result in inaccuracy estimation for $\mathbf{V}$. So the adaptive algorithm is preferred to match the adaptive ANC algorithm.

Douglas and Cichocki [5] derived a fast and efficient whitening algorithm, which can obtain faster convergence speed than no preprocessing when the mixing matrix is singular or approximately singular. The algorithm can be described as

$$\mathbf{V}(n+1) = \mathbf{V}(n) + \mu(\mathbf{I} - \tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^T(n))\mathbf{V}(n). \quad (12)$$

When the adaptive algorithm converges, the covariance matrix of whitened outputs is $\lim_{n \rightarrow \infty} \tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^T(n) = \mathbf{I}$.

Figure 2 gives the framework for multichannel ANC with robust adaptive whitening. The advantage of Eq. (12) lies in

Fig. 2 The framework for multichannel adaptive ANC with robust adaptive whitening.

good numerical stability and simple computational complexity. A variable step size technique is used to trade off between convergence speed and minimal Mean Square Error (MSE). The adaptive whitening algorithm is equivalent to adaptive Principal Component Analysis (PCA), so some robust adaptive PCA algorithm also can be applied to adaptive whitening preprocessing only if the temporal samples are replaced by spatial samples.

2.3. Adaptive whitening for convolutive mixture

The previous instantaneous mixture is an ideal assumption. In real environment, the reference sensor not only receives the noise from a direct path, but also includes the delayed and scaled component due to reverberation and reflection. So that the j th sensor signal can be written as

$$x_j(n) = \sum_{i=1}^I \sum_{k=0}^K B_{ji}(k)s_i(n-k), \quad (13)$$

where $B_{ji}(k)$ is the amplitude decay from the i th source to the j th sensor at the k th tap. For J reference signals, Eq. (13) can be written as in matrix form,

$$\mathbf{x}(n) = \sum_{k=0}^K \mathbf{B}(k)s(n-k). \quad (14)$$

In the convolutive mixture, the covariance matrix is more complex. For convenience, we will analyze the convolutive mixture in z -transform, and the algorithm performance can be evaluated by the cross-spectrum density matrix, which is defined as

$$\mathbf{S}_{xx}(z) = \mathbf{B}(z)\mathbf{S}_{ss}(z)\mathbf{B}^H(z), \quad (15)$$

where $\mathbf{S}_{ss}(z)$ is the cross-spectrum density matrix of noise sources and is diagonal $\mathbf{S}_{ss}(z) = \text{diag}\{[S_{11}(z), \dots, S_{II}(z)]\}$ for uncorrelated noises, $\mathbf{S}_{ss}(z) = \mathbf{I}$ for white noises, and $\mathbf{B}(z)$ is the z -transform for $\mathbf{B}(k)$.

From Eq. (15), it can be seen that $\mathbf{S}_{xx}(z)$ is not only related to the transfer function of different channels, also to the cross-spectrum density matrix of the sources. In fact, $\mathbf{S}_{ss}(z)$ can be decomposed into two factors and each factor can be included in the channel transfer function. According to spectrum decomposition theory, $\mathbf{S}_{ss}(z)$ can be factored into

$$\mathbf{S}_{ss}(z) = \mathbf{F}(z)\mathbf{F}^H(z). \quad (16)$$

So Eq. (15) can also be written as

$$\begin{aligned} \mathbf{S}_{xx}(z) &= \mathbf{B}(z)\mathbf{F}(z)\mathbf{F}^H(z)\mathbf{B}^H(z) \\ &= \mathbf{B}(z)\mathbf{F}(z)[\mathbf{B}(z)\mathbf{F}(z)]^H, \end{aligned} \quad (17)$$

where $\mathbf{B}(z)\mathbf{F}(z)$ can be considered as a new channel transfer function and input noises can be simplified into white noises with unit covariance matrix. So the original sources are assumed into white noises in the following section.

