A note on the acoustic properties of a double-leaf permeable membrane

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1. Introduction

The authors have previously studied the acoustic properties of membrane materials for building purposes and double-leaf membrane structures of various configurations made of these materials [1–7]. Membrane materials exhibit different acoustical properties depending on their acoustic permeability. Therefore, different combinations of permeable and impermeable membranes produce various double-leaf membrane structures with different acoustic properties. Double-leaf membranes with only impermeable leaves [5] and with impermeable and permeable leaves [6,7] have been investigated. However, those with two permeable leaves were only studied under the normal incidence condition [8,9], and their acoustical properties for diffuse field incidence should be studied.

In this study, a double-leaf membrane structure with two permeable leaves (DLPM) is analysed theoretically using a Helmholtz-Kirchhoff integral formulation to obtain a general solution, and we discuss its acoustic properties for diffuse field incidence through numerical examples.

2. Theoretical considerations

Figure 1 shows the geometry and model used for the analysis of a DLPM. The two permeable membranes (PM1 and PM2) are assumed to be of infinite extent, with surface densities m_1, m_2 (kg/m²), flow resistances r_1, r_2 (Pa s/m), and tensions T_1, T_2 (N/m), respectively. A plane sound wave of unit pressure amplitude p_i is assumed to be incident to PM1 at the angle of incidence θ .

Using a Helmholtz-Kirchhoff integral, the surface pres-

sure of the illuminated side of PM1 is expressed by the following equation:

$$p_1(x, 0) = 2p_i(x, 0) + \frac{i}{2} \int_{-\infty}^{\infty} [\rho_0 \omega^2 w_1(x_0) + ik_0 A_{m1} \Delta p_1(x_0)] H_0^{(1)}(k_0 |x - x_0|) dx_0 \quad (1)$$

Here, $A_{mj} = \rho_0 c_0 / r_j$ ($j = 1, 2$), where $\rho_0 c_0$ is the characteristic impedance of the air, $k_0 = \omega / c_0$ is the wavenumber in the air, ω is the angular frequency, w_j is the vibration displacement of the j th membrane, and Δp_j is the surface pressure difference between both sides of the j th membrane.

A similar expression is obtained for the surface pressure on the transmitted side (back side) of PM2:

$$p_3(x, D) = -\frac{i}{2} \int_{-\infty}^{\infty} [\rho_0 \omega^2 w_2(x_0) - ik_0 A_{m2} \Delta p_2(x_0)] H_0^{(1)}(k_0 |x - x_0|) dx_0. \quad (2)$$

The sound pressure and particle velocity in the air cavity are written in the following forms:

$$p_2(x, z) = (X e^{ik_0 Z \cos \theta} + Y e^{-ik_0 Z \cos \theta}) e^{ik_0 x \sin \theta} \quad (3)$$

$$v_2(x, z) = \frac{\cos \theta}{\rho_0 c_0} (X e^{ik_0 Z \cos \theta} - Y e^{-ik_0 Z \cos \theta}) e^{ik_0 x \sin \theta}. \quad (4)$$

Here, X and Y are the amplitudes of the pressures of the wave propagating in $+z$ and $-z$ directions, respectively, which are to be determined. From Eqs. (1)–(4) and the equations of motion of PM1 and PM2, using the Fourier transform technique, the reflected pressure p_r and transmitted pressure p_t are solved as follows:

$$p_r(x, z) = \left[1 + \frac{i \rho_0 \omega^2 L_1(k_0 \sin \theta) - k_0 A_{m1} J_1 \{H_1 L_1(k_0 \sin \theta) + H_2 L_2(k_0 \sin \theta) + H_3\}}{k_0 \cos \theta} \right] \exp[i(k_0 \sin \theta x - k_0 \cos \theta z)] \quad (5)$$

$$p_t(x, z) = \left[\frac{-i \rho_0 \omega^2 L_2(k_0 \sin \theta) + k_0 A_{m2} J_2 \{I_1 L_1(k_0 \sin \theta) + I_2 L_2(k_0 \sin \theta) + I_3\}}{k_0 \cos \theta} \right] \exp[i(k_0 \sin \theta x + k_0 \cos \theta z)], \quad (6)$$

