TECHNICAL REPORT

A limitation in the use of spatially stationary strain variance estimation in the regularization of shear modulus reconstruction

Chikayoshi Sumi* and Toshiki Itoh

*Department of Information and Communication Sciences,
Faculty of Science and Technology, Sophia University,
7-1 Kioi-cho, Chiyoda-ku, Tokyo, 102-8554 Japan*

(Received 9 April 2008, Accepted for publication 23 March 2010)

Abstract: To yield spatially uniform stability in shear modulus reconstruction, an effective method for setting appropriate spatially varied regularization parameters using the variances of measured strain tensor components was previously reported by us. In the method, the strain variances are evaluated at each position using multiple field measurements or in real time using a single field measurement with the Ziv-Zakai lower bound (ZZLB, i.e., V_z) when using all cross-correlation-based methods. However, a stationary assumption of the displacement or strain measurement error can also be used as an alternative method of real-time reconstruction using an estimated variance (i.e., V_d or V_s). In this study, a comparison of real-time spatially variant regularizations using the above three estimated strain variances performed on an agar phantom reveals that stationary assumptions cannot be used when the target region has a large spatial variation in the shear modulus distribution (e.g., when stress concentration regions are generated in front of and behind a stiff inclusion or tumor), although the ZZLB can be used if sufficient care is taken owing to its low local correlation. When a target has a released tissue boundary, none of the estimated variances are useful. In such regions, none of the estimated variances should be used directly, because the shear modulus is not spatially smooth.

Keywords: Shear modulus reconstruction, Regularization, Strain variance, Stationary measurement error, Spatial stability, Displacement/strain measurement, Agar phantom

PACS number: 43.80.Qf, 43.80.Vj, 43.35.Yb, 43.60.Lq [doi:10.1250/ast.31.360]

1. INTRODUCTION

It is remarkable that the pathological state of human soft tissues highly correlates with their static and low-frequency mechanical properties, particularly shear elasticity. Accordingly, many researchers including ourselves have been developing strain- or displacement-measurement-based shear modulus reconstruction methods [1–8] and using various ultrasonic (US) strain/displacement measurement methods, e.g., the Doppler method and autocorrelation method [9,10], the cross-correlation method [11–13], the sum-squared difference (SSD) method [14], our developed multidimensional cross-spectrum phase gradient method (MCSPGM), i.e., 3-dimensional (3D) or 2D CSPGM [15–17], the multidimensional autocorrelation method (MAM) [18,19] and the multidimensional Doppler method (MDM) [18,19].

For shear modulus reconstruction, various reconstruction methods have also been developed [1–6] in addition to

magnetic resonance (MR)-based techniques [7,8]. The reconstruction methods fall into two approaches, i.e., direct [1–4,7,8] and iterative [5,6] inversion approaches. In the iterative inversion approach, a generated displacement field is estimated using an estimate of the shear modulus distribution, and the difference between the measured and estimated displacement fields is minimized by iteratively updating the estimated shear modulus distribution. In the direct inversion approach, equilibrium equations (i.e., partial differential equations, PDEs) are directly inverted. In both approaches, to model target soft tissues, i.e., to derive simultaneous algebraic equations, the finite-element method (FEM) or finite-difference method (FDM) is used.

In our case, we have been developing direct 1D [1,20,21], 2D [1,20,22] and 3D [20,23,24] shear modulus reconstruction techniques as diagnostic tools for various *in vivo* tissues, e.g., breast [20,21] and liver [20] tissues. In the 1D, 2D and 3D techniques, 1D, 2D and 3D models (reconstruction methods) are respectively used together with the 1D axial strain field, and 2D and 3D strain tensor fields obtained by differentiating the measured 1D axial

*e-mail: c-sumi@sophia.ac.jp

displacement field, and 2D and 3D displacement vector fields, generated by extracorporeally applied pressure or vibration or spontaneous heart motion. A relative shear modulus and a nonabsolute modulus can also be obtained [22,25]. For such *in vivo* tissues, such reconstructions can also be used to confirm the effectiveness of simple, minimally invasive therapy techniques (e.g., chemotherapy, cryotherapy and thermal therapy) [20].

We previously reported the effectiveness of regularization for stabilizing strain (displacement) measurement methods [26–28] and shear modulus reconstruction methods [20,23,27,29–31] for decreasing the instabilities of the measurement and reconstruction due to the contamination of echo data. Such regularization enables the incorporation of the *a priori* knowledge that the targets are spatially smooth. Because the target distributions are low-frequency deformations and mechanical properties, the regularization also increases the accuracies of the measurement and reconstruction and yields useful and unique results. Thus, such regularization solves the so-called ill-conditioned problems (i.e., nonexistence, nonuniqueness and instability). In studies by other groups, regularization is also applied to the measurement of displacement using the optical flow algorithm [32] and Young’s modulus reconstructions using the Newton-Raphson method [5,6].