Whitening for convolute mixture is to find a linear transform $\mathbf{v}(z)$ to make outputs $\tilde{\mathbf{x}}(n) = \mathbf{V}(z)\mathbf{x}(n)$ uncorrelated. So the cross-spectrum density matrix of whitened outputs can be written as

$$\mathbf{S}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}(z) = \mathbf{V}(z)\mathbf{B}(z)\mathbf{F}(z)[\mathbf{V}(z)\mathbf{B}(z)\mathbf{F}(z)]^H. \quad (18)$$

There are two kind of whitening algorithms. One is the spatial whitening, where outputs are uncorrelated with each other and the spectrum density matrix of outputs $\mathbf{S}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}(z)$ is diagonal. Another is the spatio-temporal whitening, where outputs are uncorrelated in spatial and time domain and $\mathbf{S}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}}(z)$ is unit matrix, that is, outputs are uncorrelated independent identical distribution (i.i.d.) white noises.

Choi [6] proposed two fast spatio-temporal decorrelation algorithms, which are used in blind deconvolution, and excellent performance is obtained. Douglas [7] extended them and improved their performance by constraining the whitening filter be a causal FIR filter. If the whitening filter is a FIR filter with L coefficients and $\mathbf{V}_p$ is defined as the p th tap weight matrix, the adaptive learning algorithm in time domain is

$$\begin{aligned} \mathbf{V}_p(k+1) = & \mathbf{V}_p(k) + \mu \left\{ \Lambda_0(k)\mathbf{V}_p(k) \right. \\ & \left. - \tilde{\mathbf{x}}(k-L) \sum_{r=0}^L \mathbf{V}_{L-r}^T(k) \tilde{\mathbf{x}}(k-p-r) \right\} \end{aligned} \quad (19)$$

where $\Lambda_0(k) = \text{diag}\{\tilde{x}_1^2(k), \dots, \tilde{x}_J^2(k)\}$. The detail derivation and performance analysis can be referred to [7].

3. A simple simulation

A simple computer simulation is used to test the effects of the mixing matrix and the robust adaptive whitening algorithm in multichannel ANC system. We define CASE[2,2,2] as an ANC system which has 2 reference sensors, 2 loudspeakers and 2 error sensors. Two original noises are i.i.d. white noises, and the transfer functions of the primary path are assumed to be

$$\begin{aligned} h_{11}(n) &= [0, 0.9, 0.5, 0.3, 0.1]; \\ h_{12}(n) &= [0, 0.8, 0.5, 0.2, 0.5]; \\ h_{21}(n) &= [0, 0.7, 0.4, 0.3, 0.3]; \\ h_{22}(n) &= [0, 0.6, 0.3, 0.2, 0.4]. \end{aligned}$$

The ideal secondary path models are

$$\begin{aligned} c_{11}(n) &= [0.5, 0.4, 0.1]; & c_{12}(n) &= [0.7, 0, 0.2]; \\ c_{21}(n) &= [0.7, 0.4, 0.2]; & c_{22}(n) &= [0.5, 0, 0.1]. \end{aligned}$$

Because we focus on the mixing matrix, so it is assumed that the estimated secondary path model is accurate and $\hat{\mathbf{C}}(n) = 0.95 \cdot \mathbf{C}(n)$. In this simulation, the length of data samples is 5000, and the adaptive control filter length is 10 and the fixed step size 0.01 is used.

If the reference sensor is close to the corresponding noise and two sensors are separable, so $\mathbf{B}$ is assumed a unit matrix

Fig. 3 The error convergence curve of MFxLMS algorithm for different mixing matrix without adaptive whitening (average for 100 times). (a) the mixing matrix is $\mathbf{I}$; (b) the mixing matrix is $\mathbf{B}_1$; (c) the mixing matrix is $\mathbf{B}_2$; (d) the mixing matrix is $\mathbf{B}_3$.