Where H_j, I_j, J_j , and L_j are complicated functions including various parameters related to the membranes and the air cavity, which cannot be presented here for lack of space. From the above equations the absorption and transmission coefficients, α and τ , are obtained as follows:

$$\alpha = 1 - |p_r(x, 0)|^2, \tau = |p_t(x, D)|^2. \quad (7)$$

To evaluate the sound absorption performance, it is necessary to evaluate the energy dissipated in the structure

only: therefore, the difference between the absorption and transmission coefficients, $\alpha - \tau$, is used throughout. For the diffuse field incidence the field-incidence average (average over 0–78 degrees of a half sphere) is taken.

3. Numerical examples and discussion

A typical example of calculated results of $\alpha - \tau$ for a DLPM is shown in Fig. 2. At high frequencies, fluctuations with alternating peaks and dips appear, as observed in

Fig. 1 Geometry of the model of a DLPM used for theoretical analysis.

Fig. 2 Typical example of α - τ for DLPM. $m_1 = m_2 = 1.0 \text{ kg/m}^2$, $r_1 = r_2 = 816 \text{ Pa s/m}$, $D = 0.05 \text{ m}$.

conventional porous absorbers, which demonstrates the effect of the standing wave in the air cavity. This shows that PM2 behaves as the back wall in a conventional porous sound-absorbing system. The decrease in α - τ at low frequencies is caused by the decrease in acoustic resistance of the membrane due to the sound-induced vibration, which is commonly observed in various permeable materials [3].

Figure 3 shows the effect of the surface density of the membranes on α - τ for a DLPM: (a) shows that of PM1 (m_1) and (b) shows that of PM2 (m_2). In general the acoustic resistance of PM1 increases with increasing m_1 : thus, a monotonic increase in α - τ can be expected. However, the value of α - τ decreases at mid frequencies but increases at low frequencies with increasing m_1 . This is due to the somewhat complex behaviours of the absorption and transmission coefficients of the entire system: both coefficients decrease with increasing m_1 , but their rates of change and their frequency dependences are different. Therefore, the behaviour of α - τ of a DLPM with changing m_1 is rather complex. On the other hand, the effect of the surface density of PM2 is simple: it only appears at low to mid frequencies, and α - τ increases with increasing m_2 . This is considered to be caused by the decrease in transmitted energy and the increase in the acoustic resistance of PM2.

Figure 4 shows the effect of the flow resistance of the membranes on α - τ for a DLPM: (a) shows that of PM1 (r_1) and (b) shows that of PM2 (r_2). When r_1 is extremely small,

Fig. 3 Effect of (a) m_1 and (b) m_2 . (1) 0.25, (2) 0.5, (3) 1.0, (4) 2.0 and (5) 4.0 kg/m^2 . The other parameters are the same as those in Fig. 2.

Fig. 4 Effect of (a) r_1 and (b) r_2 . (1) 0.001, (2) 100, (3) 816, (4) 1,000 and (5) 10,000 Pa s/m . The other parameters are the same as those in Fig. 2.

only PM2 acts as a sound-absorbing element, exhibiting the same absorption characteristics as a single permeable membrane (Fig. 4(a)(1)). On the other hand if r_1 is extremely large,

the system becomes a double-leaf membrane with an impermeable leaf on the illuminated side and a permeable leaf on the back side. Therefore, the characteristics exhibit a resonance peak due to the membrane-type absorption, which is similar to the behaviour of an impermeable double-leaf membrane [4] (Fig. 4(a)(5)). Regarding the effect of r_2 , when it is extremely small the system behaves similarly to a single permeable membrane with PM1 only (Fig. 4(b)(1)), but when it is extremely large PM2 behaves as an impermeable membrane, and the system behaves as a double-leaf membrane with a permeable membrane on the illuminated side and an impermeable membrane on the back side, and the characteristics becomes those of a double-leaf membrane with a permeable and impermeable leaves [5] (Fig. 4(b)(5)).

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