Regarding the regularizations of strain measurement and shear modulus reconstruction, we further proposed to set suitable regularization parameters at each position (i.e., spatially variant regularization parameters) [20,23,27,28,31], because the measurement accuracies of displacements and strains spatially vary. That is, although the cross-validation method (i.e., using the independence of the raw vectors of the matrix) is used in such a modulus reconstruction for a region of interest (ROI) (e.g., [5]), our measurement and reconstruction should be properly regularized at each position according to the measurement accuracies of displacements and strains, respectively.

Because the measurement accuracies of strains can be evaluated using the correlation coefficient obtained after phase matching, the regularization parameters were preliminarily set to be proportional to the reciprocal of the power of the correlation coefficient [27]. Although the regularization effectively increased the spatially uniformity of the stability and accuracy of the strain measurement and shear modulus reconstruction, in the shear modulus regularization, the use of the variances of the measured strain tensor components was more effective in obtaining uniformly stable and more accurate results than that of the correlation coefficient [31]. When using displacement vector measurement methods (i.e., MCSPGM, MAM and MDM) without lateral modulation methods (LMMs) [18,20,33–36] or a multidirectional synthetic aperture method (MDSAM) [18,20], because the stability and

accuracy of lateral/elevational displacement measurements are much lower than those of axial displacement measurement [15–19], the regularization of only lateral/elevational displacements is also effective (i.e., displacement-component-dependent regularization) [28]. Spatially variant regularizations using an echo correlation coefficient [27] and displacement variances [28] have also been reported. Because both MDSAM and LMMs require a special data acquisition system, the regularization is effective.

In this paper, for 1D shear modulus reconstruction, we report regularizations using two estimates of axial strain variance evaluated under a local stationary assumption [37] of the measurement error of an axial displacement or strain. Alternatively, strain tensor variances are also experimentally evaluated using multiple field measurements by ensemble averaging [28,37], or a single field measurement [31] when using the cross-correlation method, by using theoretically derived axial [38–40], lateral/elevational [41] and shear [42] strain variances based on the Ziv-Zakai lower bound (ZZLB) [43,44]. Because the ZZLB can be used for all cross-correlation-based displacement measurements, the theoretically derived strain tensor variance [38–42] can also be used for 2D reconstruction by strain measurement using 2D CSPGM [31]. In [31,37], our theoretically derived strain variances are also described using the ZZLB [including Cramer’s lower bound (CRLB)] and differential filter coefficients. Although multiple field measurements can be used to cope with the decreases in the accuracy and stability of the reconstruction inherent to each US system (e.g., errors determined by US parameters such as US frequency, bandwidth and beamforming parameters), such a single field measurement is suitable for a practical, real-time *in vivo* reconstruction. Because the local stationary assumptions of the axial displacement and strain measurement errors also yield two other different estimates of axial strain variance from the single field measurement, after briefly reviewing our derived axial strain variance [31,37] using CRLB and describing the two stationary estimates (Section 2, Fig. 1), a comparison of the regularizations using the three estimated axial strain variances is performed on an agar phantom (Section 3). Finally, conclusions are given in Section 4.

2. ESTIMATIONS OF AXIAL STRAIN VARIANCE USING SINGLE FIELD MEASUREMENT

2.1. Use of CRLB

When using 2D CSPGM to measure an axial displacement distribution in an ROI for 1D shear modulus reconstruction using a spatially variant regularization, the variance of the measured axial strain component $\varepsilon_a(x, y)$ of the point of interest (x, y) in the ROI, expressed by

Fig. 1 Three estimations V_z [Eq. (4)], V_d and V_s , of variance of axial strain using assumption Ad [Eq. (2)], i.e., a constant variance of axial displacement within a differential-filtering region centered on a point of interest, and proportionality of the covariance of displacement to the variance; assumption Az , i.e., spatially invariant US frequency and bandwidth; and assumption of local stationary axial displacement or strain measurement error.

$$\begin{aligned}\sigma_s^2(\varepsilon_a(x, y)) &= E[(\varepsilon_a(x, y) - E[\varepsilon_a(x, y)])^2] \\ &= \sum_k \sum_l \sum_{k'} \sum_{l'} \{h_a(k, l)h_a(k', l') \\ &\quad \times \text{cov}[u_a(x - k, y - l), u_a(x - k', y - l')]\} \quad (1)\end{aligned}$$

using an expectation operator $E[\cdot]$, a covariance operator $\text{cov}[\cdot]$ and a differential filter $h_a(k, l)$ [22] (i.e., Eq. (5) in Appendix A in [31] or Eq. (2) in [37]), is approximated as being proportional to the variance of the measured axial displacement $u_a(x, y)$ of the point of interest (x, y) by using the assumption that the variances of the axial displacements within a region covered by a differential filter (smaller than the target lesion, e.g., a circle with a radius of about 1 mm) are the same as that of the point of interest (x, y) in addition to assuming that each covariance of the displacements is linearly dependent on the variance in conjunction with the overlap of the two local regions used for the displacement measurements (i.e., the covariances are proportional to the variance, see Appendix B in [31]), i.e.,

$$\sigma_s^2(\varepsilon_a(x, y)) \propto \sigma_s^2(u_a(x, y)). \quad (2)$$

Hereafter, both the above assumptions are referred to as assumption Ad .