Fig. 4 The error convergence curve of MFxLMS algorithm for different mixing matrix with adaptive whitening in Eq. (12) (average for 100 times). (a) the mixing matrix is $\mathbf{I}$; (b) the mixing matrix is $\mathbf{B}_1$; (c) the mixing matrix is $\mathbf{B}_2$; (d) the mixing matrix is $\mathbf{B}_3$.

that means no cross-channel. The error convergence curve with 100 iterations is shown in Fig. 3(a), whose convergence speed is completely decided by the primary and secondary path. As we change the mixing matrix, the eigenvalue spread of covariance matrix $\mathbf{R}_{\mathbf{x}\mathbf{x}}$ will change, so the conditional number of the mixing matrix is used to test singularity, and the conditional number is higher and the eigenvalue spread is larger. The following mixing matrixes are used to test MFxLMS algorithm,

$$\begin{aligned} \mathbf{B}_1 &= \begin{bmatrix} 0.9260 & 0.8640 \\ 0.0602 & 0.7536 \end{bmatrix}, & \text{cond}(\mathbf{B}_1) &= 3.0; \\ \mathbf{B}_2 &= \begin{bmatrix} 0.1343 & 0.1246 \\ 0.8541 & 0.4637 \end{bmatrix}, & \text{cond}(\mathbf{B}_2) &= 22.1; \\ \mathbf{B}_3 &= \begin{bmatrix} 0.8869 & 0.3227 \\ 0.7233 & 0.2698 \end{bmatrix}, & \text{cond}(\mathbf{B}_3) &= 253.0. \end{aligned}$$

The corresponding convergence curves are displayed in Figs. 3(b), (c) and (d). Compared with Fig. 3(a), these figures imply that the convergence speed will decrease and the minimum MSE will increase as the conditional number of $\mathbf{B}$ increases.

The proposed adaptive whitening algorithm of Eq. (12) is used to preprocess the reference signals, and outputs of the whitening algorithm are inputs of ANC FxLMS algorithm shown in Fig. 2, the error convergence curves with different mixing matrix are shown in Fig. 4, where (a) $\mathbf{B}$ is unit matrix with no whitening, (b), (c) and (d) correspond three different mixing matrix mentioned above. As it can be seen, all convergence speed and minimum MSE are almost the same, that is, the adaptive whitening algorithm eliminates the mixing matrix effects. Comparing Fig. 4 with Fig. 3, some interesting results can be found. When the mixing matrix with small conditional number is used, the adaptive whitening algorithm will not affect ANC algorithm's performance evidently; but it will play a very important role when the mixing matrix with large conditional number is used, so the whitening algorithm is robust.

4. Conclusion and future research

Two robust adaptive whitening algorithms are used to preprocess the reference signals of the multichannel ANC

system for the instantaneous mixture and convolutive mixture, and a simple computer simulation proves their robust performance. However, there are some complex problems needed to study in detail, especially for the convolutive mixture. Weinstein [8] provided some possible methods to multichannel decorrelation, but some problems, just like the condition of the solution existence and the condition for the algorithm convergence, still need to be settled down in future works.

References

- [1] S. J. Elliott, *Signal Processing for Active Control* (Academic Press, London, 2001).
- [2] S. J. Elliott, "Optimal controllers and Adaptive controllers for multichannel feedforward control of stochastic disturbances," *IEEE Trans. Signal Process.*, **48**, 1053–1060 (2000).
- [3] R. Fraanje, M. Verhaegen and N. Doelman, "Increasing the robustness of a preconditioned filtered-X LMS algorithm," *IEEE Signal Process. Lett.*, **11**, 285–288 (2004).
- [4] G. Chen, M. Abe and T. Sone, "Improvement of convergence speed in the filtered-x LMS algorithm with correlated reference signals," *IEICE Trans.*, **J80-A**, 309–316 (1997).
- [5] S. C. Douglas and A. Cichocki, "Adaptive step size techniques for decorrelation and blind source separation," *Proc. 32nd Asilomar Conf. Signals, Systems, and Computers*, pp. 1191–1195 (1998).
- [6] S. Choi, A. Cichocki and S. Amari, "Two spatio-temporal decorrelation learning algorithms and their application to multichannel blind deconvolution," *Proc. ICASSP*, 99, pp. 1085–1088 (1999).
- [7] S. C. Douglas, H. Sawada and S. Makino, "Natural gradient multichannel blind deconvolution and source separation using causal FIR filters," *Proc. ICASSP*, 04, pp. 477–480 (2004).
- [8] E. Weinstein, M. Feder and A. V. Oppenheim, "Multi-channel signal separation by decorrelation," *IEEE Trans. Speech Audio Process.*, **1**, 405–413 (1993).