Under assumption Ad , the axial strain variance is estimated from a single displacement field measurement. That is, CRLB

$$\begin{aligned}\sigma_s^2(u_a(x, y)) &= \frac{3}{2\pi^2 T(B_a^3(x, y) + 12B_a(x, y)f_a^2(x, y))} \\ &\quad \times \left\{ \left(1 + \frac{1}{\text{SNR}_c(x, y)} \right)^2 - 1 \right\} \quad (3)\end{aligned}$$

can be used as the variance of the axial displacement (i.e., Eq. (8) in [31] or see [38–44]), where T is the axial length of the local region used in 2D CSPGM, B_a is the axial bandwidth of an equivalent rectangular spectrum, f_a is the US frequency and $\text{SNR}_c = \frac{\text{SNR}_p \text{SNR}_s}{1 + \text{SNR}_p + \text{SNR}_s}$ is a combined signal-to-noise ratio (SNR) using the local echo (sono-

graphic) SNR (i.e., SNR_s) and the correlation SNR (i.e., SNR_p expressed with the local correlation coefficient ρ as $\frac{\rho}{1-\rho}$). Furthermore, because the axial size of the ROI used in the phantom experiment is small, as described later, in this study, we determine CRLB using SNR_s and ρ under the assumption that the US parameters such as f_a and B_a are spatially invariant. Hereafter, all the additional US assumptions are referred to as assumption Az . Thus, using the assumptions Ad and Az , the variance of the axial strain $\varepsilon_a(x, y)$ is estimated as

$$\sigma_s^2(\varepsilon_a(x, y)) = k \left\{ \left(1 + \frac{1}{\text{SNR}_c(x, y)} \right)^2 - 1 \right\}, \quad (4)$$

where k is a proportionality coefficient. To obtain $\text{SNR}_c(x, y)$, SNR_s is locally estimated using the same local region of 2D CSPGM at each position (x, y) [31] for estimating the echo variance as a function of depth (e.g., using Lagrange interpolation) by the multiple transmission of US to a homogeneous phantom and the power of the target echo signal (i.e., a single echo data), whereas SNR_p is estimated using the actually measured local correlation ρ . Hereafter, the estimated axial strain variance is referred to as V_z (Fig. 1).

2.2. Use of Local Stationary Assumptions of Displacement and Strain Measurement Errors

Two other estimates of axial strain variance can also be obtained from a single field measurement using the local stationary assumptions of axial displacement and strain measurement errors.

First, as in Subsection 2.1, by using assumption Ad [Eq. (2)], the axial strain variance can be estimated using the differential filter coefficients together with the axial displacement variance, where the axial displacement variance is estimated under the local stationary assumption of the axial displacement measurement error instead of the ZZLB. That is, the axial strain variance obtained is proportional to the axial displacement variance estimated by spatial averaging. Hereafter, this estimated axial strain variance is referred to as V_d (Fig. 1).

Alternatively, the axial strain variance can also be estimated directly from the measured axial strain distribution under the same local stationary assumption of the axial strain measurement error. Hereafter, this estimated axial strain variance is referred to as V_s (Fig. 1).

Here, note that because the spatial variations of the strains are generated in the stress concentration regions in front of and behind a stiff inclusion (or tumor) and at the boundary of regions whose shear moduli have a large difference [20,21], in such regions the assumption of the local stationary strain measurement error yields a large V_s . Inherently, both the shear moduli of the inclusion and the stress concentration regions are estimated to be smaller

than their actual values (i.e., an artifact of 1D reconstruction) [20,21]. Usually, an artifact is mitigated by setting a suitable reference point (i.e., having a reference strain and a reference shear modulus) in the stress concentration regions [20,21]. However, regularization using such errors in the estimated variance further affects the reconstruction. That is, an overly smoothed reconstruction will be obtained. Moreover, we should also mention in advance that, as shown in the next section (i.e., an agar phantom experiment), V_s and V_d differ owing to the use of the above-mentioned assumption Ad in V_d [Eq. (2)]. That is, although V_d is similarly affected in such regions owing to the use of the same local stationary assumption of the displacement measurement error for a large spatially varying displacement, the effect is not as intense on V_d as on V_s .

Also, V_z differs from V_d , even though the same assumption Ad is used. This is because V_d and V_z are respectively obtained using the local stationary assumption of the axial displacement measurement error and CRLB (i.e., not using such a local stationary assumption). Thus, the effect of the generation of such stress concentrations or the existence of a boundary has not been confirmed in V_z , although a large strain also causes V_z to be large owing to the low local correlation used. The behaviors of the estimated variances V_s , V_d and V_z in such a boundary region will be investigated in more detail in the next phantom experiment.

In addition, as shown in Section 3, in the central region of the stiff inclusion (tumor) both V_s and V_d were smaller than V_z , particularly V_d . As shown by simulations in [31] [i.e., Fig. 6(a) in Appendix B], when the strain is small, the strain and displacement measurement errors are predominated not by the generated strain but by the echo SNR. In such a case, the actual strain measurement errors will be smaller than V_z , which is predicted by ensemble averaging (Fig. 6(b) in [31]).

3. AGAR PHANTOM EXPERIMENT

Next, the differences among the three estimates of axial strain variances V_s , V_d and V_z [i.e., obtained using SNR_c in Eq. (4)], and their effects on spatially, variably regularized shear modulus reconstructions are compared using the same rf-echo data as that used in [31] obtained from an agar phantom. That is, for three estimates of axial strain variance, V_d , V_s and V_z , the regularization parameter α_{2g} in Eq. (1) in [31] (i.e., for the L_2 -norm of the Laplacian term of $\mathbf{u} = \mathbf{g}$) is set proportionally to the variances at each position in an ROI (other regularization parameters are set to zero). Thus, the following functional is minimized with respect to an unknown discrete shear modulus distribution, i.e., expressed as a vector $\mathbf{g}$ using the finite-difference approximation,

Fig. 2 Experimental setup.

$$I[\mathbf{g}] = \|\mathbf{b} - \mathbf{A}\mathbf{g}\|^2 + \alpha_{2g} \|\mathbf{D}^T \mathbf{D}\mathbf{g}\|^2, \quad (5)$$

where the first term is the difference between the right and left sides of the algebraic equations derived from the 1D equilibrium equation (ordinary differential equation) [20], i.e., $\mathbf{A}\mathbf{g} = \mathbf{b}$, $\mathbf{D}$ is the Laplacian operator and T is the transpose operator.

The agar phantom [60 mm (height) $\times$ 100 mm (lateral width) $\times$ 40 mm (elevational width)] used had a circular cylindrical inclusion at a depth of 15 mm (dia. = 15 mm) of a different shear modulus (see Fig. 2). Elasticity was controlled by adjusting the concentration of the agar (cylindrical inclusion, 7.0%; surrounding region, 3.0%). To control the US attenuation, graphite powder was added (concentration, 3.0%). The resultant phantom respectively had shear moduli of 10.8 and 4.8×10^5 N/m² in the inclusion and the surrounding area; thus, the relative shear modulus was 2.3 (loading tests were performed on block phantoms with the same concentrations). A linear array-type transducer with a nominal frequency of 7.5 MHz (LNR5539, Aloka Co., Ltd., Tokyo Japan) was used together with an US imaging equipment (SSD5500, Aloka). Rf-echo data were digitized with 12-bit resolution at a sampling rate of 30 MHz. The echo variance profile in the axial direction used for evaluating V_z is shown in Fig. 1 in [31]. The target phantom was uniformly compressed in the axial direction from the upper surface of the phantom using the transducer, which was mounted on a large plate. For the deformed phantom, a new rectangular ROI (38.8×14.4 mm²) and a reference line (reference value, 1.0) were respectively set at depths of 2.5 and 38 (focus depth) mm.

Figures 3(a) and 3(b) respectively show B-mode and linear-gray-scale strain images of the ROI. 2D CSPGM [15,16] was used (local region size, 0.8 mm $\times$ 1.6 mm) together with phase matching. The generated strains are smaller than 2.5%. The cutoff frequency of the differential filter [22] was 0.681 mm^{-1} (filter shape, a circle with a radius of about 1.2 mm). Figure 3(c) shows the linear-gray-scale correlation image obtained after phase matching (i.e.,

Table 1 Means of measured strains [Fig. 3(b)] and those of estimated strain standard deviations (SDs) [Figs. 4(a) to 4(c)] V_s , V_d and V_z [i.e., Eq. (4) using SNR_c] evaluated in regions A to F depicted in the B-mode image [Fig. 3(a)]. SDs are normalized by the mean V_s in region D.

Region	A	B	C	D	E	F
Mean strain [%]	-1.282	-0.265	-0.833	-0.639	-0.711	-0.342
Mean V_s [%]	1.19	0.139	0.254	0.144	0.120	0.332
Mean V_d [%]	0.267	0.067	0.168	0.144	0.158	0.101
Mean V_z [%]	0.294	0.145	0.121	0.144	0.129	0.135

Fig. 3 For agar phantom (Section 3), (a) B-mode image, in which a reference line (for shear modulus reconstruction) and six rectangular regions A to F are shown, (b) strain image obtained using 2D CSPGM (cross-spectrum phase gradient method) and (c) correlation image (i.e., image of peaks ρ of the local cross-correlation functions), in which the means in regions A to F are shown.

an image of the distribution of the peak values of the local cross-correlation functions). Six rectangular regions, A to F, are depicted in the B-mode image [Fig. 3(a)], whose correlation values are also depicted in the correlation image [Fig. 3(c)]. Region D is centered on the reference line. Note that only the same region B in the inclusion is used as that in [31]. However, recall that according to Eq. (4), SNR_s is high in the near-field region (i.e., A), whereas in the deep region (i.e., D), SNR_s is low owing to the attenuation of US [31]. Moreover, in the near-field region (i.e., A), ρ and SNR_ρ are low owing to the improper beam formation [$\rho = 0.73$, see Fig. 3(c)]. Thus, as confirmed in the strain image [Fig. 3(b)], both regions A and D have unstable strain data. For V_s , multiply overlapped local areas are set over the respective regions A to F, which are four times as large as the local region used in 2D CSPGM (i.e., $1.6 \text{ mm} \times 3.2 \text{ mm}$) and the strain measurement error is assumed to be stationary in the respective local areas. See also Table 1, in which the evaluated mean strains and mean standard deviations (mean SDs obtained from V_s) are shown (mean SDs, region A, 1.19%; region D, 0.144% > region E, 0.120%). Note that the mean SD of region C is also large (0.254%) owing to the generation of stress concentrations. There might be no such effect on

Fig. 4 For agar phantom, normalized standard deviation (SD) images obtained from (a) V_s , (b) V_d and (c) V_z [Eq. (4)] such that the mean SDs of region D obtained from V_d and V_z are the same as that obtained from V_s (i.e., 0.144 percent).

strains in region A because region A was set far from the inclusion compared with region C.

Next, the SD images obtained from V_s , V_d and V_z are shown in Figs. 4(a) to 4(c), respectively, which are normalized such that the mean SDs of region D become the same as mean V_s , i.e., 0.144%, whereas in Figs. 5(a) to 5(c), the corresponding shear modulus reconstructions are shown. Both images are shown on a log gray scale (thus, the SD images can be easily read as variance images). For V_d , the same local areas are also used as those for V_s . For comparison, Figs. 5(d) to 5(f) respectively show the reconstructions obtained (d) by spatially variant regularization with the regularization parameter α_{2g} set proportional to the square of the correlation ρ^2 [27], (e) by spatially invariant regularization (i.e., using a spatially uniform α_{2g}) [20,29] and (f) by not performing regularization, i.e., using the strain ratio [19,21]. Furthermore, means of other normalized V_d and V_z (mean strain SDs) spatially evaluated in regions A to F are also summarized in Table 1 [i.e., those corresponding to Figs. 4(b) and 4(c)], whereas the means and SDs of all the reconstructed shear moduli [Figs. 5(a) to 5(f)] in regions A to F are summarized in Table 2. For all the regularizations except for the strain ratio, α_{2g} is set such that the SDs of the reconstructed shear moduli in region D are almost the same, i.e., 0.03 (Table 2). Statistics of the shear moduli obtained using

Table 2 Means and SDs of shear moduli evaluated in regions A to F obtained by reconstructions through spatially variant regularizations using the estimates of variances V_s , V_d , V_z [i.e., Eq. (4) using SNR_c] and Eq. (4) with respective SNR_s and SNR_ρ instead of SNR_c , and using the square of the correlation ρ^2 , and through spatially invariant regularization and strain ratio, respectively.

	A	B	C	D	E	F
V_s	0.86 (0.04)	1.09 (0.04)	0.82 (0.03)	0.98 (0.03)	0.96 (0.07)	1.10 (0.08)
V_d	0.64 (0.09)	2.49 (0.18)	0.85 (0.05)	0.97 (0.03)	0.93 (0.05)	1.82 (0.37)
V_z	0.63 (0.08)	2.22 (0.06)	0.80 (0.05)	0.97 (0.03)	0.93 (0.07)	1.77 (0.32)
SNR_s	0.52 (0.19)	2.21 (0.16)	0.81 (0.05)	0.98 (0.03)	0.93 (0.06)	1.79 (0.34)
SNR_ρ	0.75 (0.03)	1.54 (0.03)	0.75 (0.04)	0.98 (0.03)	0.87 (0.06)	1.36 (0.19)
ρ^2	0.59 (0.10)	2.25 (0.06)	0.82 (0.05)	0.97 (0.03)	0.93 (0.06)	1.75 (0.34)
Invariant	0.54 (0.12)	2.24 (0.06)	0.82 (0.05)	0.97 (0.03)	0.93 (0.06)	1.74 (0.34)
Ratio	0.56 (0.40)	2.92 (9.18)	0.80 (0.10)	0.96 (0.28)	0.85 (0.21)	1.39 (62.0)

Fig. 5 For agar phantom, reconstructions of relative shear moduli obtained by spatially variant regularization using variances, i.e., (a) V_s , (b) V_d and (c) V_z , and (d) using square of correlation ρ^2 , and (e) by spatially invariant (i.e., uniform) regularization and (f) strain ratio, respectively.

either SNR_s or SNR_ρ [31] instead of SNR_c [Eq. (4)] are also shown in Table 2. If necessary, see the corresponding variance and regularized shear modulus images, i.e., Figs. 3(b) and 3(c) and Figs. 4(b) and 4(c) in [31] vs Figs. 4(c) and 5(c) (those corresponding to SNR_c).

As confirmed from the strain image [Fig. 3(b)] and its SDs for regions A to F obtained from V_s [Table 1 and Fig. 4(a)], the strain SDs in the near-field region A and the deep region D evaluated from V_d and V_z (i.e., using SNR_c) are also larger than those in region E (e.g., for region A: V_d , 0.267% > 0.158% and V_z , 0.294% > 0.129%; for region D: V_z , 0.144% > 0.129%). See also the V_d and V_z images [Figs. 4(b) and 4(c) vs Fig. 4(a)]. Thus, the regularizations using V_s and V_d yield spatial stability in the shear modulus reconstructions of regions A and D similar to that obtained using V_z [31], which can be seen by comparing the obtained reconstructions [Figs. 5(a) and 5(b)] with that obtained using V_z [Fig. 5(c)]. From Table 2, the shear modulus SDs for region A are 0.04 (V_s), 0.09 (V_d), 0.08 (V_z), 0.03 (SNR_ρ) < 0.19 (SNR_s), 0.10 (ρ^2), 0.12 (invariant) and 0.40 (strain ratio).

Thus, both regularizations using V_s and V_d also yield spatially uniform stability (i.e., a spatially uniform SD) in the shear modulus reconstructions, similar to that [31]

obtained using V_z (also compare the uniformity of SD in Table 2 with those of SNR_s , SNR_ρ , ρ^2 , spatially invariant regularization and strain ratio). However, as mentioned in Subsection 2.2, the local stationary assumptions of axial strain and displacement measurement errors respectively yield V_s and V_d in the stress concentration region (i.e., region C) that are larger than V_z , particularly V_s (0.254% and 0.168% vs 0.121%). As a result, the regularizations using V_s and V_d yield larger mean shear moduli and smaller SDs in region C than those using V_z (in Table 2, means, 0.82 and 0.85 vs 0.80; SDs, 0.03 and 0.05 vs 0.05). That is, the artifact formation owing to the generation of a stress concentration is reduced. That is, although the accuracy of the 1D reconstruction itself decreases at region C, the reconstruction accuracy of the shear modulus increases.

However, for region B centered on the stiff inclusion, the regularization using V_s yields a much smaller mean and SD shear modulus than those using V_z (1.09 and 0.04 vs 2.22 and 0.06), although the central region of the stiff circular inclusion (i.e., region B) has a smaller V_s than V_z (mean, 0.139% vs 0.145%). In contrast, the regularization using V_d yields a larger mean and SD shear modulus (2.49 and 0.18) than those using V_z when V_d is smaller than V_z (0.067% vs 0.145%). See also the obtained reconstructions [Figs. 5(a) and 5(b) vs Fig. 5(c)]. In addition to the effect on V_s and V_d for a small strain as mentioned at the end of Subsection 2.2 (0.139% and 0.067 vs 0.145%), this is also because the local stationary assumptions of axial strain and displacement measurement errors respectively yield a larger V_s and a smaller V_d than V_z in the inner region (i.e., F) at the boundary between the inclusion and the surrounding region (0.332% and 0.101% vs 0.135%). That is, at the boundary, both the inner (i.e., F) and outer (i.e., C) regions were estimated to have a larger V_s than V_z (for C, 0.254% vs 0.121%), whereas the inner (i.e., F) region was estimated to have a smaller V_d than V_z . Moreover, V_d in region C is not as large as V_s , although it is larger than V_z (0.168% vs 0.254% vs 0.121%). Thus, at such boundary

regions, the effect of the local stationary assumption is more intense on V_s than on V_d . See also the variance images [Figs. 4(a) and 4(b) vs 4(c)].

4. DISCUSSION AND CONCLUSIONS

Through an agar phantom experiment, the effects of spatially variant regularizations using three estimates of axial strain variance on the 1D shear modulus reconstruction were compared. The estimates used were obtained using the ZZLB (i.e., V_z), and local stationary assumptions of strain and displacement measurement errors (i.e., V_s and V_d , respectively). It was found that all the regularizations were effective for yielding spatial uniformity in the reconstruction.

However, it was also confirmed that the stationary assumption of the strain measurement error (i.e., V_s) should not be used for spatially variant regularizations, because V_s is estimated to be excessively large when the target region has both low and high shear moduli, i.e., when the spatial variation of a generated strain is large. In the experiment, the shear moduli in the stiff inclusion and the stress concentration regions generated in front of and behind the inclusion were excessively smoothed.

V_d was more effective than V_s in obtaining accurate reconstruction values, because the effect of the inhomogeneity of shear modulus is smaller on V_d than on V_s . However, the V_d obtained using 2D CSPGM was estimated to be significantly smaller than V_z as well as V_s in the high-shear-modulus region (i.e., mean for region B, 0.067% vs 0.145% and 0.139%), although, as mentioned in Subsection 2.2, the prediction that V_d and V_s would be smaller than V_z was previously verified by ensemble evaluations in simulations [31]. In this study, the significant decrease in V_d obtained by the stationary assumption of the displacement measurement error resulted in a large shear modulus SD (0.18 vs V_s , 0.04 and V_z , 0.06).

However, V_z did not yield a seriously erroneous reconstruction, although V_z was also affected by the inhomogeneity of the shear modulus, because in the calculation, a correlation coefficient evaluated using a local region was used. However, for 2D CSPGM, a more accurate estimate of the strain variance should be realized than that using CRLB such that the effects of the inhomogeneity of the shear modulus and the small strain are mitigated.

When the boundary is released, stationary assumptions will not be useful, although the ZZLB can be used if sufficient care is taken. As mentioned in [28,31,37], in such regions, the estimated variances should not be used directly, because the shear modulus is not spatially smooth. In the near future, regularizations using such evaluations will be reported together with that using an ensemble evaluation. Regularizations of the multidimensional shear

modulus reconstruction [23,35,36] will also be specifically reported elsewhere. Also, suitable methods of estimating the variance will be developed for other displacement measurement methods.

REFERENCES

- [1] C. Sumi, A. Suzuki and K. Nakayama, "Estimation of shear modulus distribution in soft tissue from strain measurement," *Jpn. J. Med. Electr. Biol. Eng.*, **31**(Suppl.), 390 (1993) (in Japanese).
- [2] H. Ponnekanti, J. Ophir, Y. Huang and I. Cespedes, "Fundamental mechanical limitations on the visualization of elasticity contrast in elastography," *Ultrasound Med. Biol.*, **21**, 533–543 (2001).
- [3] K. P. Raghavan and A. E. Yagle, "Forward and inverse problems in elasticity imaging of soft tissues," *IEEE Trans. Nucl. Sci.*, **41**, 1639–1648 (1994).
- [4] A. R. Skovoroda, S. Y. Emelianov and M. O'Donnell, "Tissue elasticity reconstruction based on ultrasonic displacement and strain images," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **42**, 747–765 (1995).
- [5] F. Kallel and M. Bertrand, "Tissue elasticity reconstruction using linear perturbation method," *IEEE Trans. Med. Imaging*, **15**, 299–313 (1996).
- [6] M. M. Doyley, S. Srinivasan, S. A. Pendergrass, Z. Wu and J. Ophir, "Comparative evaluation of strain-based and model-based modulus elastography," *Ultrasound Med. Biol.*, **31**, 787–802 (2005).
- [7] R. Sinkus, J. Lorenzen, D. Schrader, M. Lorenzen, M. Dargatz and D. Holz, "High-resolution tensor MR elastography for breast tumor detection," *Phys. Med. Biol.*, **45**, 1649–1664 (2000).
- [8] T. E. Oliphant, A. Manduca, R. L. Ehman and J. F. Greenleaf, "Complex-valued stiffness reconstruction for magnetic resonance elastography by algebraic inversion of the differential equation," *Magn. Reson. Med.*, **45**, 299–310 (2001).
- [9] L. S. Wilson and D. E. Robinson, "Ultrasonic measurement of small displacements and deformations of tissue," *Ultrason. Imaging*, **4**, 71–82 (1982).
- [10] M. Yamakawa and T. Shiina, "Strain estimation using the extended combined autocorrelation method," *Jpn. J. Appl. Phys.*, **40**(Part 1, No. 5B), 3872–3876 (2001).
- [11] S. Yagi and K. Nakayama, "Local displacement analysis of inhomogeneous soft tissue by spatial correlation of rf echo signals," in *Proc. WFUMB Meeting*, p. 113 (1998).
- [12] L. N. Bohs and G. E. Trahey, "A novel method for angle independent ultrasonic imaging of blood flow and tissue motion," *IEEE Trans. Biomed. Eng.*, **38**, 280–286 (1991).
- [13] J. Ophir, I. Cespedes, H. Ponnekanti, Y. Yazdi and X. Li, "Elastography: a quantitative method for measuring the elasticity of biological tissues," *Ultrason. Imaging*, **13**, 111–134 (1991).
- [14] Y. Zhu and T. J. Hall, "A modified block matching method for real-time freehand strain imaging," *Ultrason. Imaging*, **24**, 161–176 (2002).
- [15] C. Sumi, A. Suzuki and K. Nakayama, "Phantom experiment on estimation of shear modulus distribution in soft tissue from ultrasonic measurement of displacement vector field," *IEICE Trans. Fundam.*, **E78-A**, 1655–1664 (1995).
- [16] C. Sumi, "Fine elasticity imaging on utilizing the iterative rf-echo phase matching method," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **46**, 158–166 (1999).
- [17] C. Sumi, "Ultrasonic axial strain measurement for lateral tissue deformation," *Ultrasound Med. Biol.*, **33**, 1830–1837 (2007).

- [18] C. Sumi, "Displacement vector measurement using instantaneous ultrasound signal phase — Multidimensional autocorrelation and Doppler methods," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **55**, 24–43 (2008).
- [19] C. Sumi and T. Ebisawa, "Phantom experiments of axial strain measurements using multidimensional autocorrelation method, multidimensional Doppler method and direct strain measurement method," *Acoust. Sci. & Tech.*, **30**, 117–123 (2009).
- [20] C. Sumi, "Usefulness of ultrasonic strain measurement-based shear modulus reconstruction for diagnosis and thermal treatment," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **52**, 1670–1689 (2005).
- [21] C. Sumi and H. Matsuzawa, "Shear modulus reconstruction by ultrasonically measured strain ratio," *J. Med. Ultrason.*, **34**, 171–188 (2007).
- [22] C. Sumi, A. Suzuki and K. Nakayama, "Estimation of shear modulus distribution in soft tissue from strain distribution," *IEEE Trans. Biomed. Eng.*, **42**, 193–202 (1995).
- [23] C. Sumi, "Reconstructions of shear modulus, Poisson's ratio and density using approximate mean normal stress $\lambda \varepsilon_{\alpha\alpha}$ as unknown," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **53**, 2416–2434 (2006).
- [24] C. Sumi, "Effective shear modulus reconstruction obtained with remaining approximate mean normal stress unknown," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **54**, 2394–2402 (2007).
- [25] C. Sumi, "Relative shear modulus reconstruction for visualization with no geometrical artifact," *Acoust. Sci. & Tech.*, **31**, 347–359 (2010).
- [26] C. Sumi, "Stabilization of tissue displacement vector measurement utilizing ultrasonic cross-spectrum," *Tech. Rep. Jpn. Soc. Ultrason. Med.*, **101**(4), pp. 31–41 (2001) (in Japanese).
- [27] C. Sumi, "Spatially variant regularization for tissue strain measurement and shear modulus reconstruction," *J. Med. Ultrason.*, **34**, 125–131 (2007).
- [28] C. Sumi, "Regularization for ultrasonic measurements of tissue displacement vector and strain tensor," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **55**, 787–799 (2008).
- [29] C. Sumi, A. Suzuki and K. Nakayama, "Stabilization on estimation of shear modulus distribution in soft tissue from strain measurement," *Jpn. J. Med. Electr. Biol. Eng.*, **33**(Suppl.), 408 (1995).
- [30] C. Sumi and K. Nakayama, "A robust numerical solution to reconstruct a globally relative shear modulus distribution from strain measurements," *IEEE Trans. Med. Imaging*, **17**, 419–428 (1998).
- [31] C. Sumi, "Regularization of tissue shear modulus reconstruction using strain variance," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **55**, 297–307 (2008).
- [32] C. Pellot-Barakat, F. Frouin, M. Insana and A. Herment, "Ultrasound elastography based on multiscale estimations of regularized displacement fields," *IEEE Trans. Med. Imaging*, **23**, 153–163 (2004).
- [33] C. Sumi, "Shear modulus reconstruction by strain tensor measurement using lateral modulation," *Acoust. Sci. & Tech.*, **30**, 124–131 (2009).
- [34] C. Sumi, "Beamforming for realizing designed point spread function" in *Proc. IEEE Int. Ultrason. Symp.*, pp. 1557–1562 (2007).
- [35] C. Sumi, A. Tanuma and T. Noro, "Effective lateral modulations with applications to shear modulus reconstruction using displacement vector measurement," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **55**, 2607–2625 (2008).
- [36] C. Sumi and A. Tanuma, "Comparison of parabolic and Gaussian lateral cosine modulations in ultrasound imaging, displacement vector measurement and elasticity measurement," *Jpn. J. Appl. Phys.*, **47**(5B), 4137–4144 (2008).
- [37] C. Sumi and K. Sato, "Evaluation of variances of ultrasonically measured strain tensor components," *Acoust. Sci. & Tech.*, **28**, 352–356 (2007).
- [38] I. Cespedes, M. Insana and J. Ophir, "Theoretical bounds on strain estimation in elastography," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **42**, 969–971 (1995).
- [39] T. Varghese and J. Ophir, "A theoretical framework for performance characterization of elastography: The strain filter," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, **44**, 164–172 (1997).
- [40] S. Srinivasan, F. Kallel and J. Ophir, "Estimating the Elastographic signal-to-noise ratio using correlation coefficients," *Ultrasound Med. Biol.*, **28**, 359–368 (2002).
- [41] E. E. Konofagou, T. Varghese and J. Ophir, "Theoretical bounds on the estimation of transverse displacement, transverse strain and Poisson's ratio in elastography," *Ultrason. Imaging*, **22**, 153–177 (2000).
- [42] A. T. Kumar, J. Ophir and T. A. Krouskop, "Noise performance and signal-to-noise ratio of shear strain elastograms," *Ultrason. Imaging*, **27**, 145–165 (2005).
- [43] E. Weinstein and A. Weiss, "Fundamental limitations in passive time delay estimation — Part I: Narrow-band systems," *IEEE Trans. Acoust. Speech Signal Process.*, **31**, 472–485 (1983).
- [44] E. Weinstein and A. Weiss, "Fundamental limitations in passive time delay estimation — Part II: Wide-band systems," *IEEE Trans. Acoust. Speech Signal Process.*, **32**, 1064–1078 (1984).